Advertising and Consumers’ Communications

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Abstract

Until recently, brand identities were built by the firms via brand-image advertising. However, the flourishing of consumer communication weakened the grip of firms on their brands. The interaction between advertising and consumer communications and their joint impact on brand identity is the focal point of this paper.

We present a model in which consumer preference for functional attributes may correlate with the identity they desire to project on themselves. This correlation is known to the firm but not to consumers. Both the firm and the consumers can communicate their view of the brand identity, and the brand identity is determined endogeneously based on the composition of consumers who purchase it (i.e., what types of people consume this brand).

We find that sometimes by refraining from advertising the firm can strengthen the identity of its brand. This result is based on the following intermediate finding – advertising can diminish the endogeneous informativeness of consumer communications by making it one-sided. Furthermore, it turns out that refraining from brand image advertising may be optimal when the firm is especially well positioned to create a strong identity – i.e., when the consumers who like its functional attribute are relatively homogeneous in their desired identity.
1 Introduction

Many brands are valued by consumers not only for their functional attributes, but also for the identity that consumers can project on themselves while consuming the brand (Aaker 1997 and 1999). For example certain brands communicate that their user is gay (Williams 2007). Given this, firms may be interested in creating self-expressive (identity) attributes of their products. Advertising used by firms for this purpose is termed, here, “brand image advertising”.

However, these self-expressive attributes are not under the direct control of the firm. One of the reasons for this is that the identity that the brand will project on the user depends on the brand’s use by different types of consumers – i.e., on who is using it (Berger and Heath 2007). Another reason is that the product users can also communicate to help establish the identity they would prefer the product to have. For example, when the urban consumers embraced Timberland’s boots and shoes, the identity of the brand that used to be associated with rural culture experienced a dramatic change. Although Timberland proclaimed that it would like to stay away from associating its product line with any image and to promote the functional value only, its brand became a fashion and an identity statement in certain demographics due to its use and due to the communications of the brand users (Walker 2008). The optimal advertising strategy of the firm in the presence of consumer communication and their joint impact on the brand identity is the focal point of this paper.

Consumer desire to affect the brand identity is not a recent phenomenon. In addition to the Timberland case mentioned above, the New Coke fiasco in 1985 provides another well known example. In 1985, Coca-Cola Company replaced the original formula of its flagship soft drink. While a majority of consumers preferred the new taste in blind tests, the new product sparked consumer outrage. According to Wikipedia, many of the angry consumers were Southerners who considered the drink a fundamental part of their regional identity. While Coke promoted the new formula as “the same thing only better,” they viewed the company’s decision to change the formula
through the prism of the Civil War, as another surrender to the “Yankees”. According to Oliver (1985), it is widely believed that the failure of New Coke was due to outraged customers, such as retiree Gay Mullin who used upwards of $50,000 of his own money to promote his opinion that New Coke does not represent the same values as the original.

More recently, consumer communication is becoming even more easy through channels enabled by the Internet, such as Facebook and YouTube. To give an example, the “Starbucks Ad” posted on YouTube (http://www.youtube.com/watch?v=VnbT7qt6RF4) starts with a young blonde model holding a Starbucks cup and saying “I don’t know anybody who doesn’t love a Frappuccino on a hot summer day” and ends when she is summarizing how she thinks about the coffee by “. . . they’re freaking delicious!” Sounds like a hip commercial for Starbucks. Right? Think again. This is actually a user-generated video with a quite ironic subtext about the social image of the company.

Such ads are just one way in which consumers communicate in order to affect brand identity. If in the past brands’ identity were created by firms, today the consumers are deeply involved in this process.

The observation that both firms and consumers influence brand identity raises many interesting questions. In this study we focus only on one of them – the impact of consumer communication on firms’ marketing decision. Specifically, we study the impact of consumer communication on the most important tool used by firms to build and maintain brands – their brand image advertising. We present a model that addresses the following questions. Does the existence of consumer communication change the incentives to send ads and if yes, in what way? How do ads affect consumer communication and its informativeness? If firm’s advertising affects the informativeness of consumer communication, how should the firm adapt its advertising strategy? How do the firm’s incentives depend on the interrelation between consumer preferences for functional and self-expressive attributes?

Because of the impact of consumer communication on brands’ identity, firms and marketers
are sailing in new territories, and to the best of our knowledge there is no theoretical framework to assist them in this new and challenging journey. We present a model that captures the major ingredient of this new environment and identifies the interesting relationship between consumer communication and image advertising.

In the model, a monopolist sells a product over two periods. The product is characterized by two attributes: a functional one and a self-expressive one. The self-expressive attribute is endogenously determined by the type of people who will be using the product. There are two different types of consumers with this respect, and the self-expressing attribute directly depends on the proportion of consumers of each type by the end of the second period. Thus, the brand’s identity is unknown when consumers are purchasing the product.

Since the functional attribute is known, the uncertainty about the self-expressive attribute would have been resolved if the correlation between consumers’ functional and self-expressive preferences was common knowledge. Specifically, given this correlation it is immediate to derive the demand for the brand from each identity type and thus to figure out the identity of the brand. Since firms conduct research on their consumers and their tastes, it is reasonable to assume that the firm knows this correlation. However, it is not as reasonable to assume that consumers know it. Therefore, we assume that the distribution of consumer preferences is private information of the firm.

The sequence of events is as follows. The firm, who has a product with a specific functional attribute, decides whether to engage in brand-image advertising, and if so, on the advertising content. The “brand image advertising” which take the form of, say, a youthful and lively commercial surrounding the product, can be perceived as the firm’s claim or prediction about the proportion of consumers of that type that will buy its brand by the end of the second period. Based on the functional attribute and the ad, consumers decide whether to buy the brand. Those who bought the brand can communicate their type to others. This is the consumer communication component
of the model. In the second period, the rest of the consumers decide whether to purchase the brand. Note that they can base their decision on three elements: the functional attribute of the product, the brand-image advertising and the consumer communication. At the end of the second period, consumers realize their utility based on their preferences for the functional and image attributes and the realized image of the brand.

The setting of the model is described in section 2. In the first part of section 3 we consider the benchmark case in which consumer communication does not exist. We find that in equilibrium of this case, the firm is always using advertising to build the identity of its brand. Furthermore, the consumers are finding the content of the ad to be reliable about the type of consumers for which the brand is more appropriate. However, the ad is not reliable about the proportion of consumers from each type that will eventually buy the brand. For example, if there are two types of consumers “red” and “blue” and the ad would say “all of our consumers will be red”, the consumers will believe that the brand is more suitable for red consumers but they would not trust the statement about the magnitude of this effect. The stated magnitude will not be reliable since (as shown later) the firm’s profit is higher the more homogenous its identity is. This provides the firm with an incentive to exaggerate in its ad.

The existence of consumer communication, as demonstrated in the second part of section 3, can have a significant impact on the firm’s advertising decision. It turns out that when consumers are allowed to communicate, the firm will refrain from advertising in some cases. To understand this result one need to appreciate the incentives of the firm in such a case. It turns out that in equilibrium when consumers of both image type talk, consumer communication is more informative than the firm’s advertising. Specifically, the consumers who did not buy in the first period can usually extract from the communications of those who did the exact appeal of the brand for the red and blue consumers (i.e., they can infer the correlation between consumers’ functional and self-expressive preferences). However, it turns out that there is a situation in which consumer
communication can be uninformative about the correlation – when it is one sided (i.e., when the communicating consumers are either all red or all blue).

We further show that in some cases by sending an ad in the first period the firm can decrease the heterogeneity of its first period consumer to such a degree that the consumer communication is one sided. In other words, sometimes by sending an ad the firm can make the consumer communication uninformative. Another way to look at this is the following. Sometimes by refraining from sending an ad the firm can facilitate the informativeness of consumer communication. Under which conditions does the firm prefers that the consumer communication will be informative? As pointed above, the firm’s profit is higher the more homogenous its identity is. Thus, the incentive of the firm to refrain from advertising is the highest when the correlation between consumers’ functional and self-expressive preferences is high. Putting all these pieces together we present in section 3 an equilibrium in which when the correlation is high, the firm does not send an ad and when it is low it does.

This equilibrium has an intuitive appeal. It means that when consumers’ preferences on the functional attributes is a good predictor of their identity, the firm can rely on its consumers to spread the word on the brand because their message is going to be clear (i.e., quite homogeneous) and credible. Furthermore, advertising in such a case can only harm by homogenizing the early consumers too much so that the consumer communication is one sided and thus uninformative. Interestingly, in the last couple of decades we have seen the emergence of various new brands (or the re-emergence of older ones) who satisfy at least two of the conditions above – (a) they either refrained from using ads or used it in a very limited form and (b) they have a very clear image.

Consider two examples from the beverage market – Starbucks, and Red Bull. Both firms have spent very little on ads during their diffusion phase (Ries and Ries 2004). Furthermore, both have a very clear image. Starbucks’s image (which has also been criticized frequently) is of a social and environmental responsible company, while Red Bull has a wild image (wild party, wild spots
etc). Moreover, it seems that two additional characteristics of the above equilibrium apply to these companies: (1) their diffusion benefited from interactions among consumers which was facilitated by the firm (theme coffee houses in Starbucks case, and delivering free samples to student parties in Red Bull case), and (2) there seems to be a high correlation between the functional and self-expressive preferences (where this is interpreted as a high consistency between the identity and the functional characteristics of the brand). For example, the functional attribute of Red Bull – an energy drink – is also highly consistent with the wild identity that it aims to project. In other words, wild people are more likely to need an energy drink. While these examples can be interpreted in various ways and do not constitute a proof of the validity of the results of the model, it is encouraging to find that the idea that a clear-image-brand can be build without advertising is at least consistent with some business practices.

We believe that the model highlights some interesting insights about the impact of consumer communication on the incentives to use brand-image advertising. Specifically, we find that in some cases by refraining from advertising the firm can strengthen the image of its brand. As we discussed, this result is based on the following intermediate finding – brand-image advertising can diminish the informativeness of consumer communication by homogenizing the early consumers. It seems that these insights have been missing from the discussion of these issues by both academic scholars and practitioners.

To the best of our knowledge, this is the first work on the interaction between marketing communications (i.e., brand-image ads) and consumers’ communications. Furthermore, surprisingly, there is very little theoretical work that lays foundation for brand-image advertising. A notable exception is Wernerfelt (1990) that considers the firms’ decision to invest in brand-image advertising. Since in that setting, consumers are undifferentiated in their preference for the functional attribute of the product, the image is assumed to be related to advertising spending. Alternatively, in this paper, we consider how the image is endogenously affected by the correlation between user
preferences for the functional and self-expressive attributes and how the firm’s advertising decision is affected by the possibility of consumers’ communication.

An important aspect of our model is that one of the attributes of the brand is being determined in equilibrium based on the type of consumers who purchased it. This aspect is not new. Becker (1990) and Karni and Levin (1994) have allowed the image of the brand to depend on consumption decisions. In their case it was the popularity of the restaurant that comprised its image. However, the composition of the demand (i.e., which people are visiting the restaurant) did not matter. This assumption is modified in the later models of conspicuous goods (Bagwell and Bernheim 1996, Amaldoss and Jain 2005, Balachander and Stock 2009). In Amaldoss and Jain (2005), for example, snobs wish to differentiate themselves from the “conformists”. In these models price enabled the differentiation between the different segments of consumers in the two types of products (prestige and non-prestige goods). In our model, this is done via communication of the firm and/or consumers.

While there is extensive analytical marketing literature on advertising (e.g., Mahajan and Muller 1986, Desai 1997, Fruchter and Kalish 1997, Rajiv, Dutta and Shar 2002, Dukes and Gal-Or 2003, Shaffer and Zettelmeyer 2004, Villas-Boas 2004, Iyer, Soberman, and Villas-Boas 2005, He and Chen 2006, etc.), this literature normally models advertising either as informative about the product existence or directly assumes how advertising affects the consumer utility or demand. While we do not assume that advertising has a direct effect on the utility, in the equilibrium of our model, advertising content is informative. Furthermore, in equilibrium, the advertising decision of the firm sometimes serves as a signal (about the correlation between consumers’ functional and self-expressive preferences). The idea that advertising can serve as a signal is not new. Nelson (1974) suggested that advertising spending can serve as a signal on brands’ quality. This idea was later formulated in several settings (Kihlstrom and Riordan 1984, Milgrom and Roberts 1986, Simester 1995) and tested (Horstmann and MacDonald 1994 and Ackerberg 2001). Recently, Anand and Shachar (2009) suggested that the targeting of the ads can also serve as a signal on the horizontal
attributes of a brand, and Mayzlin and Shin (2009) presented a case in which the absence of information in the ad can serve as a quality signal. Our paper adds to this literature by considering how advertising would affect consumer communication decision which affects the demand as well.

As discussed above, the correlation between consumers’ functional and self-expressive preferences play an important role in our model and results. This correlation is also important in Muller and Shachar (2009) who study the economic incentive to introduce a self-expressive attribute and the market consequences of such introduction. There are many differences between the two approaches, the main being that Muller and Shachar (2009) do not consider communication of either firms or consumers and in their model, the identity of the brand is fully controlled by the firm.

2 Model setup

This section describes the setup of the model. The next section has two parts. The first presents the equilibrium when consumers’ communications is too costly and thus communications is done only by the firm. One can think about this scenario as describing the role of advertising in the past. The second part presents the equilibrium when the cost of consumers’ communication is not too high and thus the identity of the brand is influence by both advertising and consumer communication.

A monopoly firm produces one product at zero marginal cost and sells it in two periods to consumers of total mass two. Consumers derive utility from the product itself, which we will call the functional value, \( u_f \), and from self-expression the product enables, which we will call the image value, \( u_i \).

Consumers are heterogeneous in their tastes. With regard to functional value, there are two segments, each of unit mass. Within one segment, which we call \( L \)-functional segment, the consumer functional value, \( u_f \), is positive and distributed uniformly on \([0, V_F]\). Within the other segment, which we call \( R \)-functional segment, consumers’ functional value is negative and distributed
uniformly on $[-V_F, 0]$. This means that the functional attribute of the brand is more appealing to the $L$-functional segment.

With regard to the value of self-expression, there are again two segments, also each of unit mass. One segment, which we call $L$-identity segment, consists of consumers who derive a positive utility from projecting the $L$-identity and negative utility from projecting $R$-identity. The other ($R$-identity) segment has the reverse preferences. As discussed below, there is no one-to-one mapping from the functional segments to the identity segments. For example, belonging to the $L$-functional segment does not imply that this consumer belongs to the $L$-identity segment.

Following prior evidence (Berger and Heath 2007) that the identity of a product depends on the composition of its consumers, we will consider the self-expressive attribute as endogenously dependent on the set of consumers who purchased the brand. Specifically, if we denote by $D_R$ the mass of consumers from $R$-identity segment who bought the product, and by $D_L$ the mass of consumers from $L$-identity segment who purchased it, then the image of the product is defined as

$$I = 2\frac{D_L}{D_L + D_R} - 1.$$  

This expression linearly maps the proportion of the demand coming from the $L$-identity segment to the interval $[-1, 1]$, so that $I = 1$ indicates that everyone who buys the product is $L$, while $I = -1$ indicates that everyone who bought the product is from $R$-image segment.

The self-expressive utility, $u_i$, is formulated as $u_i = I \cdot v_i$ for $L$-identity consumers, and $u_i = -I \cdot v_i$ for $R$-identity consumers, where $v_i$ – the consumer value of self-expression – is distributed uniformly on $[0, V_i]$. In other words, projecting the right identity increases the utility and projecting the wrong identity decreases the utility and $v_i$ represents the magnitude of this effect, which is also the value of projecting exactly the right identity. While this model captures the social utility of brand image through an exogenous variable $v_i$, this could be consistent with a more primitive formulation of the consumer utility function. For example, as suggested by Kuksov (2007), $v_i$ can

1 Note that this is equivalent to one segment of consumers with values of the functional attribute distributed uniformly on $[-V_F, V_F]$. Dividing this distribution into the two segments is helpful for the conceptual discussion below.
capture the consumer value of social matching with other consumers, where the value is higher if the consumer identity-types match. As argued in Kuksov (2007), one can then justify that brands with images provide more credible information transmission than pure conversations and may be valuable to consumers. While this formulation may be appealing, we do not want to restrict the value of self-expression to this particular mechanism and thus model it as an exogenous variable.

Although there isn’t a one-to-one mapping between the functional and identity segments, these segments are not independent. It is reasonable to assume that the functional and self-expressive attributes are not independent for at least two reasons. First, it is likely that people with different personalities/identities have different functional needs. For example, is the action packed environment of Club Med suitable for a low-key person? Second, some functional attributes disable the signaling of some identities. For example, can owning a Hummer signal environmental caring identity? Accordingly, denote the mass of the $LL$-subsegment consisting of consumers who are both in $L$-functional and $L$-identity segments by $q$, and to use a symmetric setup, assume that the mass of $RR$-subsegment is also $q$. Accordingly, the mass of cross-matched subsegments $RL$ and $LR$ is $1 - q$ each. Note that in the subsegment notation, we use the first letter to indicate the functional segment and the second to indicate the identity segment.

Assume that $q$ may take one of the following four values: $q_l$, $1 - q_l$, $q_h$, or $1 - q_h$, for some $1/2 < q_l < q_h$. In other words, (a) the correlation between the functional and self-expression attributes can be either high, $q_h$, or low, $q_l$, and (b) $LL$ and $RR$ can be either more likely than $LR$ and $RL$ (i.e., $q \in \{q_l, q_h\}$) or less likely (i.e., $q \in \{1 - q_l, 1 - q_h\}$).

Since marketers conduct research on their consumers and their tastes, it is reasonable to assume that the firm knows the distribution of consumer preferences. On the other hand, it is unreasonable to assume that the consumers have this information. Therefore, we assume that

\[2\] Note that in such framework, since the value of the brand image comes from the expected change in the type of the partner found, expected consumer utility depends on the brand image even if the brand image is never observed by the consumer.
the distribution of consumer preferences is private information of the firm. Notice, also, that if this distribution was perfectly known to consumers, then in equilibrium, there would be no uncertainty about the brand identity and there would be no value of communication either of the firm or of consumers. We assume that the consumer prior belief about $q$ (i.e., before receiving any communication) is that it is equally likely to be any one of the possible values: $q_l$, $1 - q_l$, $q_h$, or $1 - q_h$. Thus, the expectation of the prior belief about $q$ is exactly $1/2$. It’s worth noting that other than consumers uncertainty on $q$ everything else is known to both the consumers and the firms. Specifically, they know the above description of the market as well as the exact value of the parameters $V_F$, $V_I$, $q_l$ and $q_h$.

Consumers’ uncertainty might be resolved, at least partially, by advertising. Technically, a firm would normally advertise about the image of the product, but since in our case, the uncertainty about image stems from uncertainty about $q$, what rational consumers would be trying to infer from any advertising is the value of $q$. Thus, we directly assume that the firm can send an ad with a declaration about the value of $q$ (e.g. the ad can say “$q$ is equal to $q_h$”). Once again, this formulation captures brand-image advertising well since it represents the firm’s effort to associate an identity with its product, which is exactly what such ads are trying to do (e.g. projecting a liberated image in the case of Virginia Slim). Furthermore, the firm is allowed to send any declaration – i.e., the declaration is not restricted to be truthful. However, since consumers do not forget the content of the ad, the firm is not allowed to change the content of the ad after sending it. This restriction immediately implies that if the firm advertises in the first period, advertising in the second period serves no purpose. In other words, the firm advertises in the first period, or in the second period, or in none of them.

The firm’s decision to send an ad or not, and the content of the ad can be used by the consumers in updating their prior beliefs on $q$. Furthermore, while some consumers like to be the first to buy a new product (Rogers 2003), others tend to wait and get some additional information
from these early consumers. To account for this we assume that there are two time periods in which consumers may buy. A consumer considering to purchase the brand in the first period can only have the firm’s message (or absence thereof) to use in her belief updating, whereas a consumer considering to purchase the brand in the second period can use both the firm’s message (or its absence) and the first-period consumer messages (if any).

We assume that a consumer who bought in the first period communicates her preferred identity (i.e., which identity segment she is in) if and only if her value of projecting image \( v_i \) is high enough, i.e., \( v_i \geq v_i^c \) for some \( v_i^c > 0 \).\(^3\) Since this generates either infinitely many or no consumer messages, we assume that the second-period consumers do not observe each of the messages, but only the proportion of \( R \) to \( L \) messages. We discuss and justify this assumption further in the context of the model’s solution.

To abstract from the pricing decision and concentrate on the firm’s incentives to advertise, we consider the price as exogenously fixed at some value \( p > 0 \). Furthermore, we assume that the price is the same in both periods. There is no doubt that endogenizing the price can enrich the model, but given that this is the first model to incorporate communication of both the firm and the consumers, it makes sense to focus on the communication aspect of this interaction. The previous theoretical analysis of image advertising has adopted a similar modeling strategy. For example, in his “variety model” Wernerfelt (1990) also abstracted from price considerations.

Following previous studies that demonstrated the social rewards from early adoption behavior (Arnould 1989, McCracken 1988 and Fisher and Price 1992), we assume that the total value of the product to a consumer in the second period is lower than its value in the first period. Specifically, if a consumer bought in the first period, her utility is \( u_1 = u_f + u_i - p \), where \( u_f \) and \( u_i \) are defined above. If she bought in the second period, her utility is \( u_2 = \delta(u_f + u_i) - p \). Conceptually, we consider the two periods not as equal, but as “before a significant mass of consumers bought to

\(^3\)For now, we treat this value as exogenous, but we discuss endogenizing this value from the consumer incentives to communicate in Section 4.
enable consumer communication” and “all the time after”. Therefore, we do not restrict $\delta$ to any range except $\delta \in (0, 1)$.

Before describing the sequence of events in this game, let’s summarize the formulation of the utility and the information set. The utility in period $t$ is:

$$u_t = \delta_t (u_f + u_i)$$

where $\delta_1 = 1$ and $\delta_2 = \delta$ and the following table presents the preferences and sizes of each of the subsegments

<table>
<thead>
<tr>
<th>Subsegment</th>
<th>mass</th>
<th>$u_f$ distributes</th>
<th>$u_i =$</th>
<th>$v_i$ distributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL</td>
<td>$q$</td>
<td>$U[0, V_F]$</td>
<td>$I \cdot v_i$</td>
<td>$U[0, V_I]$</td>
</tr>
<tr>
<td>LR</td>
<td>$1 - q$</td>
<td>$U[0, V_F]$</td>
<td>$-I \cdot v_i$</td>
<td>$U[0, V_I]$</td>
</tr>
<tr>
<td>RL</td>
<td>$1 - q$</td>
<td>$U[-V_F, 0]$</td>
<td>$I \cdot v_i$</td>
<td>$U[0, V_I]$</td>
</tr>
<tr>
<td>RR</td>
<td>$q$</td>
<td>$U[-V_F, 0]$</td>
<td>$-I \cdot v_i$</td>
<td>$U[0, V_I]$</td>
</tr>
</tbody>
</table>

and where $q$ may take one of the following four values: $q_l, 1 - q_l, q_h,$ or $1 - q_h$. Furthermore, the value of $q$ is private information of the firm.

The sequence of events is the following: 1) the firm, knowing $q$, decides whether to advertise and if so, which message to advertise; 2) consumers decide whether to buy; 3) those who bought communicate according to the rule described above; 4) the second period starts and the firm decides whether to advertise if it has not done so in the first period; 5) the consumers who have not yet bought decide whether to buy; 6) the identity of the brand is determined, consumers fully learn this identity and their utilities realize.

For simplicity, we assume that the firm does not discount future profits, and consumers are risk neutral. Furthermore, assume $p < V_F$, so that at least some consumers buy even if the product is not associated with any identity.

We use perfect Bayesian equilibrium as the solution concept and look for equilibria where consumers believe that the firm’s advertising decision as a function of $q$ is symmetric around $q = 1/2$.

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4 Note also that index “i” stands for the identity rather than a consumer. We do not have a subscript indexing consumers.
This is a natural assumption given the symmetric setup and will lead to symmetric advertising strategy actually being uniquely optimal.

When $V_F$ is small enough, this model could result in multiple equilibria some of which involve consumer coordination on the image, especially in the absence of the firm’s advertising. To avoid multiple equilibria, we assume that in the absence of the firm’s advertising, consumer expectation of the image is 0 (i.e., consistent with expectation of $q$ being $1/2$), and look for equilibrium where firm’s advertising $L(R)$-identity in period 1 increases demand from $L(R)$-identity segment. Note that if advertising is not costly, an equilibrium where consumer demand is uncorrelated with advertising and advertising uncorrelated with the image always exists (see, for example, the discussion in Kuksov 2007).

We analyze the model under two scenarios: (1) $v_C^c > V_I$ and (2) $v_C^c < V_I$. The first scenario represents the state of communications when consumers’ communications was very costly and thus marketing communications was done only by the firms. We would refer to this case as “no consumer communication”. The second scenario stands for the state of communications when the cost of consumers’ communications is low enough and thus marketing communications is done by both the firm and the consumers. We would refer to this case as “with consumer communication”.

3 Model Analysis

3.1 No consumer communication

In this subsection we wish to identify the incentive of the firm to send an ad and to characterize its impact and informativeness. To achieve this aim we start by describing the demand when the firm does not send an ad and then proceed to the case in which it does. The comparison between these two cases would enable us to identify the conditions under which image ad is optimal.

\footnote{Technically, the firm’s advertising decision could induce communication where it was absent without advertising. However, in our model this does not happen. That is, “no consumer communication” and “with consumer communication” are well defined as not dependent on the firm’s advertising decision.}
**No ad:** since the firm does not send an ad in either period and there is no consumer communication, consumers do not learn anything between the periods. Thus, the expected value of $u_f + u_i$ is the same in both periods. However, since consumers who purchase the product in the second period do not enjoy the full value of $u_f + u_i$ all the demand will be concentrated in the first period.

Based on the symmetric nature of the prior beliefs (which implies that the expectation of the prior belief about $q$ is exactly 1/2), the identity expected by the consumers (denoted by $\hat{I}$) is 0 for all consumers. Therefore, consumers make decisions based on their functional value only, and as a result consumer demand is

$$D_{noAd} = \frac{V_F - p}{V_F}.$$  \hspace{1cm} (2)

**With an ad:** Recall that if the firm advertises, it does it either in the first or in the second period. We start by considering the case of an ad in the first period. In such a case, like in the one above, consumers do not learn anything between periods and thus all the demand is concentrated in the first period.

Next we want to show that, in such a case, the firm has an incentive to project a homogeneous identity. Specifically, we wish to show that as the expected identity is further from zero (i.e., more homogeneous) the demand is higher. Without loss of generality, consider consumer expectation to be that the brand is more appealing to the $L$-identity consumers. In other words, consumers expect $I$ to be positive.

While the functional attribute of the brand is appealing to the $LL$ and $LR$ subsegments, the self-expressive attribute is attractive to the $LL$ and $RL$ subsegments. Thus, the demand is a sum of demands from these three subsegments ($LL$, $RL$, and $LR$). Accordingly, the stronger the $L$-identity of the brand, the higher the demand from $LL$ and $RL$ subsegments and the lower the demand from $LR$ subsegment.

An interesting case to examine is where the expected identity is in the same direction as the actual correlation, i.e., the consumers expect the brand to be more appealing to the $L$-identity
consumers, and these consumers have indeed a higher than average utility from the functional
attribute of this brand. More formally, in such a case, $\hat{I} > 0$ and $q > 1/2$. Below we would refer to
this case and to its symmetric analog (i.e., $\hat{I} \leq 0$ and $q \leq 1/2$) as “directionally consistent beliefs”.
The main reason that this case is interesting is that, as we will show later, beliefs are likely to
be directionally consistent. It turns out that in such a case, the more homogeneous the expected
identity is, the higher the demand. This is stated in following lemma:

**Lemma 1.** When consumers do not receive any new information after the first period and beliefs
are directionally consistent, the total demand increases in the absolute value of $\hat{I}$. Furthermore, the
demand is higher when beliefs are directionally consistent than when they are exactly the opposite
(e.g. when $q > 1/2$ the demand is higher when consumers expect the identity to be $\hat{I} > 0$ than when
they expect it to be $-\hat{I}$).

The proofs of all lemmas and propositions is in the Appendix. The logic of this result is the
following. Consider the case of $q > 1/2$. When $\hat{I}$ increases, the $L$-identity consumers find the brand
more appealing and the $R$-identity ones find it less so. Thus, the demand from the subsegments $LL$
and $RL$ increases while the demand from the $LR$ consumers decreases. It turns out that the increase
in the demand due to the $LL$ consumers is larger than the decrease due to the $LR$ ones. The main
reason for this is that the size of the $LL$ subsegment is larger than the size of the $LR$ one.\(^6\) This
is related to the assumption that there is a correlation between the functional and self-expressive
preferences. Specifically, this assumption implies that the size of the $LL$ subsegment is larger than
the size of the $LR$ one (i.e., $q > 1/2 > (1 - q)$).

Note that the condition that consumers do not receive any new information after the first
period is satisfied if the firm advertises in the first period, so that it does not in the second, and
when there is no consumer communication.

\(^6\)In additional to this, the decrease in the market share among the $LR$ consumers is smaller or equal to the increase
in the market share among the $LL$ ones.
Next, consider the case in which the firm does not advertise in the first period but does in the second. In such a case, it is easy to show that the demand in the first period comes from a subset of the consumers in the subsegments $LL$ and $LR$—only those who do not care much about the self-expressive attribute (i.e., those whose $v_i$ is not too high). The consumers with strong enough preference for the identity are worried that the identity of the brand might be the opposite to the one that they desire. Thus, they prefer to wait and decide whether to buy in the second period as additional information might be available.

Considering again the case in which $\widehat{I} > 0$ and $q > 1/2$, the demand in the second period comes only from the $LL$ and $RL$ subsegments. Notice that $LR$ consumers who did not buy in the first period will certainly not buy in the second period, after learning that $I$ is positive. Since the only consumers who purchase in the second period value the $L$-identity, it is immediate that the second-period demand increases with the expected identity, $\widehat{I}$. Thus, we again have that if the firm advertises (in this case in the second period), it prefers consumers to expect that the identity would be as homogeneous as possible.

These findings imply that if the functional attribute is more appealing to the $L$-identity consumers, the firm prefers that consumers would know that. In other words, if the ad would say “$q > 1/2$” the consumers would believe it and update their prior accordingly. However, since the firm always prefers consumers to believe that the identity is as homogeneous as possible, consumers would not trust the magnitude reported in the ad.

The results from the first and the second period imply that truthful advertising about whether the product is suitable for the $L$ or $R$ identity segment is better than a deceitful one. Since ads are directionality truthful, they are used by consumers in updating their beliefs. In other words, brand-image ads are both effective and informative.7

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7 An equilibrium with uninformative advertising, where consumers ignore the advertisement and the firm’s advertising decision is independent of the information it has, always exist. However, it seems natural that the agents would coordinate on the equilibrium with informative advertising if such exists.
The following proposition summarizes this discussion:

**Proposition 1.** Consumers believe the ad content about the sign of \((q - 1/2)\), but not of the value. In other words, advertising can only be informative about which consumers the brand is most suitable for, but not of the magnitude of this relationship.

Note that the above corollary is only true as far as advertising is not a costly signal. In other words, if the firm would be able to choose a more or less expensive advertising, the amount of advertising could be informative about the strength of the correlation.

Comparing the profits of the firm when it advertises or not and when it advertises in the first period versus the second leads to the following result:

**Proposition 2.** The firm strictly prefers to advertise in the first period to not advertise at all, and sometimes it prefers to advertise in the second period rather than the first.

These results demonstrate the role of image advertising in the past. A caveat of image ads, though, is that they are not informative about the magnitude of the correlation between the functional and the self-expressive preferences. This is, as we will soon see, where consumer communication can have an interesting role.

### 3.2 Ads with consumer communication

This subsection examines the impact of consumer communication on the incentive of the firm to send image ads. The existence of consumer communication enriches the considerations of the firms when it comes to whether it wants to send an ad or not. Specifically, the firm needs to take into account that by sending an ad it might affect the composition of buyers in the first period and as a result it is also influencing the content of the consumer communication. Thus, we start the discussion by exploring this effect – the impact of an ad on the content of the consumer communication.

It turns out that by sending an ad the firm can (in some cases) change the consumer communication from informative to uninformative. Consider, first, the case in which the firm does not
advertise in the first period. In such a case, in the first period, consumers do not know which identity segment is more likely to buy the brand and thus the expectation of $I$ and thus of $u_i$ is zero. Accordingly, the demand in the first period comes from those consumers in the subsegments $LL$ and $LR$ who do not care much about the self-expressive attribute (i.e., those whose $v_i$ is not too high). The other consumers in these subsegments care more about the identity of the brand and thus they would prefer to wait until the second period and find out whether the identity is suitable for them or not. Let’s focus on the $LR$ subsegment and denote by $\bar{v}_i$ the threshold $v_i$ between those $LR$ consumers who buy in the first period and those who do not.

Now, let’s consider the case in which the firm does advertise in the first period. Such an ad would attach an $L$ identity to the brand and thus would decrease the demand among the $LR$ consumers. Specifically, the threshold between those who buy in the first period and those who do not would be $v_i$ where $v_i < \bar{v}_i$. In other words, if the brand is known to have an $L$ identity, only those $LR$ consumers who care very little about identity would buy it.

When $v^c_i \in (v_i, \bar{v}_i)$ the drop in the demand among the $LR$ consumers mutes their voice in the consumer communication. Specifically, since $v^c_i < \bar{v}_i$ it means that when the firm does not advertise some $R$ buyers care enough about the identity of the brand to express themselves via consumer communication. However, since $v^c_i > v_i$ when the firm advertises, none of the $R$ buyers care enough about the identity in order to participate in the consumers’ communication. In such a case, the consumer communication is one sided.

In this model a one sided consumer communication is uninformative about the correlation while a two sided is. The reason for this is our assumption that the second-period consumers do not observe each of the consumer communication messages, but only the proportion of $L$ to $R$ messages. It is easy to show that since the consumers know all the parameters of the model other than $q$, they can extract $q$ from the proportion of $L$ to $R$ messages as long as this proportion is not

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8 Consumer communication is always informative about whether the brand is more appealing to the $L$ or $R$ identity segments. We refer to it as uninformative when it does not provide information about the strength of the relationship.
zero or one. In other words, when the consumer communication is two sided (i.e., the proportion of $L$ to $R$ messages is strictly smaller than one and bigger than zero) the consumer communication fully reveals $q$. On the other hand, if the consumer communication is one sided, this proportion is either zero or one and thus the consumer communication is uninformative.

How reasonable is our assumption that consumers do not observe each message but rather only their relative proportion (i.e., only the proportion of $L$ to $R$ messages)? There are various ways to support this assumption. Here are a couple of them. First, consider the case of YouTube. Because of its volume, it is unreasonable to assume that consumers are actually watching all videos on each topic. It is much more likely that consumers watch a sample of these videos. Thus, they will never know how many videos are from each side of the $L$-$R$ identity spectrum but rather only their relative proportion. Of course, this phenomenon is not restricted to YouTube. The volume issue is relevant to other forms of expression and other forums of communications. Second, even if consumers were able to watch every video on a topic, they are still likely to use the relative proportion. The reason is that in order to calculate the proportion of $L$ and $R$ separately one needs to know the exact size of the market – e.g. how many consumers like energy drinks? It is unreasonable to assume that consumers know that, and thus unreasonable to assume that they know the proportion of $L$ and $R$ separately. But they can calculate the ratio of $L$ to $R$ in the consumer communication even without knowing the size of the market. Thus, we adopt this assumption.

Given this assumption the result that a one sided consumer communication is uninformative is immediate (as discussed above). However, how reasonable is this result in a more general context – i.e., not necessarily within the exact assumptions of this model? There are at least two additional reasons to believe this result. First, a one sided consumer communication implies that all communicating users are sharing the same perspective on the brand. Under such conditions the incentive for users to generate one more content diminishes quickly. Furthermore, the diminishing incentive does not necessarily depend on the popularity of the product but rather on the absolute
number of messages. In other words, when “enough” people expressed their view, others do not find it attractive anymore, and “enough” is an absolute number. Therefore the volume of the one sided consumer communication cannot be used to reveal $q$. Second, consumers may not be aware of the very existence of consumer communication on a certain topic unless the topic is “controversial” – i.e., unless different opinions are present in the discussion.

We are ready to return to our discussion of the effect of advertising on the informativeness of the consumer communication. We have found that for $v_i^c \in (\underline{v}, \bar{v}_i)$ advertising has an impact on the informativeness of the consumer communication. However, outside of this range it does not. Specifically, when $v_i^c > \bar{v}_i$ none of the $R$ buyers participate in consumers’ communications either with ads or without ads. Thus, for a high enough $v_i^c$, consumer communication is uninformative with or without ads. Accordingly, when $v_i^c < \underline{v}$, consumer communication is informative regardless of the firm’s advertising decision in the first period.

Before summarizing this result in a proposition, it should be noted that the above discussion does not account for the potential signaling role of advertising. In other words, while the content of the ad is uninformative about $|q - 1/2|$ (as stated in Proposition 1), it is possible that in equilibrium the firm’s decision to advertise (versus not) might depend on $|q - 1/2|$ and as a result it can serve the consumers as a signal on it. We refer to beliefs as “action independent” if they do not account for the potential signaling role of advertising and use it in the following proposition:

**Proposition 3.** For action independent beliefs, advertising in the first period weakly decreases the informativeness of consumer communication.

By “weakly”, we mean that depending on the model parameters, advertising either does not change the informativeness of consumer communication or decreases it.

This proposition establishes that in certain conditions the firm can mute the consumer communication by sending an ad. Another way to think about this is that in certain conditions the firm can enhance the voice of the consumer communication by refraining from advertising. Under which
conditions the firm is more interested in keeping consumer communication informative? Recall that the information in the consumer communication is much more precise than in the ad. Specifically, while an ad can only provide information about the sign of \((q - 1/2)\), consumer communication can provide exact information about \(q\). Since the firm’s profit is higher the stronger the expected identity, the firm would prefer an informative consumer communication only if it will lead to a stronger identity compared to advertising. This would be the case if and only if \(q = q_h\).

It turns out that this incentive can be strong enough to change the firms’ advertising decision. Specifically, we have the following proposition:

**Proposition 4.** For some parameter values we have the following two

1. When consumer communication is not possible, the firm advertises regardless of whether \(q = q_l\) or \(q = q_h\).

2. When consumer communication is possible, the firm advertises if and only if \(q = q_l\)

Notice that while Proposition 3 demonstrated the effect of ads on the informativeness of consumer communication for action independent beliefs, for the parameter values in Proposition 4 this effect exists also in equilibrium (i.e., for action dependent beliefs, where the decision to send an ad affects the beliefs).

Interestingly, when the firm is especially well positioned to create a strong identity (i.e., \(q = q_h\)), it refrains from advertising. The reason, as pointed out above, is that in such a case the firm would rather rely on consumers’ communications since it is a more informative and thus more effective form of communication. Furthermore, the firm realizes that its ad would make the consumer communication uninformative and thus ineffective.

The informativeness of advertising in this equilibrium is quite unique. When a firm does not send an ad, the consumers cannot know which identity type is likely to be attracted to this brand. However, they know that the identity of this brand will be very homogeneous (since the firm does
not advertise when \( q = q_h \).

On the other hand, when a firm advertises the consumers are fully informed. The content of the ad informs them about the type of consumers who are attracted to this brand (i.e., \( L \) or \( R \) identity segments) and the decision of the firm to advertise informs them that the identity of the brand will not be sharp (since the firm advertises when \( q = q_l \)).

The proposition is focusing on a subset of the parameter space. Of course for parameters outside of this specific subset there are other types of equilibria. We focus on this subset since it highlights an incentive (or a disincentive) to advertise that is due to the existence of consumer communication. We discuss the reasonability of this incentive in the next section.

4 Conclusion

The story behind the “Marlboro Man” and the success of Philip Morris in changing the feminine identity of its cigarette to a manly one is well-known. These days in which marketers and especially ad agencies were powerful enough to attach an identity to a brand via image advertising are celebrated in a successful TV show called “Mad Men”. These days are over. Consumers’ role in determining the identity of their brands has increased steadily during the last part of the 20th century and riding the power of consumer communication it become highly significant in the first decade of the 21st century. Because of these changes, firms and marketers are sailing in new territories. This study presents a theoretical framework that might assist them in this new and challenging journey.

We present a model that captures the major ingredient of this new environment and highlights some interesting insights about the impact of consumer communication on the incentives to use image advertising. Specifically, we find that sometimes by refraining from image advertising the firm can strengthen the image of its brand. This result is based on the following intermediate finding – image advertising can diminish the informativeness of consumer communication by homogenizing the early consumers. Furthermore, it turns out that refraining from image advertising may be
optimal when the firm is especially well positioned to create a strong identity (i.e., $q = q_h$). The reason is that in such a case not only the identity created by consumer communications is clear and reliable, but also if the firm would advertise, the consumer communication would become uninformative. It seems that these insights have been missing from the discussion of these issues by both academic scholars and practitioners.

The idea that consumer communication is not “all evil” from the firm’s point of view has sunk in, and firms are even encouraging this type of communication in various ways, such as buzz-marketing and brand communities. The results of our model can improve these practices in various ways. First, when it comes to building brand identity encouraging consumers’ communications is optimal only when there is a high correlation between consumers’ functional and self-expressive preferences.

Second, in such a case it might be optimal to refrain from advertising in the diffusion of the brand (and instead to encourage consumer communications). As pointed earlier, some firms are already adopting such a strategy. For example, Red Bull refrained from advertising and encouraged buzz by delivering free samples to student parties. While some firms are already adopting such a strategy, the common wisdom in the diffusion literature is that advertising should be used in the early days of a product to stimulate sales and word-of-mouth. There is no doubt that early advertising can be optimal when a product category is introduced and/or when the aim of the firm is to position its product as functional. However, this study questions the optimality of such a strategy when it comes to image advertising and building a brand.

The model presented here is clearly a first step toward a better understanding of the role that consumer communication will play in marketing. The next step should probably be to offer a theoretical foundation for consumer motivation to communicate in such a setting. While it is relatively easy to justify our assumption that consumers talk if and only if their value from
the identity exceeds a certain threshold, enriching the model by fully endogenizing consumers’
communication seems promising.

While our model does not speak exactly about the balance of power between consumers and
firms in building and maintaining the brand identity, it does offer a preliminary perspective on this
important issue. Analytical modeling of this broader issue can be quite interesting. Specifically, the
shift of power from firms to consumers has shaken many executives and firms and theoretical effort
to identify the optimal strategies that firms can take is valuable. In this context an interesting
example of a successful collaboration comes from the powerful brand Star Wars. In 1999, Joe
Nussbaum directed a short film by the name George Lucas in Love – a homage and spoof of both
Star Wars and Shakespeare in Love. He financed the film with money that his grandparents left
him. Nussbaum recruited several fellow University of Southern California graduates to help him
with his movie. The film got a lot of attention and when it was released on VHS in 2000, The
New York Times reported that the film made it to number 1 on Amazon.com’s Top 10 sales chart,
beating out sales of Star Wars Episode I: The Phantom Menace for at least a day. In this example,
the user generated content has become almost as important for the Star Wars’ brand identity as
the content created by the firm (in this case, George Lucas).

Interestingly, Lucas’s philosophy has almost always been to view his consumers as collaborators
(Jenkins 2008). Furthermore, such type of collaborations is not rare in the entertainment industries
(Jenkins 2008). Future theoretical research can examine under which conditions is such balance of
power optimal and why is it relatively more frequent in the entertainment industries.

\footnote{Suppose each consumer represents a small but non-zero part of the market. Then if a first-period consumer
reports her preferred brand identity, she strictly increases the beliefs of others about it. This shifts the identity of
the brand toward her. The value that such a consumer attaches to such a shift is, by assumption, strictly increasing
in the value that she places on the brand identity, \( v_i \). Therefore, if all consumers face an equal cost \( c \) of engaging in
communication, the consumers who will find it optimal will satisfy \( v_i \geq v_i^c \) for some \( v_i^c \).}
5 Appendix

This appendix includes four subsections that provide the proofs of the lemma and propositions. Specifically, subsection 5.2 includes the proof of Lemma 1 and subsections 5.3, 5.5 and 5.6 present the proofs of Propositions 1, 2 and 4, respectively. Proposition 3 is established in the text.

Sections 5.1 and 5.4 describe how we calculate the model solutions in two cases in which there is no consumer communication: (i) when the firm advertises in the first period (subsection 5.1) and (ii) when it advertises in the second period (subsection 5.4). The solution presented in 5.1 is then used in subsections 5.2 and 5.5 and the one described in 5.4 is used in 5.5.

5.1 Model Solution When the Firm Advertises in the First Period and There is no consumer communication

In this case, consumers will receive no new information in the second period. Since each consumer with positive utility faces declining utility from the first to second period, all demand will be realized in the first period. Next, we present the demand from each of the four segments (LL, LR, RL and RR) as a function of $\hat{I}$, consumers’ expectation of the identity. Without loss of generality, consider $q > 1/2$. If $\hat{I} > 0$, the demand from the RR subsegment is always 0, and the demand from the LL subsegment is

$$D_{LL} = \frac{q}{V_I V_F} \int_{u_f + \hat{I} v_i \geq p} u_f v_i \in [0, V_F], v_i \in [0, V_I] du_f dv_i. \quad (3)$$

The double integral above represents a trapezoid area which is reduces to $\int_{0}^{V_I} \int_{p - \hat{I} v_i}^{V_F} du_f dv_i$ for $\hat{I}_V < p$, and $\int_{0}^{\hat{I}} \int_{p - \hat{I} v_i}^{V_F} du_f dv_i + \int_{\hat{I}}^{V_I} \int_{0}^{V_F} du_f dv_i$ otherwise. Thus, $D_{LL}$ simplifies to

$$D_{LL} = \begin{cases} \frac{q 2^{(V_F - p) + \hat{I}_V}}{2 V_F}, & \text{if } IV_I < p; \\ \frac{q 2^{\hat{I}_V V_F - p^2}}{2 IV_I V_F}, & \text{otherwise.} \end{cases} \quad (4)$$

Similarly, demands from LR and RL segments can be written as, respectively,

$$D_{LR} = \begin{cases} (1 - q) \frac{2^{(V_F - p) - \hat{I}_V}}{2 V_F}, & \text{if } IV_I < V_F - p, \\ (1 - q) \frac{(V_F - p)^2}{2 IV_I V_F}, & \text{otherwise,} \end{cases} \quad (5)$$
and
\[
D_{RL} = \begin{cases} 
0, & \text{if } IV_I < p, \\
(1 - q) \frac{(IV_I - p)^2}{2IV_I V_F}, & \text{if } p < IV_I < V_F + p, \\
(1 - q) \frac{2IV_I - 2p - V_F}{2IV_I}, & \text{otherwise.} 
\end{cases} 
\]  
(6)

The total demand is obtained as \( D_{LL} + D_{LR} + D_{RL} \).

For consumer expectations of the identity to be consistent, we must have
\[
\hat{I} = 2 \frac{D_{LL} + D_{RL}}{D_{LL} + D_{RL} + D_{LR}} - 1. 
\]  
(7)

This equation is the equilibrium condition on the value of the identity. Moving all terms to one side and taking the numerator, reduces this equation to polynomial equation of third degree in \( \hat{I} \).

The solution of the demand in the case of \( \hat{I} < 0 \) is symmetric to the one above.

5.2 Proof of Lemma 1

Differentiating the total demand derived above with respect to \( \hat{I} \), the consumer expectation of the image, and simplifying the derivative in each of the cases \( \hat{IV}_I < p, \hat{IV}_I \in [p, V_F - p], \hat{IV}_I \in [V_F - p, p], \hat{IV}_I \in [max\{p, V_F - p\}, V_F + p] \) and \( \hat{IV}_I > V_F + p \), one obtains that the derivative is always positive. Furthermore, the demand is continuous when changing between these regions. Therefore, the total demand is always positive, and the first claim is proven.

To prove that the demand is higher when beliefs are directionally consistent than when they are exactly the opposite, we focus on the case \( q > 1/2 \). In this case, we want to show that the firm prefers consumers to have expectation \( \hat{I} > 0 \) of the identity rather than \( -\hat{I} \). Notice that if the consumer expectation of the identity is \( -\hat{I} < 0 \) and \( q > 1/2 \), the demand equals the demand when the consumer expectation is \( \hat{I} > 0 \) but instead \( q \) is replaced by \( 1 - q \). Furthermore, while we assumed \( \hat{I} > 0 \) in the derivation of the demand, we did not assume \( q > 1/2 \). In other words, the calculations in the previous subsection can give us the demand with \( \hat{I} > 0 \) and also the demand with \( -\hat{I} \) (as long as we plug in \( 1 - q \) instead of \( q \)). Therefore, to show that the firm prefers \( \hat{I} > 0 \) over...
all we need to show is that the total demand derived above is lower when \( q > 1/2 \) is replaced by \( 1 - q \). This is straightforward to check.

### 5.3 Proof of Proposition 1

Lemma 1 implies that if the firm advertises in the first period, it prefers to influence consumers beliefs that the identity is positive and largest possible whenever \( q > 1/2 \), and it is negative and as negative as possible when \( q < 1/2 \). Therefore, the firm’s advertising in the first period is truthful in the direction (i.e., to which identity segment is the brand most suitable), but always the most extreme in the magnitude. Likewise, if the firm does not advertise in the first period, advertising positive identity in the second period brings demand only from \( L \)-identity segment. This is because if a consumer preferring \( R \)-identity would buy in the second period given the information that the identity is positive, she should have bought in the first period when the identity was uncertain. The second period demand from \( L \)-identity segment increases in the consumer belief about how positive identity is. Therefore, in the second period, the firm also prefers the most extreme advertising. Again, since the size of the \( L \)-identity segment is larger than that of \( R \)-identity segment if and only if \( q > 1/2 \), the firm prefers to advertise positive identity if and only if \( q > 1/2 \). Thus, advertising in the second period is truthful in the direction only as well.

### 5.4 Model Solution When the Firm Advertises in the Second Period Only

As in the case of firm advertising in the first period only, there are a number of subcases to consider to expand the repeated integrals. Since in this case, the number of regions is even larger, we will illustrate the solution only on the case \( \hat{I} \in [\max\{p, (2 - \delta)V_F - p\}, \delta V_F + p] \), where \( \hat{I} > 0 \) is the equilibrium identity. The derivations in the other cases are similar. This condition only affects how the multiple integrals are resolved into repeated integrals, not which constrains are binding.

If the firm does not advertise in the first period, the expected value of identity by the first
period consumers is 0. This is because $q - 1/2$ is symmetrically distributed around $1/2$ and we assume that the firm’s advertising strategy as a function of $q$ is symmetric around $q = 1/2$. In the second period, consumers expect to update the expectation of the identity to be either $\hat{I}$ or $-\hat{I}$ for some $\hat{I} \in [0, 1]$, which, in equilibrium will turn out to be the actual value of the identity. Given the above, the consumer’s strategy is either to 1) buy in the first period, 2) not buy in either period, or 3) buy in the second period if and only if the firm’s advertising message indicates a favorable identity to her. This is because if the consumer would buy in the second period regardless of the advertising, the consumer would be strictly better off buying in the first period instead. Therefore the marginal consumer in the first period is either 1) indifferent between buying and not buying in the first period, i.e., this consumer has $u_f = p$, or 2) is indifferent between buying in the first period or buying in the second period in the favorable identity case. The equation defining the latter marginal consumer is

$$u_f - p = \frac{\delta(u_f + \hat{I}v_i) - p}{2},$$

where $\frac{1}{2}$ represents the probability of the advertising indicating the favorable identity. Therefore, the demand in the first period comes from consumers with $(u_f, v_i)$ satisfying $u_f \geq p$ and $v_i \leq \frac{(2-\delta)u_f - p}{\delta \hat{I}}$.

In the case we consider, the mass of these consumers can be calculated as

$$D_1 = \frac{(V_F - p)((2-\delta)V_F - \delta p)}{2\delta IV_FV_I}.$$  

Furthermore, since consumers do not know in the first period whether the image is positive or negative, equal proportion of $L$ and $R$-functional segments buy in the first period. In other words, fraction $q$ of the first-period demand comes from $L$-identity segment, and fraction $(1 - q)$ comes from the $R$-identity segment.

In the second period, let us separately consider demands coming from $LL$, $LR$, $RL$, and $RR$ subsegments. Since, without loss of generality, we consider actual realization of $\hat{I} > 0$, no consumers from $RR$ segment buy. Furthermore, all consumers from $LR$ subsegment who may want
to buy in the second period should have bought in the first period, since these consumers’ utility decreased by $\delta$ and they received unfavorable information about the identity. The demand from the $LL$ subsegment comes from consumers with $\delta(v_f + \tilde{I}v_i) \geq p$ and who have not bought in the first period, i.e. $v_i > \frac{(2-\delta)v_f-p}{\delta I}$. In the case we consider, the expression for this demand can be simplified to

$$D_{LL} = q\frac{2p(V_F - p) - (2 - \delta)V_F^2 + 2\delta I\tilde{I}V_F}{2\delta IV_I V_F}. \quad (10)$$

Finally, demand from $RL$ subsegment comes from consumers who have $-v_f + \tilde{I}v_i \geq p$. Note that none of the consumers in this segment could have bought in the first period because their expected utility from buying in the first period was $-v_f - p < 0$. Demand from these consumers can be written as

$$D_{RL} = (1 - q)^2 \frac{\delta IV_I - p}{2\delta^2 IV_I V_F}. \quad (11)$$

For consumer expectations of the identity to be correct, we must have

$$\hat{I} = \frac{2qD_1 + D_{LL} + D_{RL}}{D_{LL} + D_{RL} + D_1} - 1. \quad (12)$$

This equation is the equilibrium condition on the value of the identity. Moving all terms to one side and taking the numerator, reduces this equation to polynomial equation of third degree in $\hat{I}$.

### 5.5 Proof of Proposition 2

The first claim is follows from observing that the demand derived in Section 4.1 when the firm advertises in the first period is higher than the demand $(V_F - p)/V_F$ given no advertising for all parameter values.

Next, we need to show that for some parameter values it is better to advertise in the second period rather than in the first and for other values the opposite hold. The following numerical example illustrates the first case (i.e., advertising in the second period is better): $p = 0.1, V_f = V_F = 10, \delta = 0.9$, and $q_l = q_h = 2/3$. Specifically, in this case, when the firm advertises in the
first period, it obtains the total demand of 0.930321, of which 0.660642 comes from LL-segment, 0.20778 comes from LR-segment, and 0.061897 comes from RL segment. If the firm advertises in the second period, it obtains the demand of approximately 0.94266, of which 0.7158 comes in the first period and 0.2268 comes in the second period (0.1808 from LL segment and 0.046 from RL segment).

An example that illustrates the opposite case in which advertising in the first period is preferable over the second period is shown in the proof of Proposition 4 below.

5.6 Proof of Proposition 4

This claim is checked by examining the solution for the following parameter values: $p = 5, V_I = V_F = 10, \delta = 0.97, q_l = 2/3, q_h = 0.8, \text{ and } v_i^c = 5.93$. In this case, without consumer communication, the firm would find it optimal to advertise in the first period regardless of $q$ (i.e., for both $q_l$ and $q_h$) and following the message of $q > 1/2$, consumer expectation of $q$ would be $(q_l + q_h)/2$. Can consumers’ beliefs change this? Let’s consider the case that advertising in the second period rather than in the first signals that $q$ is high. In such a case, the firm would indeed prefer to abstain from advertising in the first period and advertise in the second regardless of $q$. In other words, advertising in the second period cannot serve as a signal and thus such beliefs are not consistent. Therefore, in the absence of consumer communication, refraining from advertising in the first period does not signal high $|q - 1/2|$ and the equilibrium is for both firms to advertise in the first period.

If consumer communication is possible, consumer communication turns out not to be informative (since no consumer from R-identity segment communicates) if the firm sends message $q > 1/2$ in the first period. However, consumer communication turns out to be informative if the firm does not advertise in the first period (regardless of whether consumers believe $|q - 1/2|$ is high or low). Therefore, consumers find out the exact value of $q$ in the second period if and only if the firm does not advertise.
If the firm does not advertise in the first period and consumers find out the exact value of $q$ in the second period, the firm’s demand is 0.6347205901 and 0.7323025280 for $q = q_l$ and $q = q_h$, respectively. Recall that consumers’ believe that the firm advertises in the first period if and only if $q = q_l$. Now, let’s see what would be the result of deviations. If the firm advertises in the first period and consumers believe it to have $q = q_l$, the profits would be 0.6410705508 and 0.7255702328, respectively. Thus, when $q = q_h$, the firm is willing to refrain from advertising in the first period and obtain the demand of 0.7323025280 instead of getting only 0.7255702328 if it deviated. When $q = q_l$, the firm cannot benefit from mimicking this behavior since the consumer communication would reveal that $q = q_l$ and thus it would get a demand of 0.6347205901 instead of 0.6410705508.

Finally, in order to show that the firm advertises if and only if $q = q_l$, we need to rule out the possibility of pooling equilibrium. To do this, we calculate the demand when (i) $q = q_h$, (ii) the firm advertise in the first period, and (iii) consumers believe that $q = q_l$ or $q = q_h$ with equal probability (the most favorable beliefs given that when $q = q_l$ the firm always advertises in the first period). We find that the demand is 0.7322820997, which is closer but still lower than the demand (when $q = q_h$) if the firm refrain from advertising and lets consumer communication reveal the exact value of $q$. 

References


