Bailout Uncertainty, Leverage, Duration Mismatches, and Lehman’s Collapse

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Abstract

This paper develops a micro-founded general equilibrium model of the financial system composed of ultimate borrowers, ultimate lenders and financial intermediaries. The model is used to investigate the impact of uncertainty about the likelihood of governmental bailouts on leverage, interest rates, the volume of defaults and the real economy. The distinction between risk and uncertainty is implemented by applying the Gilboa-Schmeidler (1989) maxmin with multiple priors framework to lenders’ beliefs about the probability of bailout. Lehman’s collapse is conceived of as a ”black swan” event that led lenders to put a positive mass on low bailout probabilities which were assigned zero mass prior to this event.

The analysis implies that such an increase in bailout uncertainty raises interest rates, the volume of defaults in both the real and financial sectors and may, in some cases, lead to a total drying up of credit markets. On the other hand, lower ex ante

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bailout uncertainty is conducive to higher leverage - which makes the economy more vulnerable to an increase in uncertainty triggered by events such as not bailing out Lehman. A more general corollary is that larger changes in bailout uncertainty raise the amplitude of changes in leverage and the amplitude of booms and busts. The paper also investigates the impact of expansionary monetary policy on leverage and the real economy.

**Keywords and Phrases:** Risk, Uncertainty, Lehman’s default, Leverage, Financial intermediaries, Bailouts, Duration mismatches.

**JEL Classification Codes:** G01, G11, G2, G18, E3, E4, E5, E6, D81, D83.
1 Introduction

Financial sector bailouts in the US and more recently in Europe have revived the well known dilemma between restoration of confidence in the face of a panic and the costs of moral hazard. On one hand, when a panic engulfs financial markets, bailouts appear indispensable in order to restore confidence and prevent further collapses in the financial system. On the other, by subsidizing opportunistic behavior at the expense of taxpayers, bailouts encourage excessive risk taking on the part of financial institutions, borrowers and lenders, and plant the seeds of the next bubble.

Different experts in both policymaking circles as well as in academia often find themselves at odds regarding the ways to handle this problem. In spite of currently ongoing reforms in regulation this dilemma is, therefore, likely to be a central issue during the upcoming decade. Whether, and how exactly will bailout policies be deployed in the future is largely an open issue. However, due to the lack of consensus about the precise ways to deal with the (ex ante and ex post) trade-offs induced by bailouts, it is extremely likely that bailout uncertainty is likely to be non negligible in the foreseeable future. The 2008 bailout zigzags in the US (Bear-Stern versus Lehman) and current uncertainties about the reaction of EMU governments to potential sovereign debt problems of a large country like Spain attest to that.

This paper develops a micro-founded general equilibrium model of the financial system and uses it in order to investigate the impact of an increase in bailout uncertainty on financial markets and the real economy. It also investigates the ex ante, leverage expanding, moral hazard problems created by perceived generous governmental bailout policies.

As is well known since Knight’s (1921) work risk and uncertainty are distinct concepts. Modern formulations of this distinction in the context of pecuniary returns conceptualize risk as some measure of spread for a known distribution of the stochastic return. Uncertainty, on the other hand, is a situation in which individuals are unsure about the probability distribution of returns and entertain the possibility that several alternative probability distributions have positive measure. An increase in uncertainty is then viewed as an enlargement of the set of plausible probability distributions with positive measure. Ellsberg
(1961) and others have demonstrated by means of experiments that individuals are averse
to ambiguity in the sense that, other things the same, they prefer a lottery with a known
probability distribution to a lottery in which several distributions are believed to be possible.

Gilboa and Schmeidler (1989) (GS in the sequel) conceptualize an investor’s uncertainty
by postulating that he possesses a subjective set of probability measures, or multiple priors,
over outcomes. Under several axioms they show that, if the investor is averse to ambiguity
his action is determined by the Gilboa-Schmeidler max-min ambiguity aversion criterion.
That is, for each possible action the investor assumes that the worst (by the expected utility
criterion) possible distribution will realize and chooses his action so as to attain maximum
expected utility over this set of worst outcomes.

This paper utilizes the GS notion of uncertainty and the associated max-min behavioral
criterion to analyze the impact of an increase in uncertainty about governmental bailout
policy on financial markets, the aggregate level of credit and, through them, on the real
economy. The riskiness of bailouts at the level of an individual creditor is captured by
a binomial distribution in which conditional on default by a borrower there is a bailout
with probability, \( p \), or there is no bailout with probability \( 1 - p \). Bailout uncertainty then
means that individuals entertain the view that several alternative binomial distributions,
each characterized by a different value of \( p \) possess positive mass. In this context an increase
in uncertainty means that there is an enlargement in the set of possible bailout distributions.

Prior to Lehman’s collapse financial market’s beliefs about the probability of bailout have
been relatively optimistic due to Bear-Stern’s bailout in March 2008 as well as to the implicit
US government guarantees of Fannie Mae and Freddie Mac’s liabilities (Meltzer (2009)). In
terms of the GS framework this means that the family of binomial bailout distributions with
positive mass was concentrated in the relatively high range of \( p \)’s.

Taleb (2007) has popularized the notion of a “black swan” event. Such an event is
perceived to have zero mass before it realizes for the first time. However, once it realizes,
individuals assign to it (a possibly small) but positive mass. We view Lehman’s collapse in
mid September 2008 as such a “black swan” event. That event, deemed unthinkable, prior
to this collapse had realized after all and this reduced the lowest perceived probability of
bailout with positive mass.

The behavior of credit default swap (CDS) spreads during the two weeks following Lehman’s collapse provides a dramatic illustration of the sensitivity of bailout expectations to public signals. In the aftermath of this collapse credit markets experienced substantial waves of deleveraging, totally drying up in some cases, and both the level and variability of CDS spreads went through the roof. Table 1\(^1\) shows the behavior of Citibank’s CDS spread index during the period just preceding Lehman’s default and the final approval of the TARP bailout package at the beginning of October 2008.

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>CDS Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-14/9</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>15/9</td>
<td>Lehman files for chapter 11</td>
<td>250</td>
</tr>
<tr>
<td>16-17/9</td>
<td>Paulson suggests TARP to Congress</td>
<td>250</td>
</tr>
<tr>
<td>18-19/9</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>22-23/9</td>
<td>Paulson &amp; Bernanke address Congress</td>
<td>450</td>
</tr>
<tr>
<td>24-25/9</td>
<td></td>
<td>350</td>
</tr>
<tr>
<td>29/9</td>
<td>Congress rejects TARP proposal</td>
<td>Almost 450</td>
</tr>
<tr>
<td>3/10</td>
<td>Amended TARP approved by Congress</td>
<td></td>
</tr>
<tr>
<td>5-10/10</td>
<td>Aftermath of approval</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 1: **Chronology of CDS spread around Lehman’s collapse**

The table demonstrates the sensitivity of the CDS spread to ongoing public signals. In particular, following rejection of the proposed TARP bailout package by Congress in September 2008 the CDS spread goes up and following its approval in early October it goes down supporting the view that financial markets participants are quite sensitive to news

\(^{1}\)Source: Cochrane and Zingales (2009).
about the likelihood of bailout. Our view is that, following Lehman’s collapse and the ensuing public debate among policymakers about the wisdom of governmental bailouts, the lower bound on the set of binomial distributions with perceived positive mass permanently went down, say, from $\pi_0$ to $\pi_1$ (here $\pi_t$ is the lower bound of distributions with perceived positive mass in period $t$).

The analysis in the paper shows that lenders’ expected utility is lower the lower is $p$. In conjunction with the GS max-min criterion this increase in bailout uncertainty implies that, once a ”black swan” event like Lehman’s collapse materializes, lenders become more reluctant to lend, sending shock waves through both financial and real markets. One objective of the paper is to trace some of the mechanisms through which the consequent changes in perceptions affect short term credit within the financial system, as well as credit to the real sector. Another related objective is to analyze the impact of expansionary monetary policy on leverage and risk appetite. The paper’s framework makes it possible to trace out both the exante and the expost consequences of (perceived) generous bailout policies. Exante, a more generous bailout policy increases moral hazard in all segments of the financial system and induces an overall expansion of credit. But expost the maintenance of a generous bailout policy may become necessary just to avoid a crisis even if government no longer desires to maintain high bailout levels.

Important features of the model include:

(i) An individual tradeoff between return seeking through higher levels of leverage and higher probability of total loss at the individual level.

(ii) Exante and expost relations between the worst probability of bailout and leverage at the aggregate level.

Following Keynes, Akerlof and Shiller (2009) attribute changes in expectations to exogenous animal spirits. By contrast this paper takes the view that changes in expectations can be traced back to new information in noisy but relevant public signals.

Importantly, once it realizes the impact of a ”black swan” event effect persists even if there subsequently is an increase in the relative size of the mass on higher values of $p$.

Borio C. and M. Drehmann (2009) convincingly argue that such a credit buildup raises the likelihood of a financial crisis.
(iii) **Duration mismatches:** Borrowers need financing for two periods but get only one period loans from financial intermediaries in each period.

(iv) The model’s focus is on the segment of the **shadow banking system** (like SIV and hedge funds) in which funds are secured only for short periods. Accordingly, financial intermediaries are assumed to borrow for only one period.

The financial markets model in the paper can be thought of as a microfounded version of general equilibrium approaches to monetary theory and policy (Brunner and Meltzer (1997), Tobin (1969)). It features large numbers of each of the following (identical within each group) 3 types of agents: Borrowers (Bs), Financial Intermediaries (Fs) and Lenders (Ls) each with one unit of equity capital. The model’s borrowers are long term investors in plant and equipment and/or in real estate. The lenders are meant to represent pension and mutual funds that invest the accumulated savings of individual consumers. They split their funds between a well diversified portfolios of loans to financial intermediaries and a risk free asset. Although they engage in some risk spreading financial intermediaries are not fully diversified. The return on the risk free asset is determined by the monetary authority. The initial masses of those three types of agents are $M_B$, $M_L$ and $M_F$ for Bs, Fs and Ls respectively.

Figure 1 presents a bird’s eye view of the model’s financial system. In the figure $z_F$ and $z_L$ represent the fractions of funds Fs and Ls allocate to risky loans, and $r_B$ and $r_L$ are the rates paid by Bs and received by Ls respectively. $R_A$ and $R_I$ are aggregate and individual components of the total gross return to a typical borrower (see Equation (2) below). Finally, the probability that the aggregate component assumes the value $R_A$ rather than zero is $q_A$ and $q_I$ is the probability that the individual component assumes the value $R_I$ rather than zero.
The rest of the paper is organized as follows. Section 2, 3 and 4 introduce a typical borrower, a typical financial intermediary and a typical lender and characterize the optimal microeconomic behavior of each type of agent. Government’s bailout policy is specified in Section 4. General equilibrium of the financial system and the determination of market rates are discussed in section 5. Section 6 analyzes the impact of an exogenous decrease in perceptions about the likelihood of bailout on financial markets and utilizes it to explain some of the events observed following Lehman’s collapse. Section 7 discusses the exante choice of leverage by borrowers in general equilibrium including, in particular, the impact of perceived bailout policy and the associated moral hazard problem. This is followed by concluding remarks in Section 8. A central result of the paper (implied by the discussion in sections 6 and 7 and elaborated in the conclusion) is that higher bailout uncertainty raises the amplitude of booms and busts. Most proofs are in the Appendix.
2 The Typical Investor-Borrower

2.1 Basic structure

There are 3 periods labeled 0,1 and 2. All real investment decisions by investors-borrowers (Bs) are made in period 0 and are long term in the sense, that once chosen, the project’s size cannot be adjusted. The typical investment project yields a stochastic gross return, $\tilde{R}$, (which may be positive or negative) in each of periods 1 and 2. All real projects have the same distribution of returns. The distributions of $\tilde{R}$ in the two periods are identical and independent across periods but, due to an aggregate common shock, not across different projects within a given period. Further details about the specification of returns appears below. Projects are financed by a combination of equity and of leverage supplied by financial intermediaries (Fs) to Bs.

At period 0, each borrower-entrepreneur owns one unit of equity capital. The initial financing structure (equity-1 versus leverage-$L_B$) is chosen by each B in period 0 along with the project’s size, denoted by $x$, where since B’s initial equity capital is 1, $x = 1 + L_B$. Loans by Fs to Bs are given only for a single period. Consequently a B’s project is financed by two consecutive one period loans. If a B gets a bad draw in period 1, he must seek refinancing in that period. In such a case B depends on the availability and the cost of credit in period 1. If excluded from the credit market in period 1 when he needs refinancing a B defaults and loses the entire investment project including his equity. The borrower possesses the quasi-linear utility function

\[ u(W_B) = \begin{cases} 
W_B & \text{if } W_B \geq 0 \\
-P & \text{if } W_B < 0, \quad 0 < P 
\end{cases} \]

(1)

where $W_B$ is his period’s 2 terminal wealth after servicing all debts and $P$ is a, non negative, penalty inflicted on him if he defaults on his debt service in either of periods 1 or 2. Since there is limited liability actual wealth cannot be lower than zero but since it is defined net of debt service a negative $W_B$ indicates that B defaults. Hence, the content of the second line in equation (1) is that B suffers the penalty, $P$, whenever he defaults.
2.2 The distribution of returns on a typical project

A project’s gross return in each period is the sum of an aggregate and of an individual idiosyncratic shock

\[ \tilde{R} = \tilde{R}_A + \tilde{R}_I, \tag{2} \]

where the aggregate (economy-wide shock), \( \tilde{R}_A \), is binomially distributed as follows

\[
\tilde{R}_A = \begin{cases} 
R_A \text{ (Expansion) w.p. } q_A \\
0 \text{ (Contraction) w.p. } 1 - q_A 
\end{cases} \tag{3}
\]

and the idiosyncratic shock, \( \tilde{R}_I \), is also binomial with:

\[
\tilde{R}_I = \begin{cases} 
R_I \text{ w.p. } q_I \\
0 \text{ w.p. } 1 - q_I 
\end{cases} \tag{4}
\]

\( \tilde{R}_A \) and \( \tilde{R}_I \) are independent across periods and mutually independent within a period. The idiosyncratic shock, \( \tilde{R}_I \), is independent across projects. Returns, whether positive or negative, are cash flows. Equations (2)-(4) imply that the distribution of \( \tilde{R} \) is

\[
\tilde{R} = \begin{cases} 
R_A + R_I \equiv R^1 \text{ w.p. } q_Aq_I \equiv q_1 \\
R_I \equiv R^2 \text{ w.p. } (1 - q_A)q_I \equiv q_2 \\
R_A \equiv R^3 \text{ w.p. } q_A(1 - q_I) \equiv q_3 \\
0 \equiv R^4 \text{ w.p. } (1 - q_A)(1 - q_I) \equiv q_4 
\end{cases} \tag{5}
\]

Assumption 1: The distribution of the gross return, \( \tilde{R} \), on a representative project is ranked

\[ 0 < 1 < R_A < \mu_B < R_I < R_A + R_I \]

where \( \mu_B \equiv E\tilde{R} \).

Construction of \( B' \)'s equilibrium: In what follows we derive a set of conditions on the exogenous parameters of the model that induce the borrower to choose a level of leverage

\[ \text{We do not use the indirect notation, } R^i \text{ and } q_i, i = 1, \ldots, 4, \text{ in the text. However, it is used in the Appendix to reduce large expressions to manageable size.} \]

\[ \text{Obviously, Assumption 1 is implied by the weaker assumption } 1 < R_A < \mu_B < R_I. \]
such that he defaults and loses the project already in period 1 if and only if (iff) the realized
return in that period is zero. When B is solvent in period 1 he may still default in period 2.
Given the equilibrium level of leverage we develop, he defaults in period 2 after being solvent
in period 1 iff his first period’s return is \( R_A \) and his second period return is either \( R_A \) or 0.

2.3 **Borrower’s financial requirements in period 1**

To maintain the project alive till period 2 a borrower needs to service period’s 1 debt (incurred
in period 0). If period’s 1 return is sufficiently high he may secure this amount partially or
totally from internal cash flows. Otherwise he needs to refinance in period’s 1 credit market.
In case he incur a loss (\( \tilde{R} < 1 \)), since returns are cash flows the borrower also needs to borrow
in order to cover the loss. Depending on the realization of the project’s return in period 1 a
borrower may or may not need external financing to carry his project through to the second
period. We assume that when the economy booms and B is individually lucky so that his
return in that period is \( R_A + R_I \) he can rely entirely on internal finance to maintain the
project alive and to service period’s 0 debt and that any excess is distributed as dividend.
But at lower returns B needs some external finance to repay the debt and to maintain the
project alive. The amount needed to service the debt is \( r_{B1}L_B \) and the amount needed to
cover losses (if any) is \( (1 + L_B)(1 - \tilde{R}_{B1}) \), where \( r_{B1} \) is the rate at which F’s lend to B’s in
period 1.\(^7\) Hence B’s total financial requirements in period 1 are

\[
FR = (1 + L_B)(1 - \tilde{R}_{B1}) + r_{B1}L_B.
\]  

2.4 **Borrower’s solvency conditions**

2.4.1 **Period’s 1 solvency condition**

A borrower that needs external funds is solvent in period 1 if and only if he obtains the
refinancing required to maintain the project till period 2. Financial intermediaries will offer
the required credit if and only if the cash flow expected from the project in period 2 suffices

\(^7\)When \( 1 - \tilde{R}_{B1} < 0 \) some of period’s 1 profits are used to service period’s 0 debt.
to cover period's 1 debt service, that is

\[(1 + L_B)\mu_B \geq FRr_{B2}.\] (7)

Rearranging this expression is equivalent to

\[K(\tilde{R}_{B1}, \mu_B)L_B \geq r_{B2} \left(1 - \tilde{R}_{B1}\right) - \mu_B,\] (8)

where

\[K(\tilde{R}_{B1}, \mu_B) \equiv \mu_B - r_{B2} \left(1 + r_{B1} - \tilde{R}_{B1}\right).\] (9)

Notice that since \(r_{B1}\) and \(r_{B2}\) are taken as given by the individual borrower, they are omitted from the list of arguments in \(K(\cdot)\). It is shown below that if the individually optimal level of leverage, \(L_B\), is positive (which we assume to be the case) then \(K(\mu_B, \mu_B)\) must be positive (see Lemma 1). In conjunction with Assumption 1, this implies that \(K(\tilde{R}_{B1}, \mu_B) > K(\mu_B, \mu_B)\) for period 1 returns that are above the mean (either \(R_A + R_I\) or \(R_I\)). In those two cases Equation (7) implies that the borrower is solvent at any level of leverage. However, when period 1 returns are below the mean (either \(R_A\) or 0), \(K(\tilde{R}_{B1}, \mu_B)\) is likely to be negative and we proceed on the assumption that this is the case.\(^8\) In such cases Equation (7) can be rearranged as

\[L_B \leq \frac{r_{B2}(1 - \tilde{R}_{B1}) - \mu_B}{K(\tilde{R}_{B1}, \mu_B)} \equiv L^c_B(\tilde{R}_{B1}),\] (10)

implying that when period 1’s return is either \(R_A\) or 0 the borrower will be solvent in that period if and only if the leverage is bounded from above by \(L^c_B(\tilde{R}_{B1})\) for either \(\tilde{R}_{B1} = R_A\) or for \(\tilde{R}_{B1} = 0\). The value \(L^c_B(\tilde{R}_{B1})\) is referred as the B’s critical leverage level conditional on the return of his project at time 1, \(\tilde{R}_{B1}\). Note that \(L^c_B(R_A) > L^c_B(0)\) implying that the higher the realization of \(\tilde{R}_{B1}\) the higher the level of leverage that B can assume and still remain solvent.

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\(^8\)More precisely, this will be the case if \(r_{B1}\) and \(r_{B2}\) are sufficiently large in comparison to \(\mu_B\) and \(R_A\).
2.4.2 Period’s 2 solvency condition and a condition for positive leverage

A borrower is solvent in period 2 iff his cash flow in that period is larger than the required debt service

\[(1 + L_B)\tilde{R}_{B2} \geq FRr_{B2}.\] (11)

or equivalently

\[W_B(\tilde{R}_{B1}, \tilde{R}_{B2}; L_B) \equiv A(\tilde{R}_{B1}, \tilde{R}_{B2}) + K(\tilde{R}_{B1}, \tilde{R}_{B2})L_B \geq 0,\] (12)

where

\[A(\tilde{R}_{B1}, \tilde{R}_{B2}) \equiv \tilde{R}_{B2} + r_{B2} (\tilde{R}_{B1} - 1).\] (13)

Lemma 1: B’s optimal leverage is positive iff \(K(\mu_B, \mu_B) > 0\).

Proof. Applying the expected value operator, as of period 0, to Equation (12)

\[EW_B(\tilde{R}_{B1}, \tilde{R}_{B2}; L_B) = A(\mu_B, \mu_B) + K(\mu_B, \mu_B)L_B.\]

Consequently B chooses a positive level of leverage iff \(K(\mu_B, \mu_B) > 0\). 

2.5 Individual borrower’s optimization

Not surprisingly the individual borrower faces a tradeoff between expected returns and default probability. In the large, by raising leverage, he raises the expected value of terminal equity but also the chances of a default at which he incurs the penalty \(P\). But the discreetness of returns implies that the probability of default is a step function of leverage. In conjunction with Equation (10) this further implies that the optimal level of leverage (and by implication also the optimal project’s size) is one of the following two values

\[L_B = \begin{cases} L_B^c(R_A) - \varepsilon & \equiv L_{B3} \\ L_B^c(0) - \varepsilon & \equiv L_{B4} \end{cases},\] (14)

where \(\varepsilon > 0\) is an infinitesimally small number.
To see the origin of this result consider a case in which $L_B$ is set initially at $L_B^c(0) - \varepsilon$. The condition in (10) implies that in such a case B is solvent in period 1 even if he gets the worse payoff of 0. Hence, the probability of default in period 1 is 0. Once he raises leverage a bit above $L_B^c(0)$ but maintains it below $L_B^c(R_A)$ B’s period 1 default probability jumps to $(1 - q_A)(1 - q_I)$, and once he raises it above $L_B^c(R_A)$ it jumps again to $(1 - q_A)(1 - q_I) + q_A(1 - q_I)$. However within the interval $(L_B^c(0), L_B^c(R_A))$ the probability of default is constant at $(1 - q_A)(1 - q_I)$. Hence, once he crosses the $L_B^c(0)$ threshold the borrower can only gain by raising leverage almost up to the next threshold, $L_B^c(R_A)$. By doing that he raises the expected value of final wealth without raising the probability of default. It follows that the optimal level of leverage must be just below one of the thresholds at which the probability of default jumps. Since, for period 1, the only two relevant thresholds are those in Equation (16) optimal leverage equals one of those.\(^9\)

Recalling that the choice of project size and of leverage is done in period 0, before the resolution of uncertainty in either of periods 1 and 2, and using B’s utility function in Equation (1), B’s problem is to choose a leverage level, $L_B$, so as to maximize the following expected utility

$$Eu\left[W_B(\bar{R}_{B1}, \bar{R}_{B2}; L_B)\right] = \sum_{i \in S} \sum_{j \in S} W_B(R^i, R^j; L_B) q_i q_j - Pr[D | L_B] P$$

(15)

where $S$ designates the solvency state and $Pr[D | L_B]$ is the overall probability of default (D) conditional on leverage level $L_B$.\(^{10}\) Here

$$W_B(R^i, R^j; L_B) \equiv A(R^i, R^j) + K(R^i, R^j)L_B \quad i = 1, \ldots, 4 \quad j = 1, \ldots, 4.$$  

(16)

**Proposition 1:** Provided the following constraints hold

\(^{9}\)Note that, if there is no default in period 1, the total probability of default is the sum of the probabilities of default in periods 1 and 2 and this probability also changes in steps. However, if B defaults already in period 1, this total probability of default is irrelevant since the project does not survive to period 2. In such a case the overall probability of default is identical to period’s 1 default probability.

\(^{10}\)For notational convenience we use here the compact notation, $q_i$, (defined in Equation (5)) for the probabilities of different states of nature.
(ii) \( W_B(R_A, R_A; L_{B3}) < 0, W_B(R_A, 0; L_{B4}) > 0, W_B(0, R_A + R_I; L_{B4}) < 0, \)

(iii) \( K(\mu_B, \mu_B) > 0, K(R_A, \mu_B) < 0, K(0, \mu_B) < 0, \mu_B > r_{B2}, \) and

(iv)
\[
E u [W_B(\cdot; L_{B3})] > E u [W_B(\cdot; L_{B4})],
\]  
then

(a) B’s optimal leverage is
\[
L^*_B = L_{B3} = \frac{\mu_B - (R_A - 1)r_{B2}}{r_{B2}^e (1 + r_{B1}) - \mu_B - r_{B2}^e R_A - \varepsilon},
\]  
where \( r_{B2}^e \) is period’s 0 forecast of period’s 1 borrowing rate.

(b) The condition in Equation (17) is more likely to be satisfied, the larger \( R_A \) and, provided \( q_I > \frac{1}{2} \), the larger is \( q_I \). When \( q_I = 1 \) the condition is always satisfied.

**Proof.** Since \( K(\mu_B, \mu_B) > 0 \), Lemma 1 implies that optimal leverage is strictly positive. Condition set (iii) of the proposition implies that \( L_{B3} \) and \( L_{B4} \) are positive. Since the probability of default is a step function of leverage, \( L_{B3} \) and \( L_{B4} \) are also the only two candidates for optimal leverage. When the condition in Equation (17) is satisfied the expected value under \( L_{B3} \) is larger than under \( L_{B4} \) so that the optimal level of leverage is \( L_{B3} \). The rest of the proof is presented in the Appendix. ■

The reader is reminded that the optimal level of leverage \( L^*_B \) implies that:

1. B defaults in period 1 iff the return in that period is 0,
2. He defaults in period 2 iff his first period return is \( R_A \) and his second period return is either \( R_A \) or 0.

### 3 Financial intermediaries

For reasons that will become apparent later it is convenient to open this section with a forward look at the relation between various equilibrium rates of interest.
3.1 A forward look at general equilibrium relations among various rates

Let $r_f$ be the risk free rate, $r_L$ the equilibrium rate paid by financial intermediaries to lenders, and $r_B$ the equilibrium rate paid by borrowers to financial intermediaries.$^{11}$

**Proposition 2:** *In a general equilibrium with risk aversion on the part of borrowers, financial intermediaries and lenders, and positive levels of leverage in both the real and the financial sectors, the following inequalities hold*

$$r_f < r_L < r_B.$$  

**Proof.** The proof is a direct consequence of the fact that all three types of agents are risk averse and that leverage levels are positive. ■

3.2 The typical intermediary

There is a large number of financial intermediaries (Fs) each of which possesses one unit of core funds consisting of a combination of equity and of long term (two periods) debt. A typical F can also raise short term (one period) funds from lenders through various deposits including certificates of deposit (CDs). Since the focus of our analysis is on changes in the availability of short term credit in the face of new information the amount of short term leverage assumed by a typical F is determined endogenously while the sum of equity and of long term debt is taken to be exogenous.

Total financial resources of a typical F consist of the core funds and of short term leverage, $L_F$. The financial intermediary diversifies his total resources between the risk free asset whose rate, $r_f$, is a policy instrument and a risky portfolio of two equally sized loans to Bs. The fraction of resources invested in the risky loan portfolio to Bs is denoted $z_F$. A typical F

$^{11}$The reader is reminded that all returns are in gross terms — i.e. one plus the net return.
possesses a (generally risk averse) quadratic utility function:

$$ u(W_F) = W_F - \frac{b}{2}W_F^2, \quad W_F < \frac{1}{b}, \quad b > 0, \quad (19) $$

where $W_F$ is his terminal wealth in each period. F’s risk aversion is lower the lower is $b$. Note that when $b$ tends to zero from above F becomes almost risk neutral.

### 3.3 Distribution of returns and optimization

Total return to a financial intermediary depends on the performance of the two borrowers to whom he lends. Since borrowers are identical exante, the optimal risky portfolio of an F consists of a fifty-fifty split between loans to two borrowers. If both borrowers are solvent both of them pay the full face value, $r_B$, of the (gross) debt service and the return to F is $r_B$. If both of them default F gets 0. If one borrower is solvent and the other defaults F gets $\frac{1}{2}r_B$. The following expression summarizes the probabilities associated with each case

<table>
<thead>
<tr>
<th>Yield-$\tilde{r}_B$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>The two borrowers are solvent</td>
<td>$r_B$</td>
</tr>
<tr>
<td>One borrower defaults and one is solvent</td>
<td>$\frac{1}{2}r_B$</td>
</tr>
<tr>
<td>Both borrowers default</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2: Financial intermediaries’ return**

The wealth of a typical F at the end of each period is

$$ \tilde{W}_F = (1 + L_F) \left[ z_F\tilde{r}_B + (1 - z_F)r_f \right] - r_LL_F, \quad (20) $$

where the distribution of $\tilde{r}_B$ is given in Table 2 and $r_L$ is the gross interest rate paid by a F on its short term obligations. A representative F chooses his leverage, $L_F$, and the fraction, $z_F$, of resources invested in the risky loan portfolio to Bs so as to maximize $E\tilde{W}_F$ in each of periods 0 and 1. The following proposition characterizes the optimal policy of a typical F.
Proposition 3: Let \( r_f < r_L \). Then at an optimum with positive leverage, \( F \) invests all his resources in risky loans to Bs and

\[
L_F^* = \frac{(\gamma_F^1 + 0.5\gamma_F^2)r_B - r_L - b[\gamma_F^1 r_B(r_B - r_L) + 0.5\gamma_F^2 r_B(0.5r_B - r_L)]}{b[\gamma_F^1(r_B - r_L)^2 + \gamma_F^2(0.5r_B - r_L)^2 + (1 - \gamma_F^1 - \gamma_F^2)r_L^2]}.
\] (21)

Proof. The proof of the first statement is by contradiction. Since \( r_f < r_L \), an \( F \) with positive leverage and some fraction of the portfolio in the risk free asset can increase profits by reducing both short term leverage and, in parallel, the investment in the risk free asset. Consequently, a configuration with both positive leverage and some investment in the risk free asset cannot be a financial intermediary optimum. Hence \( z_f = 0 \). Equation (21) is obtained by maximizing the expected utility of \( F \) with respect to \( L_F \) for \( z_f = 0 \). Further details on this maximization appear in the Appendix. ■

Note that optimal \( F \)'s leverage, \( L_F^* \), is higher the lower is the risk aversion parameter, \( b \).

Proposition 4: Provided the marginal utility from the wealth, \( W_F \), of a representative \(^{12}\) \( F \) is still positive at twice the value of \( W_F \) in the full solvency state (which is the case when \( b \) is sufficiently small) then

\[
\frac{dL_F^*}{dr_L} < 0, \quad \frac{dL_F^*}{dr_B} > 0.
\]

Proof. In Appendix ■

Proposition (3) establishes the intuitive result that a typical \( F \)'s leverage is lower the higher the interest rate that has to be paid to lenders and the lower the interest rate obtained on loans to borrowers.

3.3.1 \( F \)'s solvency condition

\( F \) is solvent iff

\[
W_F(L_F) = (1 + L_F)\tilde{r}_B - r_L L_F = \tilde{r}_B + (\tilde{r}_B - r_L)L_F \geq 0.
\] (22)

\(^{12}\)Representative in the sense that all \( F \) are identical.
Since \( r_B > r_L \), F is solvent for any \( L_F \) when \( \bar{r}_B = r_B \). In the other two cases F is solvent only if \( L_F \) is sufficiently small. The precise conditions are:

\[
L_F \leq \frac{r_B}{2r_L-r_B} \quad \text{when} \quad \bar{r}_B = \frac{1}{2}r_B \\
L_F = 0 \quad \text{when} \quad \bar{r}_B = 0.
\] (23)

We focus on an equilibrium in which F’s risk aversion as characterized by \( b \) is relatively mild so that

\[
L_F^* > \frac{r_B}{2r_L-r_B}.
\] (24)

**Proposition 5:** Given the implicit condition on \( b \) in Equation (24) the financial intermediary is solvent iff the two borrowers to whom he has lent are solvent. If solvent a financial intermediary pays the full debt service to lenders. Otherwise he defaults and pays nothing.

**Proof.** Immediate from the preceding discussion.

4 The representative lender and government’s bailout policy

Through pension or mutual funds the representative lender splits his equity between a fully diversified portfolio of loans to financial intermediaries and the risk free asset.\(^{13}\) Since, ex ante, all Fs have identical distributions of returns the optimal shares of loans to different Fs are all equal. The fraction invested in the risky loan portfolio to Fs is denoted \( z_L \). The typical lender (L) possesses mean-variance (or Constant Absolute Risk Aversion - CARA)\(^{14}\) preferences

\[
u(W_L) = -\frac{1}{\alpha}e^{-\alpha W_L}, \quad \alpha \geq 0,
\] (25)

where \( W_L \) is his terminal wealth in each period and \( \alpha \) characterizes the degree of constant absolute risk aversion.

\(^{13}\)Representative lender in the sense that all lenders are identical.

4.1 Perceived government’s bailout policy

Government may repay the gross debt owed to lenders by defaulting Fs. The perceived probability that the debt service of a defaulting F is paid by government (a bailout) is denoted by \( p \). The likelihood of bailout is independent across Fs debt. In case of bailout a lender receives the full debt service, \( r_L \). In the presence of risk but no bailout uncertainty \( p \) is unique. We use the Gilboa Schmeidler’s (1989) multiple priors framework to formalize Knightian uncertainty.\(^{15}\) Accordingly, in the presence of bailout uncertainty perceptions include the set of all possible binomial distributions characterized by \( p \)’s that the lenders believe to have positive mass. The lowest value of \( p \) in the set is denoted by \( \pi \). As will become clear below this is also the worst plausible prior from a typical lender’s point of view.

The degree of uncertainty is determined by the ”size” of the set of possible priors. That is, when circumstances become more uncertain (ambiguous) the set of possible priors expands to include priors that previously were considered implausible. Consequently, provided some of the set enlargement is toward lower \( p \)’s, the worst prior, \( \pi \), is revised downward.

4.2 Representative lender’s returns and optimization

Although the risky portfolio of a lender is fully diversified the variance of the risky portfolio is positive since the returns to lenders from loans to different Fs are correlated due to the common shock, \( \tilde{R}_A \), in the return to real investments of borrowers. As explained in the previous section a financial intermediary either pays his debt in full to lenders or fully defaults. When F defaults on the debt service, government may or may not step in and pay the delinquent debt service to a lender. Consequently the lender faces a binomial distribution of returns from lending to an individual F – he either gets the full debt service, \( r_L \), (either from F or from government) or 0. Although the bailout policy of government does not affect the binomial nature of the payoffs from a single F, it does alter their distribution. Since the risky portfolio of L’s contains a large number of such binomially distributed loans

\(^{15}\)Knightian uncertainty is also also referred to as ambiguity.
the risky portfolio of Ls is normally distributed with a mean and a variance that depend on both economic \((q_A, q_I)\) and political \((\pi)\) uncertainties. Details appear in the following proposition

**Proposition 6:** For a given \(p\) the return to a lender on his (fully diversified) portfolio of loans, \(\{\tilde{r}_L\}\), is normally distributed with mean

\[
E \left( \{\tilde{r}_L\} \right) = \left( q_A + (1 - q_A)q_I^2 + p(1 - q_A)(1 - q_I^2) \right) r_L
\]

and variance

\[
Var \left( \{\tilde{r}_L\} \right) = q_A(1 - q_A)(1 - p)q_I^2(1 - q_I^2)^2 r_L^2.
\]

**Proof.** Calculation of the expected value is relatively straightforward. Derivation of the variance utilizes the fact that the variance of a fully diversified risky portfolio composed of (equally weighed) infinitely many identically distributed assets is equal to the covariance between any two assets within the portfolio. Calculation of this covariance simplifies the derivation of an explicit expression for \(Var \left( \{\tilde{r}_L\} \right)\) but still involves some messy intermediate algebra. Details on the derivation of the portfolio’s variance appear in the Appendix. ■

A representative L chooses the fraction, \(z_L\), of resources invested in the risky loan portfolio to Fs so as to maximize\(^{16}\)

\[
E \left( \tilde{W}_L(z_L) \right) = -\frac{1}{\alpha} E \left( e^{-\alpha \tilde{W}_L(z_L)} \right)
\]

in each period, where

\[
\tilde{W}_L(z_L) = z_L \tilde{r}_L + (1 - z_L)r_f.
\]

**Proposition 7:** At an individual optimum, a lender allocates the fraction

\[
z_L^* \approx \frac{E \left( \{\tilde{r}_L\} \right) - r_f}{\alpha Var \left( \{\tilde{r}_L\} \right)}
\]

\(^{16}\)The right hand side of equation (28) is obtained by using a typical lender’s utility function in equation (25).
of each single $ to the diversified risky portfolio of loans to Fs.\textsuperscript{17}

Proof. In Appendix. \qed

4.3 Partial equilibrium impact of less generous bailouts on the size of lenders’ risky portfolios and the impact of ambiguity aversion

In the absence of bailout uncertainty government’s bailout policy is characterized by a unique perceived probability, \( p \), that government will pay the debt of delinquent F’s to L’s. A more generous (towards L’s) bailout policy is characterized by a higher \( p \) and a less generous bailout policy by a lower \( p \). By changing the distribution of \( \tilde{r}_L \) the value of \( p \) affects both the mean and the variance of lenders’ risky portfolios. Obviously a more generous bailout policy raises the mean of the distribution of \( \tilde{r}_L \) a fact that is confirmed by equation (26). At least a-priori, the impact of a more generous bailout policy on the variance of this portfolio is generally ambiguous. However a glance at Equation (27) suggests that a more generous bailout policy also reduces the variance of the risky portfolio implying the first part of the following proposition.

Proposition 8:

(a) Holding \( r_L \) constant, a less generous bailout policy (lower \( p \)) induces a "flight to safety" by lenders (lower \( z^*_L \)).

(b) Provided \( r_f > \frac{r_L}{2} \) and holding \( p \) constant an increase in \( r_L \) raise the appetite for risky loans to financial intermediaries (raises \( z^*_L \)).\textsuperscript{18}

Proof. The proof of part (i) follows immediately from the preceding observations and from Equation (30). To demonstrate part (ii) differentiate \( z^*_L \) in Equation (30) with respect to \( r_L \)

\textsuperscript{17}This approximation is accurate for a small risk premium, \( E (\{\tilde{r}_L\}) - r_f \).

\textsuperscript{18}The partial equilibrium aspect of Proposition 8 derives from the facts that \( r_L \) is kept constant in part (i) and \( \pi \) is kept constant in part (ii). By contrast, the general equilibrium analysis below takes into consideration the impact of a change in \( \pi \) on the equilibrium value of \( r_L \).
and rearrange using equations (26) and (27). This calculation implies that the sign of the
\( \frac{dz^*}{dp} \) is the same as that of
\[
2r_f - (q_A + (1 - q_A)q_I^2 + p(1 - q_A)(1 - q_I^2)) r_L.
\]
Since \( (q_A + (1 - q_A)q_I^2 + p(1 - q_A)(1 - q_I^2)) < 1 \) a sufficient (but not necessary) condition
for \( \frac{dz^*_L}{dp} > 0 \) is \( r_f > \frac{r_L}{2} \). □

Since \( r_f \) and \( r_L \) are gross rates of interest the condition in part (ii) of the proposition is
always satisfied for normal ranges of interest rates.

An important corollary to Proposition 8 is that, other things the same, lower values of \( p \)
are associated with lower returns to lenders. Hence, in the presence of bailout uncertainty,
and since they are averse to ambiguity in the Gilboa-Schmeidler (1989) sense, lenders be-
have as if the probability of bailout is the lowest within the set of \( p \)'s with positive mass.
Stated differently, they choose the fraction of their portfolio invested in risky loans to Fs
so as to maximize expected utility under the assumption that bailout probability is \( \pi \). The
operational consequence of such behavior is that \( p \) should be replaced with \( \pi \) in Propositions
6 and 8.

5 General equilibrium of the financial system

Given expectations about the future, general equilibrium of the financial system is character-
ized by two market clearing conditions. One for credit from Ls to Fs and the other for credit
from Fs to Bs. These two conditions simultaneously determine \( r_B \) and \( r_L \) in each period.
In period 1 expectations about the future only involve realizations of period’s 2 returns to
borrowers. As a consequence the formulation of this equilibrium is relatively simple. But in
period 0 they also involve expectations about period’s 1 market clearing values of \( r_B \) and \( r_L \)
in period 1 (\( r^*_B \) and \( r^*_L \)). Those expectations are assumed to be model consistent in the sense
that, in period 0, financial market participants use the information available in that period
along with their knowledge of the fact that period’s 1 rates will be determined by market
clearing to derive $r^*_B$ and $r^*_L$. It is therefore convenient to characterize period’s 1 equilibrium first and to then use it to characterize period’s 0 equilibrium.

5.1 General equilibrium of the financial system in period 1

As stated above general equilibrium of the financial system in period 1 is characterized by competitive market clearing in the market for loans by Fs to Bs and the market for credit by Ls to Fs. The aggregate demand for loans by borrowers in period 1 depends on their financial requirements in that period. Those requirements depend, in turn, on a typical B’s leverage, $L^*_B$, (which had been determined in period 0) and on realized rates of return on real investments in period 1. In a state of aggregate expansion (E) borrowers’ rates of return are

$$R_A + R_I \text{ or } R_A. \quad (31)$$

In a state of aggregate contraction (C) borrowers’ rates of return are

$$R_I \text{ or } 0. \quad (32)$$

Although equilibrium on financial markets varies depending on whether the economy is in state E or in state C in period 1 the market clearing conditions are qualitatively similar.

5.1.1 Period’s 1 general equilibrium of the financial system in states of expansion (E)

In case an expansion occurs in period 1 all borrowers are solvent.\(^{19}\) Hence, all Fs are solvent too – there are no defaults on Fs’ debt to Ls and consequently no need for governmental bailouts. In particular the mass of existing Fs and Ls in period 1 is the same as it was in period 0. The mass of Bs is also the same but the demand for loans in period 1 originates only from borrowers who got adverse idiosyncratic return shocks since the other real investors do not need external financing by assumption. Given ergodicity (which we assume), the fraction

\(^{19}\)This is a consequence of the fact that $L^*_B = L_B^3$. 

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of Bs who need external financing is given by \(1 - q_t\). The market clearing conditions in case of expansion are therefore

\[
(1 - q_t)M_B ((1 + L_B^*)(1 - R) + r_B1L_B^*) = M_F (r_B1 + L_F^*(r_B2, r_L2)) \tag{33}
\]

and

\[
M_F L_F^*(r_B^2, r_L^2) = r_{L1} M_L^1 E (\{\tilde{r}_{L2}\} | \pi_1) - r_{f2} \sum \text{Var} (\{\tilde{r}_{L2}\} | \pi_1). \tag{34}
\]

The left hand side (LHS) of the first equation is the total demand for loans by Bs that need an external financing and the right hand side (RHS) is the supply of loans by Fs in period 1. Since Fs get paid in full, the equity capital of an F in period 1 is \(r_B1\) rather than 1. The LHS of the second equation is the total demand for loans by Fs and the RHS is the supply of such loans by Ls. Equations (26) and (27) show that the mean and the variance of \(\tilde{r}_{L2}\) depend on the perceived likelihood, \(\pi\), of governmental bailouts. In anticipation of the comparative statics in the next section, the dependence of those parameters on \(\pi\) is explicitly reflected in the notation. Equations (33) and (34) simultaneously determine \(r_B2\) and \(r_L2\) for given values of \(L_B^*, r_B1, r_L1, E (\{\tilde{r}_{L2}\} | \pi)\) and \(\text{Var} (\{\tilde{r}_{L2}\} | \pi)\).

5.1.2 Period’s 1 general equilibrium in states of contractions (C)

In case of contraction all borrowers need an external financing in period 1, but only those who got favorable idiosyncratic return shocks are accommodated by Fs. By ergodicity, the fraction of such Bs is \(q_t\) and the fraction of Fs that survive in period 1 is \(q_t^2\). Financial markets clearing conditions are therefore

\[
q_t M_B ((1 + L_B^*)(1 - R) + r_B1L_B^*) = q_t^2 M_F (r_B1 + L_F^*(r_B2, r_L2)) \tag{35}
\]

and

\[
q_t^2 M_F L_F (r_B2, r_L2) = \left(\gamma_{LC}^1 + \frac{\gamma_{LC}^2}{2}\right) r_{L1} M_L^1 E (\{\tilde{r}_{L2}\} | \pi_1) - r_{f2} \sum \text{Var} (\{\tilde{r}_{L2}\} | \pi_1), \tag{36}
\]

where \(\pi_1\) is the lowest probability of bailout within the set of perceived bailout probability distributions with positive mass in period 1 and \(\gamma_{LC}^1 + \frac{\gamma_{LC}^2}{2}\) is the fraction of Ls with positive
funds in that period. Equations (35) and (36) simultaneously determine \( r_{B2} \) and \( r_{L2} \) for given values of \( L_B^*, r_{B1}, r_{L1}, \gamma_{LC}^1, \gamma_{LC}^2, E(\{\tilde{r}_{L2}\} | \pi_1) \) and of \( \text{Var}(\{\tilde{r}_{L2}\} | \pi_1) \).

5.2 General equilibrium of the financial system in period 0

The choice of leverage in period 0 depends, among other things, on period 0’s forecast of period’s 1 borrowing rate, \( r_{eB2} \), which we assume to be model consistent. The operational content of this assumption is that individuals use the equilibrium conditions in equations (33) through (36) along with the information available in period 0 to form a forecast of \( r_{B2} \). Derivation of this forecast is complicated by the following two facts:

(i) In each period \( r_B \) is determined simultaneously with \( r_L \).

(ii) Expectations about period’s 1 equilibrium values of \( r_B \) and \( r_L \) differ depending on whether period 1 experiences an expansion (E) or a contraction (C).

As a consequence the model consistent forecast of \( r_{B2} \) is a weighted average of the expected equilibrium values of \( r_{B2} \) in expansion and in contraction states where the weights are the probabilities, \( q_A \), of E and, \( 1 - q_A \), of C.

This leads to 6 equilibrium conditions: Period’s 0 equilibrium conditions (2 equations that, given \( r_{eB2}^E \), determine \( r_{B1} \) and \( r_{L1} \)), period’s 1 forecasted equilibrium condition for the case of expansion (2 equations) and period’s 1 forecasted equilibrium condition for the case of contraction (2 equations). In addition there is one model consistent equation that links \( r_{eB2}^E \) to the expected equilibrium values of \( r_{B2} \) under expansions and contractions. Those 7 conditions determine the following 7 endogenous variables: \( r_{B1}, r_{L1}, r_{B2}^E, r_{L2}^E, r_{B2}^C, r_{L2}^C, r_{eB2}^E \). Here \( r_{B2}^i, r_{L2}^i, i = E, C \) are period’s 1 values of the debitory and creditory rates of F’s in states of expansion and contraction respectively, as expected in period 0. The analytical forms of the above mentioned conditions are detailed in what follows. Period’s 0 equilibrium

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20Here it is implicitly assumed that the news leading to the downward revision in \( \pi \) from \( \pi_0 \) to \( \pi_1 \) arrive after period’s 1 defaults and solvencies have realized. This implies that, given model consistent expectations, the fraction \( \gamma_{LC}^1 + \frac{\gamma_{LC}^2}{2} \) depends on \( \pi_0 \) and on \( q_1 \).
conditions are
\[ M_B L_B^* = M_B \frac{\mu_B + (R_A - 1) r_{B2}^e}{r_{B2}^e(1 + r_{B1}) - \mu_B - r_{B2}^e R_A} = M_F \left( 1 + L_F^* (r_{B1}, r_{L2}) \right) \] (37)

and
\[ M_F L_F^* (r_{B1}, r_{L1}) = M_L \frac{E \left( \{ \tilde{r}_{L1} \} \mid \pi_0 \right) - r_{f1}}{\alpha \text{Var} \left( \{ \tilde{r}_{L1} \} \mid \pi_0 \right)}, \] (38)

where \( \pi_0 \) is the lowest probability of bailout within the set of perceived bailout probability distributions with positive mass in period 0. Period’s 1 equilibrium conditions as perceived in period 0 for a state of expansion \((\tilde{R}_A = R_A)\) are

\[ (1 - q_I) M_B \{ (1 + L_B^*) (1 - R_A) + r_{B1} L_B^* \} = M_F \left( r_{B1} + L_F^* (r_{B2}^E, r_{L2}^E) \right) \] (39)

and
\[ M_F L_F^* (r_{B2}^E, r_{L2}^E) = r_{L1} M_L \frac{E \left( \{ \tilde{r}_{L2} \} \mid \pi_0 \right) - r_{f2}^e}{\alpha \text{Var} \left( \{ \tilde{r}_{L2} \} \mid \pi_0 \right)}. \] (40)

Period’s 1 equilibrium conditions as perceived in period 0 for a state of contraction \((\tilde{R}_A = 0)\) are\(^{21}\)

\[ q_I M_B \{ (1 + L_B^*) (1 - R_I) + r_{B1} L_B^* \} = q_I^2 M_F \left( r_{B1} + L_F^* (r_{B2}^C, r_{L2}^C) \right) \] (41)

and
\[ q_I^2 M_F L_F (r_{B2}^C, r_{L2}^C) = \left( \gamma_{L}^C (\pi_0) + \frac{\gamma_{L}^{2C} (\pi_0)}{2} \right) r_{L1} M_L \frac{E \left( \{ \tilde{r}_{L2} \} \mid \pi_0 \right) - r_{f2}^e}{\alpha \text{Var} \left( \{ \tilde{r}_{L2} \} \mid \pi_0 \right)}. \] (42)

Model consistent expectations of \( r_{B2} \) as of period 0 are

\[ r_{B2}^e = q_A r_{B2}^E + (1 - q_A) r_{B2}^C. \] (43)

\(^{21}\)Here the dependence of the expected fraction of lenders with positive amounts of funds in case a contraction materializes in period 1, \((\gamma_{L}^{1C} (\pi_0) + \frac{\gamma_{L}^{2C} (\pi_0)}{2})\), on period’s 0 perceived bailout probability is made explicit in the notation.
6 Expost general equilibrium impacts of a perceived decrease in the generosity of bailouts – implications for financial markets in the aftermath of Lehman’s collapse (period 1)

It may be worth to reiterate that \( \pi_0 \) is the lowest perceived bailout probability as of period 0 for both periods 0 and 1.\(^{22}\) Note that the assumption that, as of period 0 financial market participants do not expect this probability to change between those two periods is reflected in the formulation of the general equilibrium conditions for period 0 (equations (37) through (42)). Suppose now that, following a major indication of a shift in government’s bailout policy – like not rescuing Lehman — bailout uncertainty increases. In particular, the lowest perceived bailout probability with positive mass decrease from \( \pi_0 \) to \( \pi_1 \). Proposition 8 implies that, holding \( r_L \) constant, this change reduces the supply of funds to Fs by Ls. This change triggers a general equilibrium increases in both \( r_L \) and \( r_B \). Those increases raise the fraction of defaulting borrowers in period 2 and may, under some circumstances, lead to the total drying up of credit in period 1. The following proposition provides a precise formulation of these results

**Proposition 9:** Given the conditions in propositions 4 and 8 an increase in pessimism about governmental bailout policy (a decrease of the probability of bailout from \( \pi_0 \) to \( \pi_1 \)) leads to

(i) An increase in the cost of funds, \( r_{L2} \), to Fs above what it had been expected to be as of period 0,

(ii) An increase in the cost of funds, \( r_{B2} \), to borrowers above \( r^e_{B2} \), and thus reducing the profits expected for period 2 and raising the probability of borrowers’ default in that period,

(iii) When the decrease in \( \pi \) is sufficiently large, the consequent increase in \( r_{B2} \) may induce

\(^{22}\)More generally \( \pi_t \) stands for the lowest belief in period \( t \) about the probability of governmental bailout in all relevant future periods.
a total drying up of credit to borrowers whose period’s 1 return is $R_A$ and to consequent
defaults by those borrowers already in period 1.

**Proof.** As explained earlier period’s 1 general equilibrium differs depending on whether the
economy is in an expansion or in a contraction. Hence the general equilibrium comparative
statics are performed for each of those states separately. In case of expansion the comparative
statics are obtained by differentiating equations (33) and (34) totally with respect to $\pi$, and
in case of contraction by differentiating equations (35) and (36) with respect to $\pi$. The
results are summarized compactly in the following pair of equations

\begin{equation}
0 = \gamma_F(i)M_F \left[ \frac{\partial L^*_F(i)}{\partial r_B} \frac{dr_{B2}}{d\pi}(i) + \frac{\partial L^*_F(i)}{\partial r_L} \frac{dr_{L2}}{d\pi}(i) \right],
\end{equation}

\begin{equation}
\gamma_F(i)M_F \left[ \frac{\partial L^*_F(i)}{\partial r_B} \frac{dr_{B2}}{d\pi}(i) + \frac{\partial L^*_F(i)}{\partial r_L} \frac{dr_{L2}}{d\pi}(i) \right] = Q(i)r_{L1}M_L \left[ \frac{\partial z^*_L(i)}{\partial r_L} \frac{dr_{B2}}{d\pi}(i) + \frac{\partial z^*_L(i)}{\partial r_L} \frac{dr_{L2}}{d\pi}(i) \right],
\end{equation}

where $i = E, C$ and

\begin{align*}
\gamma_F(i) &= \begin{cases} 
1 & \text{for } E \\
q^2_i & \text{for } C
\end{cases} \\
Q(i) &= \begin{cases} 
1 & \text{for } E \\
\gamma^1_{L2} + \frac{\gamma^2_{L2}}{2} & \text{for } C
\end{cases}.
\end{align*}

Solving for $\frac{dr_{B2}}{d\pi}(i)$ in terms of $\frac{dr_{L2}}{d\pi}(i)$

\begin{equation}
\frac{dr_{B2}}{d\pi}(i) = -\frac{\partial L^*_F(i)}{\partial r_B} \frac{dr_{L2}}{d\pi}(i), \quad i = E, C.
\end{equation}

Substituting equation (47) into equation (45) and solving for $\frac{dr_{L2}}{d\pi}(i)$

\begin{equation}
\frac{dr_{L2}}{d\pi}(i) = -\frac{\partial z^*_L(i)}{\partial r_L} \frac{dr_{L2}}{d\pi}(i), \quad i = E, C.
\end{equation}

By Proposition 8 $\frac{\partial z^*_L(i)}{\partial r_L}$ and $\frac{\partial z^*_L(i)}{\partial r_L}$ are both positive implying, from Equation (48), that $\frac{dr_{L2}}{d\pi}(i) < 0$. By Proposition 4 $\frac{\partial L^*_F(i)}{\partial r_B} > 0$ and $\frac{\partial L^*_F(i)}{\partial r_L} < 0$ implying, from Equation (47),
that $\frac{dr_{B2}}{d\pi}(i)$ and $\frac{dr_{L2}}{d\pi}$ possess identical signs. It follows that the general equilibrium effects of a surprise decrease in $\pi$ is to raise both $r_{B2}$ and $r_{L2}$ above what those variables had been expected to be in period 0 ($r_{B2}^e$ and $r_{L2}^e$).

If the increase in $r_{B2}$ is sufficiently moderate, Bs that had expected, as of period 0, to get refinancing in period 1 still get it but at a higher price. Consequently, although the increase in $r_{B2}$ does not trigger period’s 1 defaults beyond what had been expected for the returns that actually realize, period’s 2 expected profits go down – raising the fraction of defaults in that period.

If the increase in $r_{B2}$ is sufficiently large, some of the Bs that had expected, as of period 0, to get refinancing in period 1 do not get it because the increase in $r_{B2}$ leads to a violation of the solvency condition in Equation (10). As a consequence the fraction of defaults in period 1 rises leading to a destruction of ongoing projects. Since $R_A < R_I$ this is more likely to be the case when the demand for period’s 1 refinancing emanates from borrowers whose realized rate of return is $R_A$. In such a case the period’s 1 credit markets totally dry up. ■

The comparative statics impacts in Proposition 9 accord well with actual developments following the downfall of Lehman Brothers. They are consistent with the view that much of the financial market panic in the aftermath of this collapse was due to a downward revision in perceptions about the likelihood that the US government will step in and use public funds to reduce creditors’ losses following defaults by financial intermediaries.
7 Exante general equilibrium impacts of bailout perceptions on financial markets – implications for leverage and for moral hazard (period 0)

7.1 The impact of permanently higher bailout perceptions (higher $\pi_0$)

Unlike period 1 in which the demand for credit by borrowers is insensitive to the borrowing rate, period’s 0 leverage depends on the borrowing rate in that period as well as on the borrowing rate expected to prevail in period 1. In general equilibrium both of those rates, as well as the rates at which financial intermediaries borrow from lenders depend on financial markets participants’ perceptions about the likelihood of bailout. Hence, by affecting equilibrium interest rates, perceptions about the likelihood of bailout in period 0 affect the volume of leverage in financial markets. This section investigates the impact of period’s 0 permanent beliefs about governmental bailout policy as summarized by the parameter $\pi_0$ on equilibrium interest rates and the volume of leverage. It is convenient to first note the following lemma.

**Lemma 2:** For model consistent expectations higher permanent values of $\pi_0$ (lower bailout uncertainty) are associated with lower values of $r^e_{B2}$.

**Proof.** Applying part (ii) of Proposition 9 to equations (39) and (40) reveals that

$$\frac{dr^E_{B2}}{d\pi_0} < 0.$$

Applying similarly part (ii) of Proposition 9 to equations (41) and (42) gives

$$\frac{dr^C_{B2}}{d\pi_0} < 0.$$

Using equations (49) and (50) in Equation (43) gives

$$\frac{dr^e_{B2}}{d\pi_0} < 0.$$
The following proposition summarizes the impacts of \( \pi_0 \) on the interest rates and on the volume of credit in period 0.

**Proposition 10:** Provided the direct impact of \( \pi_0 \) on \( z_{L1}^* \) is sufficiently large in comparison to the absolute value of the (negative) impact of \( \pi_0 \) on \( r_{B2}^e \), and given the conditions in propositions 4 and 8, a higher \( \pi_0 \) is associated with

(i) Lower levels of \( r_{B1} \) and of \( r_{L2} \);

(ii) Overall larger levels of credit by lenders to financial intermediaries and by intermediaries to borrowers (larger values of \( L_B^* \) and of \( L_F^* \)).

**Proof.** We start by showing that the amount of leverage demanded by B’s is an decreasing function of \( r_{B2}^e \). Partially differentiating Equation (18) with respect to \( r_{B2}^e \)

\[
\frac{\partial L_B^e}{\partial r_{B2}^e} = - \frac{r_{B1}^e u_B}{\left[r_{B2}^e (1 + r_{B1}) - \mu_B - r_{B2}^e R_A\right]^2} < 0
\]

implying that Bs demand for leverage is a decreasing function of \( r_{B2}^e \): Inspection of Equation (18) also reveals that

\[
\frac{\partial L_B^e}{\partial r_{B1}} < 0.
\]

Note that, given \( r_{B2}^e \), equations (37) and (38) determine \( r_{B1} \) and \( r_{L1} \). Totally differentiating those equations with respect to \( \pi_0 \) yields

\[
M_B \left[ \frac{\partial L_B^e}{\partial r_{B1}} \frac{dr_{B1}}{d\pi_0} + \frac{\partial L_B^e}{\partial r_{B2}^e} \frac{dr_{B2}^e}{d\pi_0} \right] = M_F \left[ \frac{\partial L_F^e}{\partial r_{B1}} \frac{dr_{B1}}{d\pi_0} + \frac{\partial L_F^e}{\partial r_{L1}} \frac{dr_{L1}}{d\pi_0} \right]
\]

and

\[
M_F \left[ \frac{\partial L_F^e}{\partial r_{B1}} \frac{dr_{B1}}{d\pi_0} + \frac{\partial L_F^e}{\partial r_{L1}} \frac{dr_{L1}}{d\pi_0} \right] = M_L \left[ \frac{\partial z_{L1}^*}{\partial r_{L1}} \frac{dr_{L1}}{d\pi_0} + \frac{\partial z_{L1}^*}{\partial \pi_0} \right].
\]

Those two equations implicitly determine \( \frac{dr_{B1}}{d\pi_0} \) and \( \frac{dr_{L1}}{d\pi_0} \). The explicit solutions are

\[
\frac{dr_{B1}}{d\pi_0} = - \frac{M_B \frac{\partial L_B^e}{\partial r_{B2}^e} \frac{dr_{B2}^e}{d\pi_0} \left[ M_F \frac{\partial L_F^e}{\partial r_{L1}} - M_L \frac{\partial z_{L1}^*}{\partial r_{L1}} \right] + M_F M_L \frac{\partial L_F^e}{\partial r_{L1}} \frac{dr_{L1}}{d\pi_0}}{M_B M_L \frac{\partial L_B^e}{\partial r_{B1}} \frac{dr_{B1}}{d\pi_0}}
\]

and

\[
\frac{dr_{L1}}{d\pi_0} = \frac{M_B M_L \frac{\partial L_B^e}{\partial r_{B1}} \frac{dr_{B1}}{d\pi_0} - M_F \frac{\partial L_F^e}{\partial r_{B1}} \left[ M_L \frac{\partial z_{L1}^*}{\partial \pi_0} - M_B \frac{\partial L_B^e}{\partial r_{B2}^e} \frac{dr_{B2}^e}{d\pi_0} \right]}{M_B M_L \frac{\partial L_B^e}{\partial r_{B1}} \frac{dr_{B1}}{d\pi_0}}.
\]
where
\[ D \equiv M_B \frac{\partial L^*_B}{\partial r_{B1}} M_F \frac{\partial L^*_F}{\partial r_{L1}} - M_B \frac{\partial L^*_B}{\partial r_{B1}} M_L \frac{\partial z^*_L}{\partial r_{L1}} + M_F \frac{\partial L^*_F}{\partial r_{B1}} M_L \frac{\partial z^*_L}{\partial r_{L1}} > 0. \] (58)

The positive sign of \( D \) follows from Proposition 4, part (ii) of Proposition 8 and equation (53). Along with Lemma 2 these facts also imply that: (i) The first term in the numerator of Equation (56) is positive and the second one negative, (ii) the first term in the numerator of Equation (57) is negative and the second one positive. Since the direct positive impact of \( \pi_0 \) on \( z^*_L \) is large in comparison to the absolute value of the (negative) impact of \( \pi_0 \) on \( r_L \)

\[ \frac{\partial z^*_L}{\partial \pi_0} >> \frac{dr^*_B}{d\pi_0}, \]

the negative expressions in each of equations (56) and (57) dominate the positive expressions implying that \( \frac{dr_B}{d\pi_0} \) and \( \frac{dr_L}{d\pi_0} \) are both negative. This establishes part (i) of the proposition.

To prove part (ii) note that since \( \frac{\partial L^*_B}{\partial r_{B2}} \frac{dr^*_B}{d\pi_0} > 0, \frac{\partial L^*_B}{\partial r_{B1}} < 0 \) and \( \frac{dr_B}{d\pi_0} < 0 \), equilibrium demand for loans by Bs is larger the higher is \( \pi_0 \). Hence, the left hand side of Equation (54) is positive implying that the right hand side is positive too. Consequently, equilibrium demand for loans by Fs is higher too when \( \pi_0 \) is larger. The upshot is that a higher level of \( \pi_0 \) is associated with larger volumes of credit to both intermediaries and borrowers. ■

The results of Proposition 10 arise through several interconnected channels. By part (i) of Proposition 8 perceptions of more generous bailout policy directly raises the fraction of their portfolio that lenders desire to invest in risky loans to financial intermediaries. This effect exerts a downward pressure on \( r_L \) and, via the reaction of Fs, also on \( r_B \). But the higher level of \( \pi_0 \) also reduces the (model consistent) borrowing rate expected to prevail in period 1 (\( r^*_B \)) and this raises period’s 0 demand for credit by borrowers. This effect creates upward pressures on both \( r_L \) and \( r_B \). Since both the demand for and the supply of funds to borrowers go up, the general equilibrium impact of a higher \( \pi_0 \) on \( r_B \) is generally ambiguous. But, under the conditions of Proposition 10 the downward impact on interest rates dominates leading to a general credit expansion.

Clearly, the belief that government may repay the debt of some delinquent financial intermediaries creates a moral hazard problem. An important implication of Proposition 10
is that, by raising leverage in the economy, the perception of a more generous bailout policy aggravates this problem and increases the likelihood of a financial crisis in period 1.

7.2 The impact of a temporary expansionary monetary policy (lower \( r_f \) in period 0)

Within the context of the model a temporary expansionary monetary policy in period 0, policy takes the form of a decrease in \( r_f \) holding the borrowing rate expected for period 1, \( r_{B2}^{e} \), constant. The following proposition summarizes the impact of such a policy.

**Proposition 11:** Given the conditions in propositions 4 and 8, a temporary decrease in \( r_f \) leads to a decrease in both \( r_{B1} \) and \( r_{L1} \) and to an increase in leverage within both the financial and the real sectors (both \( L_B^* \) and \( L_F^* \) go up).

**Proof.** Totally differentiating equations (37) and (38) with respect to \( r_f \) yields

\[
M_B \left[ \frac{\partial L_B^*}{\partial r_{B1}} \frac{dr_{B1}}{dr_f} \right] = M_F \left[ \frac{\partial L_F^*}{\partial r_{B1}} \frac{dr_{B1}}{dr_f} + \frac{\partial L_F^*}{\partial r_{L1}} \frac{dr_{L1}}{dr_f} \right], \tag{59}
\]

\[
M_F \left[ \frac{\partial L_F^*}{\partial r_{B1}} \frac{dr_{B1}}{dr_f} + \frac{\partial L_F^*}{\partial r_{L1}} \frac{dr_{L1}}{dr_f} \right] = M_L \left[ \frac{\partial z_{L1}^*}{\partial r_{L1}} \frac{dr_{L1}}{dr_f} + \frac{\partial z_{L1}^*}{\partial r_f} \right]. \tag{60}
\]

Those two equations implicitly determine \( \frac{dr_{B1}}{dr_f} \) and \( \frac{dr_{L1}}{dr_f} \). Solving for those changes

\[
\frac{dr_{B1}}{dr_f} = - \frac{M_F M_L \frac{\partial L_F^*}{\partial r_{B1}}}{D} \frac{1}{\text{Var}(\{r_f\})} \frac{1}{\alpha} > 0, \tag{61}
\]

\[
\frac{dr_{L1}}{dr_f} = - \left[ M_B \frac{\partial L_B^*}{\partial r_{B1}} - M_F \frac{\partial L_F^*}{\partial r_{B1}} \right] \frac{M_L}{D} \frac{1}{\text{Var}(\{r_f\})} \frac{1}{\alpha} > 0. \tag{62}
\]

where \( D \) is given in Equation (58) and is positive. The conditions in propositions 4 and 8 imply that the numerators of equations (61) and (62) are both positive establishing that both \( r_{B1} \) and \( r_{L1} \) go down when \( r_f \) decreases. Since \( \frac{dr_{B1}}{dr_f} > 0 \) and \( \frac{\partial L_B^*}{\partial r_{B1}} < 0 \) the left hand side of Equation (59) is negative implying that its right hand side is negative as well. But the left hand and right hand sides of Equation (59) represent, respectively, the changes in \( L_B^* \) and in \( L_F^* \) as a result of an increase in \( r_f \). It follows that when \( r_f \) decreases, both \( L_B^* \) and \( L_F^* \) go up. \( \blacksquare \)
Interpretation: The subprime crisis equivalent of period 0 of the model can be thought of as the buildup phase of the crisis. During this phase market participants believed that the set of bailout probabilities with positive mass is concentrated in a range with high values of $p$. In addition, monetary policy was loose by historical standards. Propositions 10 and 11 imply that both factors contributed to the ex ante expansion of credit and to a real investment boom, making the system more fragile to a sudden downward revision of perceptions about the likelihood of governmental bailouts.

8 Concluding remarks

A major result of our analysis is that the larger the change in bailout uncertainty the stronger the pre-crisis buildup and the deeper the ensuing crisis. The detailed mechanics of this result can be appreciated by thinking of period 0 as the pre-crisis phase during which the worst scenario perceived likelihood of bailout is high and monetary policy relatively loose leading to credit expansion and to an investment boom. Taylor (2009) argues that loose monetary policy caused, prolonged and worsened the financial crisis. Period 1 can be thought of as the phase in which, due to the arrival of some major public signal — like not rescuing Lehman — financial market operators adjust their worst scenario perceptions about the likelihood of bailout downward. By Proposition 9 this adjustment induces a general increase in market interest rates, a rise in the proportion of insolvent borrowers along with the destruction of real investments and, for some realizations of real returns, a complete drying up of short term credit markets.

By Proposition 10 the pre-crisis bubbly credit boom is larger the larger $\pi_0$. By Proposition 10 the magnitudes of deleveraging and of insolvencies (real and financial) is larger the lower is $\pi_1$. Since it measures the extent to which the set of possible bailout distributions widened between periods 0 and 1 the difference $\pi_0 - \pi_1$ is a natural proxy for the increase in bailout uncertainty. Combining this proxy with Propositions 9 and 10 yields the conclusion that higher changes in bailout uncertainty are associated with larger pre-crisis bubbles as well as with higher levels of insolvencies and destruction of real economic activity when
the bubble bursts. The crucial variable through which those effects operate is leverage. It expands more during periods of optimism about the likelihood of bailouts but, by the same token, it shrinks more violently during periods of pessimism about the likelihood of bailouts. Given $\pi_1$ the deleveraging process during period 1 involves a larger volume of insolvencies the larger is $\pi_0$. The reason is that a larger $\pi_0$ raises the exante leverage buildup in comparison to what market operators would have engaged in, had they known already in period 0 that the probability of bailout in period 1 will drop to $\pi_1$. The larger this ”excessive” credit buildup, the larger the expost volume of insolvencies in the real economy.

Somewhat analogously to Diamond and Dybvig (1983) (DD in the sequel) classic model of bank runs, a main objective of this paper was to identify circumstances that trigger a financial crises. A main result of the DD framework is that deposit insurance eliminates runs on the banks. Although there is an analogy between the role of deposit insurance in DD and bailouts in our framework, a crucial difference between these frameworks is that, up to a given limit, deposit insurance is backed by the exante certainty of a legal act while the availability (and scope) of the generalized bailouts considered here is shrouded in uncertainty and is likely to remain in this state also in the future. Besides other obvious differences two additional difference worth emphasizing are: (i) In DD liquidity shocks are exogenous while here they are related to an increase in uncertainty due to the arrival of new information about ”black swan” events. (ii) Our framework is designed to make statements about the impact of monetary policy on leverage and the economy.

Reinhart and Rogoff (2009) present broad evidence supporting the view that private financial crises are often followed by substantial reductions in tax collections and defaults on sovereign debt. Motivated by this findings and some of the results in this paper we speculate in what follows on an additional channel through which higher exante leverage buildups possibly makes the economy more crisis prone when new information arrives. Higher leverage raises the probability as well as the magnitude of potential defaults, and with it the cost of potential bailouts. The more costly is a bailout to tax payers the more reluctant is government to engage in such bailouts. As a consequence, the higher is leverage, the higher bailout uncertainty making beliefs more sensitive to news.
The punch line is that the sensitivity of expectations to various news becomes larger the larger is leverage. In terms of the Gilboa-Schmeidler (1989) uncertainty framework this means that the range of bailout probability distributions entertained by individuals becomes more sensitive to news. As a consequence, the same pessimistic new information about the likelihood of bailout is more likely to puncture a bubble the higher is leverage.
References


A Appendix

A.1 Proof of Proposition 1

A.1.1 Probability of default in period 1

The definition of $L_{B3}$ in Equation (14) implies that $W_B(R_A, \mu_B; L_{B3}) \geq 0$. Since $L_{B4} < L_{B3}$, $W_B(R_A, \mu_B; L_B) > 0$. In conjunction with Assumption 1 and the inequality $W_B(0, R_A + R_I; L_{B4}) < 0$ from part (ii) of Proposition 1, this implies that

$$\Pr[D_1 \mid L_{B3}] = \Pr[D_1 \mid L_{B4}] = (1 - q_A)(1 - q_I),$$

where $\Pr[D_1 \mid L_{Bj}]$, $j = 3, 4$ is the probability of default in period 1 when leverage is equal to $L_{Bj}$.

A.1.2 Probability of default in period 2

Inspection of the expressions for $K(\cdot)$, $W_B(\cdot)$ and $A(\cdot)$ in equations (9), (12) and (13) suggests that they all are increasing functions of $\tilde{R}_{B1}$ and of $\tilde{R}_{B2}$. Hence, given $\tilde{R}_{Bi}$, $i = 1, 2$, $K(\cdot)$ is larger the larger is $\tilde{R}_{Bj}$, $j \neq i$. In conjunction with condition (i) of the proposition this implies that $K(\cdot)$ is positive for all combinations of $\tilde{R}_{B1}$ and of $\tilde{R}_{B2}$ above the mean, as well as for $(R_A, R_A + R_I)$ and $(R_A, R_I)$ and negative otherwise. Consequently $W_B(\cdot; L_{B3}) > W_B(\cdot; L_{B4})$ for all $(\tilde{R}_{B1}, \tilde{R}_{B2})$ pairs in the first set and $W_B(\cdot; L_{B3}) < W_B(\cdot; L_{B4})$ in the complementary set.

Somewhat similarly, given $\tilde{R}_{Bi}$, $i = 1, 2$, $W_B(\cdot)$ is larger the larger is $\tilde{R}_{Bj}$, $j \neq i$. Hence, given $\tilde{R}_{Bi}$, $i = 1, 2$, $W_B(\cdot)$ is larger the larger is $\tilde{R}_{Bj}$, $j \neq i$. In conjunction with condition (ii) of the proposition this implies that $W_B(\cdot; L_{B3})$ and $W_B(\cdot; L_{B4})$ are positive for all combinations of $\tilde{R}_{B1}$ and of $\tilde{R}_{B2}$ above the mean, as well as for $(R_A, R_A + R_I)$ and $(R_A, R_I)$, and negative for all $(\tilde{R}_{B1}, \tilde{R}_{B2})$ pairs in which $\tilde{R}_{B1} = 0$. Hence, for both $L_{B3}$ and $L_{B4}$ B is solvent in period 2 for the first set of return realizations and insolvent for the second set. For the two remaining pairs, $(R_A, R_A)$ and $(R_A, 0)$, $W_B(\cdot; L_{B3}) < 0$ and $W_B(\cdot; L_{B4}) > 0$ implying that, for those realizations of returns B is solvent under $L_{B4}$ and insolvent under $L_{B3}$. In
conjunction with equation (5) and the definition of $L_{B3}$ in equation (14) those observations imply that

$$\Pr[D_2 \mid L_{B3}] = [1 - (1 - q_A)(1 - q_I)] q_A(1 - q_I)^2,$$

$$\Pr[D_2 \mid L_{B4}] = 0,$$

where $\Pr[D_2 \mid L_{Bj}], j = 3, 4$ is the probability of default in period 2 when leverage is equal to $L_{Bj}$.

A.1.3 Total default probability

Since the distribution of returns is independent across the two periods, from the vantage point of period 0, the full default probabilities, $\Pr[D \mid L_{Bj}], j = 3, 4$, are

$$\Pr[D \mid L_{B3}] = \Pr[D_1 \mid L_{B3}] + \Pr[D_2 \mid L_{B3}]$$

$$= (1 - q_I) [(1 - q_A) + (1 - (1 - q_A)(1 - q_I)) q_A(1 - q_I)]$$

and

$$\Pr[D \mid L_{B4}] = \Pr[D_1 \mid L_{B4}] + \Pr[D_2 \mid L_{B4}]$$

$$= (1 - q_A)(1 - q_I).$$

A.1.4 Derivation of the conditions which increase the likelihood that the optimal level of leverage is $L_{B3}$

As explained in the text the optimal level of leverage is $L_{B3}$ iff

$$Eu[W_B(\cdot; L_{B3})] > Eu[W_B(\cdot; L_{B4})].$$

Inserting the definition of $W_B(\widetilde{R}_{B1}, \widetilde{R}_{B2}; L_B)$ from Equation (12) into Equation (1) and rearranging it can be shown (after some tedious algebra) that the condition above is equivalent to

$$(L_{B3} - L_{B4}) \sum_{i=2}^{1} \sum_{j=2}^{1} K(R^i, R^j) q_i q_j + q_3 M > (1 - q_4)q_3(q_3 + q_4)P, \quad (100)$$

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where
\[
M \equiv (L_{B3} - L_{B4}) \sum_{j=2}^{3} K(R^3_j, R^j) q_j - \sum_{j=4}^{3} [A(R^3_j, R^j) + K(R^3_j, R^j)L_{B4}] q_j,
\]

and where, to keep the expressions within manageable size the more compact notation, \( R^i \) and \( q_i, i = 1, ..., 4 \) (defined in Equation (5)), is used.

Inspection of the inequality in (100) reveals that it is more likely to be satisfied the larger the positive difference, \( (L_{B3} - L_{B4}) \). But this difference is an increasing function of \( R_A \) implying that the inequality is more likely to be satisfied the larger is \( R_A \). Using the definitions of of \( q_i, i = 1, ..., 4 \) in terms of \( q_A \) and \( q_I \) from Equation (5) it can be shown that, provided \( q_I > \frac{1}{2} \), the term on the RHS of (100) is decreasing in \( q_I \). Hence, the larger \( q_I \), the more likely it is that \( L_{B3} \) is preferred to \( L_{B4} \). When \( q_I = 1, q_3 = 0 \) the condition in (100) reduces to
\[
(L_{B3} - L_{B4}) \sum_{i=2}^{3} \sum_{j=2}^{3} K(R^i_j, R^j) q_i q_j > 0,
\]

which is unambiguously the case since \( (L_{B3} - L_{B4}) > 0 \) and \( K(R^i_j, R^j) > 0 \) for \( i, j \geq 2 \).

QED

A.2 Derivation of optimal F’s leverage in Proposition 3

Given that \( z_f = 1 \), and noting Table 2, the 3 possible values that can be assumed by \( W_F(\tilde{r}_B, L_F) \) are given in the last column of the following table

<table>
<thead>
<tr>
<th>Yield-( \tilde{r}_B )</th>
<th>( W_F(\tilde{r}_B, L_F) = (1 + L_F)\tilde{r}_B - r_L L_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>The two borrowers are solvent</td>
<td>( r_B )</td>
</tr>
<tr>
<td>One borrower defaults and one is solvent</td>
<td>( \frac{1}{2}r_B )</td>
</tr>
<tr>
<td>Both borrowers default</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Financial intermediaries’ wealth
Substituting the possible values for $W_F(\tilde{r}_B, L_F)$ from this table along with the corresponding probabilities from Table 2 into Equation (19) and rearranging

$$Eu[W_F(\tilde{r}_B, L_F)] = \left[(\gamma_F^1 + 0.5\gamma_F^2)r_B - r_L\right]L_F - \frac{b}{2}E[W_F(\tilde{r}_B, L_F)]^2,$$  

(101)

where

$$E[W_F(\tilde{r}_B, L_F)]^2 = \gamma_F^1 [r_B + (r_B - r_L)L_F]^2 +$$

$$\gamma_F^2 [0.5r_B + (0.5r_B - r_L)L_F]^2 + (1 - \gamma_F^1 - \gamma_F^2)(r_GL_F)^2.$$  

Equation (21) in the text is obtained by taking the first derivative of Equation (101) with respect to $L_F$ and by rearranging. The second derivative of (101) with respect to $L_F$ is

$$-b \left\{ \gamma_F^1 (r_B - r_L)^2 + \gamma_F^2 (0.5r_B - r_L)^2 + (1 - \gamma_F^1 - \gamma_F^2)(r_GL_F)^2 \right\}.$$  

Since all the expressions within the curly brackets are positive the entire expression is negative implying that $L_F^*$ in Proposition 4 is the maximizer of the expected utility in equation (101). QED

A.3 Proof of Proposition 4

The condition that the marginal utility from $W_F$ is positive at twice the value of $W_F$ in the full solvency state implies

$$1 > 2b [r_B + (r_B - r_L)L_F^*].$$  

(102)

By the implicit function theorem

$$\frac{dL_F^*}{dr_j} = -\frac{\partial FOC}{\partial r_j}, \quad j = L, B,$$

where $FOC$ and $SOC$ are the first and second order conditions for the maximization of F’s expected utility. Since $SOC < 0$ the sign of $\frac{dL_F^*}{dr_j}$ is the same as the sign of $\frac{\partial FOC}{\partial r_j}$. To find the signs of $\frac{dL_F^*}{dr_j}$, $j = L, B$ it suffices, therefore, to establish the signs of $\frac{\partial FOC}{\partial r_j}$ for $j = L, B$. Differentiating $FOC$ with respect to $r_L$ and rearranging yields

$$\frac{\partial FOC}{\partial r_L} = -\left[1 + 2(1 - \gamma_F^1 - \gamma_F^2)r_GL_F^* \right]$$

$$+ 2b \left\{ \gamma_F^1 \left[ \frac{r_B}{2} + (r_B - r_L)L_F^* \right] + \gamma_F^2 \left[ \frac{r_B}{4} + (\frac{r_B}{2} - r_L)L_F^* \right] \right\}.$$  

(103)
The last term on the right hand side of this expression is a weighted average, with weights whose sum is smaller than one, of two terms each of which is smaller than the right hand side of Equation (102). Since the right hand side of (102) is smaller than one so is the last term on the right hand side of (103). Hence $\frac{\partial FOC}{\partial r_L} < 0$ establishing that $\frac{dL^*_F}{dr_L} < 0$.

Similarly,

$$\frac{\partial FOC}{\partial r_B} = \gamma^1_F [1 + br_L - 2b (r_B + (r_B - r_L)L^*_F)] + \frac{\gamma^2_F}{2} \left[ 1 + \frac{1}{2} br_L - 2b (r_B + (r_B - 2r_L)L^*_F) \right].$$

(104)

Equation (102) implies

$$1 + br_L - 2b (r_B + (r_B - r_L)L^*_F) > 0$$
$$1 + \frac{1}{2} br_L - 2b (r_B + (r_B - 2r_L)L^*_F) > 0.$$

But the right hand side of Equation (104) is a weighted average, with weights whose sum is smaller than one, of those two positive terms. Consequently, $\frac{\partial FOC}{\partial r_B} > 0$ establishing that $\frac{dL^*_F}{dr_B} > 0$. QED

A.4 Derivation of the variance of a typical lender’s fully diversified portfolio of loans to financial intermediaries (Proposition 6)

Calculation of the variance utilizes the fact that the variance of a fully diversified risky portfolio composed of identically distributed assets is equal to the covariance between any two assets in the portfolio. Calculation of this covariance is based on the joint probability distribution of any two risky loans to Fs within the lender’s portfolio. This distribution contains the following four possible realizations of $\tilde{r}_L$ across the two assets: $(r_L, r_L)$, $(r_L, 0)$, $(0, r_L)$ and $(0, 0)$. In the first case L’s return is $r_L$, in the second and the third it is $\frac{1}{2} r_L$, and in the last one it is zero.

The probability of each of those events depends on the probability that each of the two financial intermediaries considered services or does not service his debt. Whether a single F services or does not service the debt depends on whether both of his borrowers are solvent.
or insolvent. By proposition 5 a financial intermediary fully services his own debt to Ls in
the first case and defaults in the second. Consequently, and provided the perceived minimal
probability of bailout (π), is zero, the probability distribution of the states above ultimately
depends only on \( q_A \) and \( q_I \). In addition, when \( \pi > 0 \) it also depends on \( \pi \).

The following table summarizes this distribution

<table>
<thead>
<tr>
<th>( \tilde{r}_{Li} )</th>
<th>( \tilde{r}_{Lj} )</th>
<th>Probability of ( \tilde{r}<em>{Li} \cap \tilde{r}</em>{Lj} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_L )</td>
<td>( r_L )</td>
<td>( \delta_S + \delta_P \pi + \delta_D \pi \equiv \gamma^1_L )</td>
</tr>
<tr>
<td>( r_L )</td>
<td>0</td>
<td>( (1 - \pi) \left( \frac{1}{2} \delta_P + \delta_D \pi \right) \equiv \frac{1}{2} \gamma^2_L )</td>
</tr>
<tr>
<td>0</td>
<td>( r_L )</td>
<td>( (1 - \pi) \left( \frac{1}{2} \delta_P + \delta_D \pi \right) \equiv \frac{1}{2} \gamma^2_L )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( \delta_D (1 - \pi)^2 \equiv \gamma^3_L )</td>
</tr>
</tbody>
</table>

where \( \tilde{r}_{Li} \) and \( \tilde{r}_{Lj} \) denote the returns from lending to financial intermediaries \( i \) and \( j \) respectively,

\[
\delta_S \equiv q_A \left[ q^4_i + 4q^3_i (1 - q_I) + 6q^2_i (1 - q_I)^2 + 4q_I (1 - q_I)^3 + (1 - q_I)^4 \right] + (1 - q_A) q^4_i \\
\delta_P \equiv (1 - q_A) \left[ 2q^2_I (1 - q_I)^2 + 4q^3_I (1 - q_I) \right] \\
\delta_D \equiv (1 - q_A) \left[ (1 - q_I)^4 + 4q_I (1 - q_I)^3 + 4q^2_I (1 - q_I)^2 \right],
\]

and

\[ \gamma^1_L + \gamma^2_L + \gamma^3_L = 1. \]

The expression for L’s portfolio variance in Proposition 7 is obtained by using the joint
distribution of \( (\tilde{r}_{Li}, \tilde{r}_{Lj}) \) from Equation (105) in the definition of the covariance between \( \tilde{r}_{Li} \)
and \( \tilde{r}_{Lj} \) and by simplifying using the Mathematica software. QED

A.5 Proof of Proposition 7

Recall that, to keep notation simple, we use the symbol \( \tilde{r}_L \) to denote the return on a portfolio
that consist of an infinite number of loans, \( \{\tilde{r}_L\} \). A typical lender’s maximization problem
is given by

\[
\max_{z_L} E \left[ u \left( z_L (\tilde{r}_L - r_f) + r_f \right) \right],
\]
where \( u(\cdot) \) stands for the utility function. The first order condition implies

\[
E[u'(z^*_L (\bar{r}_L - r_f) + r_f) (\bar{r}_L - r_f)] = 0.
\]

Taking a second order Taylor approximation of \( \bar{r}_L \) around \( r_f \) yields

\[
E[u'(z^*_L (r_f - r_f) + r_f) (\bar{r}_L - r_f) + u''(z^*_L (r_f - r_f) + r_f) (\bar{r}_L - r_f) (\bar{r}_L - r_f) z^*_L] \approx 0.
\]

For a sufficiently small risk premium

\[
z^*_L \approx -\frac{u'(r_f) (E\bar{r}_L - r_f)}{u''(r_f) E [(\bar{r}_L - r_f)^2]} = \frac{u'(r_f)}{u''(r_f)} \left( E\bar{r}_L - r_f \right),
\]

but for constant absolute risk aversion, \( u(x) = -e^{-\alpha x} \), the coefficient of absolute risk aversion is \( -\frac{u''(r_f)}{u'(r_f)} = \alpha \), and thus

\[
z^*_L \approx \frac{E (\{\bar{r}_L\}) - r_f}{\alpha Var (\{\bar{r}_L\})}.
\]

QED