I. Case where Dual Channel Has Positive Sales in Both Channels:

For a bargaining channel to have positive sales, at least one consumer should reach an agreement in bargaining with the salesperson, which requires, according to the Nash bargaining model, a nonnegative gain for both bargainers. Formally, it requires that there exist a pair \( \{V, hc\} \), where \( V \in [0, 1] \) and \( hc \in \{hc_i, hc_h\} \), such that \( V - p^b - hc - D_c \geq 0 \) and \( B(p^b - M) \geq 0 \).

Substituting equation Error! Reference source not found. to \( p^b \), the following must hold: (1) there exists a \( hc \), where \( hc \in \{hc_i, hc_h\} \), such that \( p^f - hc - M \geq 0 \), or (2) there exists a pair \( \{V, hc\} \), where \( V \in [hc + U_c, p^f + U_c] \) and \( hc \in \{hc_i, hc_h\} \), such that \( V - hc - U_c - M \geq 0 \).

Simplifying the above, it requires that there exists a \( hc \), where \( hc \in \{hc_i, hc_h\} \), such that \( p^f - M \geq hc \). In this case, consumers with sufficiently high valuation, \( V \in [hc + U_c + M, 1] \), will buy in the bargaining channel, and other consumers will choose the outside option. However, no one buys at the fixed price.

Then, for the fixed-price channel to have positive sales, there should be a \( hc \) such that \( p^f \leq hc + M \). In this case, consumers with sufficiently high valuation, \( V \in [p^f + U_c, 1] \), will buy in the fixed-price channel, while others will choose the outside option, and no one reaches an agreement in the bargaining channel.

In summary, receiving positive sales in the bargaining channel requires \( p^f - M \geq hc \), and in the fixed-price channel it requires \( p^f \leq hc + M \). Obviously they can’t hold together if \( hc_i = hc_h \).

Therefore, for both the fixed-price channel and bargaining channel to have positive sales, two
conditions must be satisfied: (a) \(0 < \beta < 1\) so that \(hc \in \{hc_i, hc_h\}\), and (b) the prices should satisfy the following condition: \(hc_i \leq p^f - M \leq hc_h\). Condition (a) states that, for the dual channel to have positive sales in both channels, there must be two types of consumers, each with different haggling costs. The bargaining channel gets low haggling-cost consumers with valuation satisfying \(V \in [hc_i + U_c + M, 1]\), and the fixed-price channel gets high haggling-cost consumers with valuation satisfying \(V \in [p^f + U_c, 1]\). Condition (b) suggests that, for both channels to generate demand, the difference between the no-haggle fixed price, \(p^f_{dual}\), and the minimum acceptable price, \(M_{dual}\), should be within a certain range. This means that not only must \(M_{dual}\) be lower than the fixed price, \(p^f_{dual}\), to compensate for consumers’ haggling cost in the bargaining channel, it also requires that \(M_{dual}\) be low enough but not too low such that it compensates only for low haggling-cost consumers, forcing those with the higher cost to buy from the no-haggle channel, thereby ensuring that both channels generate positive sales.

II. Optimization Problems and Solutions under the Three Channel Structures

We derive below the optimization problems and solutions under the three channel structures. All notation is the same as defined in the text. Without the loss of generality, we assume that \(U_c = 0, hc_i = 0\) and \(hc_h < 1 - C\).

(1) The Bargaining-Only Channel

Consumers can either buy a product, at a bargained price, or use an outside option, with the utility function defined as below:

\[
U = \begin{cases} 
V - p^b - hc & \text{if the product is bought at the negotiated price; } hc \in \{0, hc_h\} \\
0 & \text{if the outside option is exercised}
\end{cases}
\]
where $p^b$ maximizes the following expression, according to the Nash axiomatic approach

$$\max_{p^b} \ [V - p^b - hc - D_c]^{-\alpha} \times [B(p^b - M) - D_s]^\alpha$$  

(A2)

where $D_c = 0$, $D_s = 0$. This leads to

$$p^b = M + \alpha (V - hc - M)$$  

(A3)

It can be easily derived that the high-haggling cost consumers with $V \in [M + hc_h, 1]$ and the low-haggling cost consumers with $V \in [M, 1]$ will buy the car at the bargained price. The salesperson receives a payment specified as:

$$\pi^z = B \left[ \beta \int_{hc_h + M}^1 \alpha (V - hc_h - M) dV + (1 - \beta) \int_M^1 \alpha (V - M) dV \right]$$  

(A4)

The objective problem for the seller is to maximize the profit, while ensuring that the salesperson is to receive a minimum payoff, $U_s$, from the job.

$$\max_{(B, M)} \pi^D = \pi^b - \pi^s$$

$$= \beta \int_{hc_h + M}^1 [(1 - B) \alpha (V - hc_h - M) + M - C] dV + (1 - \beta) \int_M^1 [(1 - B) \alpha (V - M) + M - C] dV$$

$$= (M - C)(1 - M - \beta hc_h) + \frac{(1 - \beta) \alpha}{2} \left[ (1 - M)^2 - 2 hc_h (1 - M) + \beta hc_h^2 \right]$$

Subject to

$$\pi^s \geq U_s$$  

(A6)

Let $\lambda$ be the Lagrange multiplier corresponding to (A6). The Lagrangian $L$ for the problem (A5) -(A6) is:

$$L(B, M, \lambda) = \pi + \lambda \left( -U_s + \pi^s \right)$$  

(A7)

As there is complete information, the seller shall extract all the economic rent from the salesperson, which suggests that $\pi^s = U_s$. 


Solving the first-order condition $\partial L / \partial M = 0$ gives

$$M^* = \frac{1}{2-\alpha} \left[ C + \alpha \right]$$  \hspace{1cm} (A8)

Then, the optimal quantity and profit are

$$Q^* = \frac{1}{2-\alpha} \left[ 1 - \beta \right]$$  \hspace{1cm} (A9)

$$\pi^* = \frac{1}{(2-\alpha)^2} \left[ (1-C)^2 - 2\beta \right] - U_S$$  \hspace{1cm} (A10)

The consumer surplus can be written as follows

$$W^* = \beta \int_{h_{c_0}}^{M} \left[ (1-\alpha) \left( V - M^* - h \right) \right] dV + (1-\beta) \int_{h}^{M} \left[ (1-\alpha) \left( V - M^* \right) \right] dV$$

$$= \frac{1-\alpha}{2(2-\alpha)^2} \left[ (1-C)^2 - 2(1-C) \beta h + \beta h^2 (4-\alpha(1-\beta)(4-\alpha)-3\beta) \right]$$  \hspace{1cm} (A11)

(2) The Dual Channel

As Appendix I shows, high haggling cost consumers with valuation $V \in [p', 1]$ buy at the fixed price, while low haggling cost consumers with valuation $V \in [M, 1]$ buy at the bargaining price. Following the model description in the text, we rewrite the maximization problem for the seller. Note that the price constraints $h_{c_0} \leq p' - M \leq h_{c_1}$ need to be satisfied:

$$\max_{(p', \beta, M)} \pi^D = \pi^f + \pi^b - \pi^s$$

$$= \beta(p' - C)(1 - p') + (1-\beta) \left[ \int_{p'}^{1} \left( M - C + \alpha \left( p' - M \right) (1-B) \right) dV + \int_{M}^{p'} \left( M - C + \alpha \left( V - M \right) (1-B) \right) dV \right]$$  \hspace{1cm} (A12)

$$= \beta(p' - C) \left[ \frac{(1-B)\alpha}{2} \left( M^2 - p'^2 + 2p' - 2M \right) + (M - C)(1-M) \right]$$
\[ \pi^S \geq U_S \quad (A13) \]
\[ 0 \leq p' - M \quad (A14) \]
\[ p' - M \leq \h c_h \quad (A15) \]

Let \( \lambda_1, \lambda_2, \lambda_3 \) be the Lagrange multipliers corresponding to (A13)-(A15), respectively. The Lagrangian \( L \) for the problem (A12)-(A15) is:

\[
L(p', B, M, \lambda_1, \lambda_2, \lambda_3) = \pi^D + \lambda_1 (-U_S + \pi^S) + \lambda_2 (p' - M) + \lambda_3 (\h c_h - p' + M) \quad (A16)
\]

Also, complete information assumption suggests \( \pi^S = U_S \). The Lagrangian \( L \) can be rewritten as

\[
L'(p', M, \lambda_2, \lambda_3) = \pi^D + \lambda_2 (p' - M) + \lambda_3 (\h c_h - p' + M) \quad (A17)
\]

The first-order conditions are

\[
\frac{\partial L}{\partial p'} = 0 \Rightarrow p' = \frac{\alpha + (1 - \alpha) \beta + \beta C + \lambda_2 - \lambda_3}{\alpha (1 - \beta) + 2 \beta} \quad (A18)
\]
\[
\frac{\partial L}{\partial M} = 0 \Rightarrow M = \frac{(1 - \alpha + C)(1 - \beta) - \lambda_2 + \lambda_3}{(1 - \beta)(2 - \alpha)} \quad (A19)
\]

\[
p' - M \geq 0, \quad \lambda_2 \geq 0, \quad \text{with complementary slackness} \quad (A20)
\]
\[
\h c_h - p' + M \geq 0, \quad \lambda_3 \geq 0, \quad \text{with complementary slackness} \quad (A21)
\]

The above conditions suggest four possible patterns of equations and inequalities. First, we can immediately eliminate the combination that \( p' - M = 0 \) and \( \h c_h - p' + M = 0 \), as \( \h c_h > 0 \).

Second, we consider \( p' - M = 0, \lambda_2 > 0, \h c_h - p' + M > 0, \lambda_3 = 0 \). However, this is also ruled out as \( p' - M = \frac{\alpha (1 - C)(1 - \beta) + 2 \lambda_2}{(2 - \alpha)(1 - \beta)[\alpha (1 - \beta) + 2 \beta]} \neq 0 \).

Third, we consider \( p' - M > 0, \lambda_2 = 0, \h c_h - p' + M > 0, \lambda_3 = 0 \). It gives the solutions below.
\[ p^* = \frac{\alpha + \beta - \alpha \beta + \beta C}{\alpha (1 - \beta) + 2 \beta} \]  
(A22)

\[ M^* = \frac{1 - \alpha + C}{2 - \alpha} \]  
(A23)

\[ Q^* = \frac{(1 - C)(2 \beta + \alpha (1 - 2 \beta))}{(2 - \alpha)(\alpha (1 - \beta) + 2 \beta)} \]  
(A24)

\[ \pi^* = \frac{(1 - C)^2 (\alpha - 2 \alpha \beta + 2 \beta)}{2 (2 - \alpha)(\alpha - \alpha \beta + 2 \beta)} - U_S \]  
(A25)

Note that \( h_C - p^* + M > 0 \) suggests that \( h_C > \frac{\alpha (1 - C)}{(2 - \alpha)(\alpha (1 - \beta) + 2 \beta)} \).

The consumer surplus in this case is

\[ W^* = (1 - \beta) \left( \int_{p^*}^{1} (V - \alpha p^*) - (1 - \alpha)M^* dV + \int_{M^*}^{1} (1 - \alpha)(V - M^*)dV \right) + \beta \int_{p^*}^{1} (V - p^*)dV \]

\[ = \frac{(1 - C)^2 \left[ 4 \beta^2 + 4 \alpha \beta (1 - 2 \beta) + \alpha^2 (1 - 7 \beta + 7 \beta^2) - \alpha^3 (1 - 2 \beta) (1 - \beta) \right]}{2 (2 - \alpha)^2 (\alpha (1 - \beta) + 2 \beta)^2} \]  
(A26)

Last, we consider \( p^* - M > 0, \lambda_2 = 0, h_C - p^* + M = 0, \lambda_3 > 0 \). It gives the solutions below

\[ p^* = \frac{1}{2} \left[ 1 + C - (\alpha - 2)(1 - \beta) h_C \right] \]  
(A27)

\[ M^* = \frac{1}{2} \left[ 1 + C - (\alpha + 2 \beta - \alpha \beta) h_C \right] \]  
(A28)

\[ Q^* = \frac{1}{2} \left( 1 - C + \alpha (1 - \beta) h_C \right) \]  
(A29)

\[ \pi^* = \frac{1}{4} \left[ (1 - C)^2 + 2 \alpha (1 - C)(1 - \beta) h_C - (2 - \alpha)(\alpha - \alpha \beta + 2 \beta)(1 - \beta) h_C^2 \right] - U_S \]  
(A30)
Comparing (A25) and (A30), it can be shown that the profit as expressed in (A25) is always higher. Therefore, the solutions (A27)–(A30) hold only when \( h_c \leq \frac{\alpha (1-C)}{(2-\alpha)(\alpha(1-\beta)+2\beta)} \).

The consumer surplus in this case is

\[
W^* = (1-\beta) \left( \int_{p^*}^{h} (V - \alpha p^* - (1-\alpha)M^*) dV + \int_{M^*}^{p^*} (1-\alpha)(V-M^*) dV \right)
+\beta \int_{p^*}^{h} (V - p^*) dV
= \frac{1}{8} \left[ (1-C)^2 - 2(1-C)\alpha(1-\beta)hc + (1-\beta)(4\alpha(1-2\beta) - 3\alpha^2(1-\beta) + 4\beta)hc^2 \right] \tag{A31}
\]

(3) The Fixed-Price-Only Channel

Consumers can either buy a product at the fixed price or use an outside option, with the utility function defined as below:

\[
U = \begin{cases} 
V - p^f & \text{if the product is bought at the no-haggle fixed price} \\
0 & \text{if the outside option is exercised} 
\end{cases} \tag{A32}
\]

Consumers with sufficiently high valuation, i.e., \( V \in [p^f, 1] \), will buy the product; otherwise they will choose the outside option. Note that consumers’ haggling cost is irrelevant in this case, since buying at the fixed price incurs no haggling cost.

The objective problem for the seller is

\[
\text{Max}_{p^f} \pi = (p^f - C)(1-p^f) \tag{A33}
\]

which yields the optimal price, quantity and profit as follows:

\[
p^f* = \frac{(1+C)}{2} \tag{A34}
\]

\[
Q^* = \frac{(1-C)}{2} \tag{A35}
\]

\[
\pi = \frac{(1-C)^2}{4} \tag{A36}
\]
The consumer surplus is

\[ W^* = \int_{p^*}^{V} (V - p^*) dV = \frac{1}{8} (1 - C)^2 \]  

(A37)

III. Proof of Proposition 1:

**Proof of Proposition 1(a).** The optimal minimum acceptable prices in the dual channel and bargaining-only channel are listed in Appendix II. We compare them with the seller’s cost.

In the bargaining-only channel, \( M_{\text{bargaining\text{-}only}} - C = \frac{(1 - \alpha)(1 - C - \beta h_c)}{2 - \alpha} > 0 \). In the dual channel, when \( h_c > \frac{\alpha(1 - C)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \), \( M_{\text{dual}} - C = \frac{(1 - \alpha)(1 - C)}{2 - \alpha} > 0 \); otherwise

\[ M_{\text{dual}} - C = \frac{1}{2} \left[ 1 - C - (\alpha - \alpha \beta + 2\beta) h_c \right] \geq \frac{(1 - \alpha)(1 - C)}{2 - \alpha} > 0 . \]

In both cases, we have \( \frac{\partial (M - C)}{\partial \alpha} \left( \frac{1 - (1 - \alpha)/(2 - \alpha)}{\partial \alpha} \right) = -\frac{1}{(2 - \alpha)} < 0 \).

**Proof of Proposition 1(b).** When \( h_c > \frac{\alpha(1 - C)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \),

\[ M_{\text{dual}} - M_{\text{bargaining\text{-}only}} = \frac{(1 - \alpha) \beta h_c}{2 - \alpha} > 0 \]. When \( h_c \leq \frac{\alpha(1 - C)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \),

\[ M_{\text{dual}} - M_{\text{bargaining\text{-}only}} = \frac{(1 - C) \alpha - [2\alpha(1 - \beta) + 2\beta - \alpha^2(1 - \beta)] h_c}{2(2 - \alpha)} \geq \frac{\alpha \beta (1 - C)(1 - \alpha)}{(2 - \alpha)^2(\alpha(1 - \beta) + 2\beta)} > 0 . \]

IV. Proof of Proposition 3:

Under the dual-channel strategy, consumers who buy in the bargaining channel all have low haggling cost and their valuation is in the range \( V \in [M_{\text{dual}}; 1] \) (see Error! Reference source not found.). Given that the bargained price is a non-decreasing function with respect to \( V \) (equation...
Error! Reference source not found.), the dispersion of bargained prices \(\text{Disp}_{\text{dual}} = \alpha \left( p_{\text{dual}}' - M_{\text{dual}} \right)\). Under the bargaining-only channel, consumers who buy here include both the high haggling-cost consumers, with \(V \in \left[ h_c + M_{\text{bargaining-only}}, 1 \right]\), and the low haggling-cost consumers, with \(V \in \left[ M_{\text{bargaining-only}}, 1 \right]\). Given that the bargained price is a non-decreasing function in \(V\) (equation (A3)), the dispersion of bargained prices \(\text{Disp}_{\text{bargaining-only}} = \alpha \left( 1 - M_{\text{bargaining-only}} \right)\). We then compare the dispersions under the two channel structures,

\[
\Delta \text{Disp} = \text{Disp}_{\text{dual}} - \text{Disp}_{\text{bargaining-only}} = \alpha \left( p_{\text{dual}}' - M_{\text{dual}} - 1 + M_{\text{bargaining-only}} \right) \quad (A38)
\]

Consider two situations: (1) when \(h_c > \frac{\alpha \left( 1 - C \right)}{(2 - \alpha)\left( \alpha(1 - \beta) + 2\beta \right)}\), equations (A8), (A22) and (A23) give

\[
\Delta \text{Disp} = \frac{-\alpha \beta \left[ (1 - C)(2 - \alpha) + (1 - \alpha)(\alpha(1 - \beta) + 2\beta)h_c \right]}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} < 0, \quad (A39)
\]

(2) when \(h_c < \frac{\alpha \left( 1 - C \right)}{(2 - \alpha)\left( \alpha(1 - \beta) + 2\beta \right)}\), equations (A8), (A27) and (A28) give

\[
\Delta \text{Disp} = \frac{\alpha}{(2 - \alpha)} \left[ -(1 - C)(2 - \alpha - \beta + \alpha \beta)h_c \right] \quad (A40)
\]

As \(h_c < \frac{\alpha \left( 1 - C \right)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} < \frac{1 - C}{2 - \alpha - \beta + \alpha \beta}\), \(\Delta \text{Disp} < 0\).

V. Proof of Proposition 4:

When \(h_c > \frac{\alpha \left( 1 - C \right)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)}\), equations (A22) and (A34) give that
\[ \Delta p^f = p_{\text{dual}}^f - p_{\text{fixed-price-only}}^f = \frac{\alpha(1 - C)(1 - \beta)}{2\alpha(1 - \beta) + 4\beta} > 0. \]

When \( h_c \leq \frac{\alpha(1 - C)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \), equations (A27) and (A34) give that

\[ \Delta p^f = p_{\text{dual}}^f - p_{\text{fixed-price-only}}^f = \frac{1}{2}(1 - \beta)(2 - \alpha)h_c > 0. \]

VI. Demand Comparison:

The demand under the three channel structures is detailed in Appendix II.

Dual channel vs. bargaining-only channel (\( M > C \)). (1) When \( h_c > \frac{\alpha(1 - C)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \),

\[ Q_{\text{bargaining-only}} - Q_{\text{dual}} = \frac{\beta\left[\alpha(1 - C) - (\alpha(1 - \beta)(2\beta)h_c\right]}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \text{. Therefore, } Q_{\text{dual}} < Q_{\text{bargaining-only}} \text{ if } \]

\[ h_c < \frac{\alpha(1 - C)}{\alpha(1 - \beta) + 2\beta} \text{ (or } \beta < \frac{\alpha(1 - C - h_c)}{h_c(2 - \alpha)} \text{.) (2) When } h_c \leq \frac{\alpha(1 - C)}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \text{, } \]

\[ Q_{\text{bargaining-only}} - Q_{\text{dual}} = \frac{\alpha(1 - C) - [(\alpha(1 - \beta)(2\alpha) + 2\beta)h_c\right]}{(2 - \alpha)(\alpha(1 - \beta) + 2\beta)} \text{. Suppose } Q_{\text{dual}} \geq Q_{\text{bargaining-only}} \text{, then } \]

\[ h_c \geq \frac{\alpha(1 - C)}{\alpha(1 - \beta)(2 - \alpha) + 2\beta} \text{. However, this is impossible given the range of } h_c \text{. Therefore, } Q_{\text{dual}} < Q_{\text{bargaining-only}} \text{ must hold. In summary, the demand in the dual channel is lower than that in } \]

the bargaining-only channel if \( h_c < \frac{\alpha(1 - C)}{\alpha(1 - \beta) + 2\beta} \text{ (or } \beta < \frac{\alpha(1 - C - h_c)}{h_c(2 - \alpha)} \text{).} \]

Dual channel vs. bargaining-only channel (\( M = C \)). \( Q_{\text{bargaining-only}} = 1 - C - \beta h_c \text{,} \)

\[ Q_{\text{dual}} = 1 - C - \beta\left(p_{\text{dual}}^f + C\right) \text{. Recall Lemma 1, } p_{\text{dual}}^f \leq h_c + M = h_c + C \text{. Therefore,} \]
\[ Q_{\text{bargaining-only}} \leq Q_{\text{dual}} \]. In other words, when \( M = C \), the total demand in the dual channel is never lower than that in the bargaining-only channel.

**Dual channel vs. fixed-price-only channel.**

\[ Q_{\text{fixed-price-only}} - Q_{\text{dual}} = -\frac{(1-C)\alpha^2(1-\beta)}{2(2-\alpha)(\alpha(1-\beta)+2\beta)} < 0 \]. In other words, the demand in the dual channel is always higher than that in the fixed-price-only channel.