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KEYWORDS: Pricing; Bundling; Co-promotion; Skimming; Price Discrimination
Product Bundling or Reserved Product Pricing?
Price Discrimination with Myopic and Strategic Consumers

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ABSTRACT

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1. INTRODUCTION

Consider the following realistic purchasing scenarios at Amazon.com:

- Scenario 1: “Cracking the SAT, 2013 Edition” (Princeton Review) listed at $15; “Book of Majors 2012” (College Board) listed at $15; price for both (i.e., the bundle) is $28.
- Scenario 2: “Cracking the SAT, 2013 Edition” (Princeton Review) listed at $15; “Book of Majors 2012” (College Board) listed at $15. Consumers can buy both at $30. However, for consumers who bought just one book, Amazon sends a personalized email offering the second at $13.

Which of these two scenarios is optimal for the retailer? This is our study’s guiding question.

Scenario 1 is rooted in bundling, the strategy of offering combinations of products as a package. It is widely used by multi-product sellers and is evident in vacation packages, grocery products, wireless plans, and personal technology. A seller with two products can offer them in their standalone form (in a strategy called pure components), as a bundle (pure bundling), or both (mixed bundling). Scenario 1 corresponds to mixed bundling (MB). MB succeeds by targeting premium priced individual products at consumers who value a specific product only and a discounted bundle at consumers who value both products (Schmalensee 1984).

Scenario 2 is form of co-promotion. We examine a particular variation of co-promotion that we call reserved product pricing (RPP hereafter). As noted in Scenario 2, a seller offering two products sets their initial prices, observes the purchase behavior of alternative segments and, accordingly, offers discounts to segments that purchased one product but not the other in the first stage. That is, the seller holds the discount on product offerings in reserve so that the segment that buys both products at the initial price cannot avail of the discount.

MB and RPP work in distinct ways and their ordering is not apparent \textit{a priori}. The tie-in effect of the bundle helps in the transfer of consumer surplus from one product to the other and benefits the seller through demand gain (Schmalensee 1984). Yet unlike MB, which is inherently
static, RPP has the benefit of gathering additional information about customers in the initial stage and leveraging that in the later stage. Despite this apparent advantage, the appeal of temporally dropping the price is reduced if consumers have rational expectations about the second stage discount and simply delay their purchase till the discount is offered.

Against this backdrop, we formulate an analytical model in which a seller has two products to offer and can adopt pure components (PC), pure bundling (PB), mixed bundling (MB) or reserved product pricing (RPP). The potential consumers are heterogeneous in their reservation prices for each product. Each consumer is either myopic (i.e., unaware of or unable to anticipate the second stage discount) or strategic (i.e., forward looking). Myopic behavior is plausible due to the large number of strategic options available to a seller, and can be explained by bounded rationality (Ellison 2006). We address the following research questions: If the market consists of a mix of myopic and strategic consumers, which strategy is optimal for the seller? How does the mix of consumers impact the optimal strategies and prices?

Our key findings: When the market consists of a mix of myopic and strategic consumers, RPP is optimal if at least half the market is myopic. The domain of optimality of RPP expands as marginal costs increase. In the limit, RPP is optimal when the market is entirely myopic whereas MB is optimal when the market it is entirely strategic. Profit under RPP is an improvement over PC (always) and PB (for the most part). MB and RPP do not emerge as equivalent strategies with strategic consumers due to the seller’s commitment problem in offering the discount under RPP. Interesting pricing results also emerge and are discussed later.

In an extension (§4), we compare MB and RPP against price skimming for a multi-product case. We find that MB and RPP can hold their ground under a wide range of conditions while also being inferior under certain conditions.
2. LITERATURE AND POSITIONING

Our study is related to three research streams. On the bundling side, our study is motivated by the analytical modeling work in economics (e.g., Adams and Yellen 1976; McAfee et al. 1989, Schmalensee 1984) and marketing (see reviews by Stremersch and Tellis 2002 and Venkatesh and Mahajan 2009). Like these studies, we focus on the optimality of PC, PB and MB from the perspective of a monopolist and on price discrimination as the primary demand side rationale for bundling. As PC and PB are nested in MB, a known result is that MB is typically the most profitable strategy for the seller. However, strong complementarity among the products, economies of scope, and low marginal costs may cause MB to converge to PB as the optimal strategy (see Bakos and Brynjolfsson 1999). Strong substitutability and asymmetric marginal costs or network externality among the products may cause MB to converge to PC (see Prasad et al. 2010; Venkatesh and Kamakura 2003).

While RPP is a form of co-promotion, our motivating examples and conceptualization are distinct from prior work on cross-market discounts such as Dhar and Raju (1998), Goic, Jerath, and Srinivasan (2011), and Gilbride, Guiltinan, and Urbany (2008). In each of these extant studies, every consumer who buys one product is eligible for a discount – a coupon, reward miles or a price cut – toward the purchase of another product. However, with RPP, the seller holds the product discount in reserve, revealing it later but only to consumers that buy either product but not both. Under RPP, the segment that is willing to buy both products at the full price does not receive any discount at all, as in the Amazon.com example noted earlier.

Our development of RPP is also motivated by markdown pricing (e.g., Pashigian 1988; Su 2007). The key finding is that if a monopolist sets a price higher than the static equilibrium price and then lowers it, profits can increase. Yet if consumers are strategic, they will wait for the
price to drop, and the seller can do no better than price at the discount immediately (Coase 1972). As in the above studies, consumers in our model can be either myopic or strategic. While RPP retains the idea of markdown pricing, our conceptualization of RPP complements extant research by considering a multi-product setting in which the seller blends inter-temporal pricing and cross selling. RPP mitigates the problem arising out of the Coase Conjecture in the following sense: The consumer under RPP cannot get a lower price on all items by waiting. Getting the discount requires the consumer to buy a regularly priced item, and so some units are assuredly sold at the regular price even in a market entirely composed of strategic consumers.

Overall, while past studies have focused on bundling (or its subtypes) or inter-temporal pricing, our objective is to bring these within a broader strategy space for the seller, and examine analytically which of these strategies work better and under what conditions.

3. MODEL AND ANALYSIS

We set up the general model consisting of a mix of myopic and strategic consumers. Myopic consumers represent proportion $\alpha$ of the market and $1-\alpha$ are strategic. Later we will examine the special cases of only myopic ($\alpha=1$) and only strategic ($\alpha=0$) consumers as corollaries to the main result.

The seller is a profit maximizing monopolist offering two products, 1 and 2. The two-product assumption is usual in normative articles on bundling (e.g., McAfee et al. 1987; Schmalensee 1984; Venkatesh and Kamakura 2003). On the practitioner side, Amazon, despite its wide product range, usually restricts its book recommendations to bundles of two or, sometimes, three products. In other categories (e.g., consumer electronics or videogames) the two-product assumption is arguably even more reasonable.
Potential consumers maximize their individual surplus and each has a demand for at most one unit of each product. The market size is normalized to 1. Consumer $k$ has a reservation price $R_{ki}$ for product $i$, where $i \in \{1,2\}$, and the reservation price for the bundle is additive in its component reservation prices. Following Carbajo et al. (1990), Matutes and Regibeau (1992), and Nalebuff (2004), among others, we assume that $(R_{k1}, R_{k2})$ is uniformly distributed over the unit square $[0,1] \times [0,1]$ to capture heterogeneity. The assumption of independently and uniformly distributed reservation prices is common in the bundling literature (e.g., Bhargava 2013; Carbajo, de Meza, and Seidmann 1990; Nalebuff 2004; Prasad, Venkatesh, and Mahajan 2010). We assume that products have identical marginal cost $c \in [0,1)$ (see Nalebuff 2004; Venkatesh and Kamakura 2003).

The strategy space consists of four strategies: PC, PB, MB and RPP. We present the results under the three bundling strategies first and then analyze RPP.

3.1. Alternative Bundling Strategies

As the products are symmetric in their marginal costs and market valuations, their prices in equilibrium are also symmetric. Under PC, each product is offered at price $P (=P_1 = P_2)$. The price of the bundle under PB is $P_{12}$. With MB, the individual products are offered at price $P$ and the bundle at price $P_{12}$. We avoid additional suffixes to denote the strategy (unless the context is unclear). Analysis with asymmetric marginal costs presents little additional difficulty and is suppressed for ease of exposition.

Bundling strategies are static and the distinction between myopic and strategic consumers does not have a bearing on the results. The PC, PB and MB results are available from extant studies (e.g., Venkatesh and Kamakura 2003). Closed form solutions for optimal prices and profits under PC and PB are provided in Table 1. Explicit solutions for optimal prices under
mixed bundling are unavailable for the commonly modeled reservation price distributions and only numerical solutions exist. The demand derivations for mixed bundling are shown in Appendix A.

Table 1: Prices, Demand and Profit under PC and PB

<table>
<thead>
<tr>
<th></th>
<th>Pure Components</th>
<th>Pure Bundling</th>
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<tbody>
<tr>
<td>Profit</td>
<td>$\Pi = 2(P - c)D$</td>
<td>$\Pi = (P_{12} - 2c)D_{12}$</td>
</tr>
</tbody>
</table>
| Demand         | $D = 1 - P$     | $D_{12} = \begin{cases} 
1 - P_{12}^2 / 2 & \text{for } P_{12} \leq 1, \\
(2 - P_{12})^2 / 2 & \text{for } P_{12} \geq 1. 
\end{cases}$ |
| Optimal price  | $P = (1 + c) / 2$ | $P_{12} = \begin{cases} 
(2c + \sqrt{(2c)^2 + 6}) / 3 & \text{for } c \leq 1 / 4, \\
(2 + 4c) / 3 & \text{for } c \geq 1 / 4. 
\end{cases}$ |

The following results may be noted from the literature: (i) PB is more profitable than PC if the products have low marginal cost ($c < 0.2$, approx.); PC is more profitable otherwise; (ii) MB is more profitable than both PB and PC (McAfee et al. 1987).

3.2. Reserved Product Pricing

Under RPP, the two products are initially offered at price $P (= P_1 = P_2)$. The second stage discounts for the products are denoted by $\delta$, symmetric across the two products and offered to consumers who bought just one product in the first stage. The stages are assumed to be separated by a few days only as in the introductory example, so a discount factor is not applied. If it is included, it would qualitatively reduce RPP profitability and hence its comparison with PC, PB and MB. Demand from myopic consumers (proportion $\alpha$) and strategic consumers (proportion $1 - \alpha$) must be distinguished. We derive the demand from the two segments separately.
Demand Derivation for Myopic Consumers in RPP

Myopic consumers maximize their surplus in each decision making stage since by definition they do not foresee or choose to ignore future price discounts. Figure 1 shows the demand derivations from different sub-segments of myopic consumers.

![Figure 1: Demand from Myopic Consumers](image)

Demand in the first stage is $2(1-P)$ at price $P$, and in the second stage is $2(1-P)\delta$ at price of $P-\delta$.

Hence: Profit contribution from myopic consumers = $\alpha[2(P-c)(1-P) + 2(P-\delta-c)(1-P)\delta]$  \hspace{1cm} (1)

Demand Derivation for Strategic Consumers in RPP

Strategic consumers have rational expectations about the second stage discount. Let $\delta^e$ denote their expected second stage discount when they make their first stage decisions. It is plausible that the expectation is formed after observing the price, so $\delta^e$ is a function of $P$.

Figure 2 shows the construction of the demand functions. Compared to myopic consumer demand, the strategic consumers do not buy both products at their regular prices in the first stage given the expected future discount. Further, there is a small triangle of strategic consumers who buy a product at its regular price despite it being higher than their reservation price because it is an entry ticket to getting the second stage discount. They calculate that a non-negative surplus
will result over the two purchases. The demand in the first stage forms the potential market for
the second stage. The seller has the freedom to deviate to a different discount $\delta$ than what is
expected though. We compute the demand in the second stage based on the potential market and
this discount, and maximize the second stage profit with respect to the discount. For rational
expectations to occur, we equate the seller’s optimal discount to the expected discount.

**Figure 2: Demand from Strategic Customers**

1st Stage: Purchases made at full price $P$. 2nd Stage: Purchases at discount price $P-\delta$.

The profit contribution of strategic consumers under RPP is given in the first stage by

$$(1-\alpha) \times 2(P-c)[P(1-P) + (1-P)^2 / 2 + \delta^2 / 4],$$

where the margin is $(P-c)$ and the term in square brackets is the demand for each product as
obtained from Figure 2 (left panel), and in the second stage by

$$(1-\alpha) \times 2(P-\delta-c)\left[\frac{(1-P+\delta)^2 - 0.5(2\delta-\delta^*)^2}{2}\right],$$

where the margin is $(P-\delta-c)$ and the term in square brackets is the demand for each product
obtained from Figure 2 (right panel) as follows:

The shaded area in Figure 2 (right panel) is a square $(1-P+\delta)^2$ minus a triangle sliced
off its bottom left corner (representing consumers who did not purchase in the first stage and
hence exited). We show that the area of this triangle is $0.5(2\delta-\delta^*)^2$. The diagonal that slices the
corner of the square is the same as the one in the left panel passing through $(P, P-\delta^*)$ and
In fact, the sum of the coordinates of every point it passes through is $2P\delta^*$ because it is a $45^\circ$ line, i.e., with slope -1, hence of the form $y=\alpha-x$ so that $y+x=\alpha$ always. Now apply this rule to the right panel. If one coordinate is $P-\delta$ the other coordinate must be $P+\delta-\delta^*$. So the coordinates of the triangle sliced off the bottom left corner of the square are $(P-\delta, P-\delta), \ (P-\delta, P+\delta-\delta^*), \ (P+\delta-\delta^*, P-\delta)$. Thus, it is a right angled isosceles triangle with base $(P+\delta-\delta^*)-(P-\delta)=2\delta-\delta^*$. Its area will be $0.5(2\delta-\delta^*)^2$ as was required to show.

Combined Profit from Mix of Myopic and Strategic Consumers ($\alpha$ myopic, $1-\alpha$ strategic)

The seller’s decision variables under RPP are the regular prices $P$ and price discount $\delta$.

These are solved over two stages. The first stage RPP optimization problem is:

$$\max_{\delta \in \{0,1\}} \alpha\left[2(P-c)(1-P) + 2(P-\delta-c)(1-P)\delta\right] + (1-\alpha)[2(P-c)\left(P(1-P) + \left(\frac{1-P}{2}\right)^2 + \frac{\delta^2}{4}\right) + 2(P-\delta-c)\left(\frac{(1-P+\delta)^2 - 0.5(2\delta-\delta^*)^2}{2}\right)].$$  

(4)

However, following backwards induction, we begin by solving for the seller’s second stage decision taking the regular price $P$ as given.

$$\max_{\delta} (P-\delta-c)\left[2\alpha(1-P)\delta + (1-\alpha)\left((1-P+\delta)^2 - 2(\delta-\delta^*/2)^2\right)\right].$$  

(5)

Taking the derivative and imposing the requirement for rational expectations that $\delta^*=\delta$ yields:

$$2(1-P)(P-c)-4\delta(1-P)-(1-\alpha)(1-P)^2-(1-\alpha)\delta^2/2 = 0.$$  

(6)

This can be solved as $\delta = (P-c)/2$ if $\alpha = 1$, else,

$$\delta = \frac{-4(1-P) + \sqrt{16(1-P)^2 - 2(1-\alpha)^2(1-P)^2 + 4(1-\alpha)(1-P)(P-c)}}{(1-\alpha)}.$$  

(7)

We insert the expression for the RPP discount back into the first stage, also setting $\delta^*=\delta$. The optimal initial price under RPP is determined from:

$$\max_{P \in [c,1]} \alpha(1-P)^2(P-c) + 2(1-P)\left[(P-c)(2P-2\delta-c)+(P-\delta-c)^2\right] \quad \text{s.t. Equation (7).}$$
The proof for the above is in Appendix B. Comparison of profits from MB and RPP lets us make the following result:

**Result 1:** When proportion $\alpha$ of consumers are myopic and the remaining $(1-\alpha)$ strategic: MB is optimal under lower $\alpha$ and RPP is optimal under higher $\alpha$. RPP’s domain of optimality is increasing in marginal cost $c$. Neither PB nor PC is optimal.

The domains of optimality of MB and RPP are tied to the levels of $(\alpha, c)$ and are delineated in the phase diagram in Figure 3.

![Figure 3. RPP vs. Bundling for a Mix of Myopic and Strategic Consumers](image)

The PC and PB solutions in Table 1 and the figure for MB in Appendix A were used to compare against RPP. The conclusion from Figure 3 is that mixed bundling, despite its pervasiveness in multi-product settings, is inferior to RPP over a large domain. In particular, when marginal costs are negligible, RPP is more profitable if about half of the market is myopic. With higher marginal costs, RPP is more profitable than MB even with a small proportion of myopic consumers.
We focus on why the domain of optimality of RPP is increasing in marginal cost $c$. At lower levels of $c$, MB is truly a product line strategy with both the bundle and the individual products appealing to distinct and sizable groups of consumers. However, when $c$ is higher, the role of the bundle in MB becomes weaker relative to that of the individual products. MB essentially converges to PC for high $c$. With MB looking more like PC under higher $c$, RPP emerges as the more optimal strategy over a wider domain.

We discuss the pricing implications after two further results for markets with all-myopic and all-strategic consumers. These results help us elaborate on the intuition.

**Result 2:** With only myopic consumers ($\alpha=1$):

1. The optimal second stage discount under RPP is $\delta = \frac{(P - c)}{2}$ and the optimal regular price of each product under RPP is:

   $$P = \frac{1 + c}{2} + \frac{(1 - c)}{2(9 - c) + 2\sqrt{(9 - c)^2 + 3(1 - c)^2}}.$$

2. RPP weakly dominates PC, PB and MB on profits.

   The proof of part 1 is in Appendix B and that for part 2 is in Figure 4 below.

![Figure 4. Profits from RPP Relative to Those from PC, PB and MB with Myopic Consumers](image-url)
As noted earlier in Result 1 and Figure 3, RPP dominates the bundling strategies – even MB – in a market comprised of myopic consumers. Incremental profits from RPP, shown in Figure 4, are the highest at low marginal cost $c$. This is partly because the higher costs provide less leeway on the depth of the second stage RPP discount. RPP has a bigger (or smaller) profit advantage relative to PC (or PB) under low $c$. The converse is true for higher $c$. This follows directly from the superiority of PB over PC (or vice versa) when $c$ is low (or high). (As shown in the proof, PC is dominated by RPP even for general reservation price distributions.)

**Result 3:** With only strategic consumers ($\alpha=0$), the results can be summarized as follows: MB weakly dominates RPP. PC is dominated by RPP. RPP is more profitable than PB for all but very low marginal costs. The results are graphed in Figure 5.

As noted in Result 1 and Figure 3, profits from MB from a market comprised of strategic consumers only are at least weakly higher than those from RPP. The magnitude of the difference is plotted in Figure 5. The two converge only when marginal cost reaches the limit. The intuition for the superiority of MB with strategic consumers is that any RPP solution can be replicated by MB but the converse does not always hold. To see why, consider a hypothetical optimal MB
solution of $30 for the bundle and $25 for each product. Let the marginal cost be $10. For RPP to be equivalent to MB, the initial RPP price should be $25 and consumers should expect that the discount in the next stage will be $20. However, a second stage discount of $20 is untenable as it would mean charging $5 for a product that costs $10. The seller will not offer such a discount, and consumers in the first stage cannot have a rational belief that the seller will do so. As a result, this MB solution cannot be implemented under RPP. On the other hand, every RPP solution can be replicated in MB by setting the same product prices and giving the same bundle discount as in RPP. Thus, MB prevails over RPP with strategic consumers.

As with the myopic consumer case, RPP dominates PC in a market with strategic consumers. PC’s inability to price discriminate yields the advantage to RPP. With PB, while it was shown to be dominated by RPP for a market with myopic consumers, the result holds except under negligible marginal costs. Two factors are at play here. First, the strategic consumers blunt (but do not fully overturn) the temporal price discrimination advantage of RPP. Second, the ability of PB to enhance demand at (near) zero marginal costs tilts the balance in its favor. However, as marginal costs increase, PB’s known problem of undersupply takes over and the relative advantage of RPP grows up to a point. While marginal costs are higher still, even RPP suffers and the profit curves from the two strategies get closer.

**Pricing Implications with RPP**

The RPP price and second stage discount are shown graphically in Figure 6. Results for the market with strategic consumers are compared against those for the market comprised of

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1 In the analysis, the marginal costs and reservation prices in the analysis are on a zero to one scale by normalizing the willingness to pay (WTP) distribution appropriately. In this numerical example if we assume that the upper support of the WTP distribution is $100, then the scales value would be 0.1 for the marginal costs, 0.25 for the component price and 0.3 for the bundle price, which should provide an easier comparison with the numbers in the analysis.
myopic consumers. While prices and discounts for a market of myopic consumers are from Result 2 (part 1), those for strategic consumers have been determined numerically. With a positive mixture of both consumer types, the prices and discounts fall between the bounds shown in Figure 6.

With respect to comparative statics, we observe in Figure 6 that as the costs increase, RPP prices increase and discounts decrease. Relative to the myopic consumer market, with strategic consumers the optimal regular price is higher and the discount lower. This is because a higher second stage discount with myopic consumers increases unit sales of the second product without hurting sales at regular price, but would significantly hurt sales with strategic consumers. Strategic consumers able to buy both products at regular price delay the purchase of the second product to avail of the discount.

4. COMPARISON WITH MULTI-PRODUCT PRICE SKIMMING

Given our interest in the inter-temporal pricing aspects of RPP, a question may arise as to how it and the alternative bundling strategies hold up against multi-product price skimming (i.e.,
markdown pricing used by the seller for each of the two products). With price skimming, the seller offers each product separately, first at a higher price and then with a discount (see Lazear 1986, Pashigian 1988). This is an application of the pure components strategy at each of two stages. Henceforth, “price skimming” refers to this two-stage application of pure components.

RPP has an element of relationship marketing. The firm has knowledge of or access to consumers, plausibly via a database of their purchases on the first product, and this simplifies targeting offers on the other products that they do not have. On the other hand, the firm pursuing price skimming does not have a clear way of targeting customers who did not purchase in their first stage. Under skimming, consumers can continue to monitor prices of the unpurchased product in the second stage, waiting for a markdown to buy it, or they can go with an outside option. These scenarios with skimming require further assumptions about consumer behavior.

Clearly both RPP and price skimming rely on price discounts and hence inter-temporal price discrimination between high and low reservation price consumers, notably those who are myopic. We may posit that the behavior of strategic consumers under price skimming will be similar to that in RPP, i.e., that they will anticipate the future price drop and not pay the higher price. Thus the profitability of this strategy is dependent even more on myopic consumers because, unlike RPP, no strategic consumer will pay the full price for any product. On the other hand, we can posit that myopic consumers will pay the price they encounter on the purchase occasion assuming that it is within their reservation price.

Two inefficiencies arise with price skimming. First, some proportion of myopic consumers who could have paid the higher price may encounter the discounted price instead (e.g., if they arrive when the product is on a sale) since the fencing of segments is difficult to make watertight. Second, a fraction of myopic consumers who could not afford the initial higher
price will be lost, because they exit the market before encountering the discounted price. Of these, suppose that in the former case the fences are indeed watertight, which is a conservative assumption since it favors price skimming, and instead focus on the latter inefficiency explicitly.

We assume that a proportion \(1-\lambda\) \((\leq 1)\) of the myopic consumers who did not purchase at the regular price will exit the market. They may even have looked unsuccessfully for the lower price and been unable to find it, and hence left the market. On the other hand, a proportion \(\lambda\) of myopic consumers who did not purchase at the regular price actually encounter the discounted price. Thus \(\lambda = 0\) means that the non-purchasing myopic consumers from the first stage have exited the market and make no second stage purchase, while \(\lambda = 1\) means that all non-purchasing myopic consumers are still accessible to the seller. (None of the strategic consumers exit.)

In the following diagram for myopic consumers only, let us consider a single product since the demand for the other product can be derived similarly.

![Figure 7: Demand under Price Skimming](image)

The demand from myopic consumers in the first stage at price \(P\) is \((1-P)\alpha\) and that in the second stage at price \(P-\delta\) is \(\alpha \delta \lambda\). The demand from strategic consumers is \((1-P+\delta)(1-\alpha)\) and it occurs at price \(P-\delta\) in the second stage. Thus the profit to the seller in the first stage is \(\sum_{i=1,2} (P_i - c_i)(1-P_i)\alpha\) and in the second stage is \(\sum_{i=1,2} (P_i - \delta_i - c_i)[\alpha \delta \lambda + (1-P_i+\delta_i)(1-\alpha)]\). In PC the seller can separately maximize the profit with respect to the price and the discount.
We can write the objective for product $i$ as maximizing the total profit from price skimming in one step:

$$\max_{\delta, \pi} \sum_{i=1,2} \pi_i = \sum_{i=1,2} (P_i - c_i)(1 - P_i)\alpha + (P_i - \delta_i - c_i)[\alpha\delta_i\lambda + (1 - P_i + \delta_i)(1 - \alpha)]$$ (8)

It can be recommended to endogenize $\lambda$ as a function of $P$ if a behavioral rationale so warrants, but this has not been done presently in the absence of a theory. The related result:

**Result 4:** When $\alpha \in [0,1)$ of consumers are myopic and the remaining $1 - \alpha$ are strategic:

1. For the price skimming strategy in which a proportion $1 - \lambda$ of the myopic consumers exit the market after the first stage:
   a. The profit is
   $$\sum_{i=1,2} \frac{(1 - c_i)^2}{4} \left(1 + \frac{\alpha\lambda^2}{4 - \alpha(2 - \lambda)^2}\right).$$
   b. The optimal price is given by $P_i = \frac{1 + c_i}{2} + \frac{\delta_i}{2} [\alpha\lambda + 2(1 - \alpha)]$ and the optimal discount
   $$\delta_i = \frac{\lambda(1 - c_i)}{4 - \alpha(2 - \lambda)^2}.$$

2. Mixed bundling, RPP and price skimming are optimal as follows: (i) Mixed bundling is optimal when $\alpha$ and $c$ are lower; (ii) RPP is not optimal when $\lambda = 1$. For smaller $\lambda$, RPP is optimal for moderate $\alpha$ and low $c$; (iii) Price skimming is optimal when $\alpha$ and $c$ are higher.

The proof of part 1 is in Appendix B. That for part 2 is in the phase diagrams below.
Figure 8 underscores that mixed bundling and RPP can be viable even when price skimming is an alternative. The principal advantage of RPP comes from the firm’s ability to identify and target consumers on the basis of their first stage purchase. With skimming, a subset of the myopic, first stage consumers exit the market (i.e., buy an outside option or simply do not return to seek the second good) and the firm’s ability to leverage its second stage discount is minimized. (Of course, in the extreme case when none of the myopic consumers exit the market, price skimming dominates RPP because the former is a close analog of first degree price discrimination.) Mixed bundling retains its domain of optimality. The essential reason is that if most of the consumers are strategic, price skimming and RPP lose their discriminating ability. The consumers simply will wait and buy the cheaper offering(s). Mixed bundling, which makes prices apparent up front but leverages second degree price discrimination, prevails.

5. DISCUSSION

Multi-product settings provide the backdrop for our study. While the role of bundling in its several forms has been examined quite extensively, it is striking that the power of mixed bundling (MB) has not been benchmarked against any alternative form of price discrimination. We have sought to make a contribution by comparing mixed bundling and its variants against reserved product pricing (RPP), a form of targeted pricing. To recall, the seller under RPP announces an initial regular price, observes the purchase pattern, and then selectively offers consumers who bought just one product a discount toward purchase of the other. The use of RPP by Amazon.com, among others, makes the comparison relevant to practitioners.

We have developed and analyzed a model in which a monopolist with two products to offer faces a market of myopic and/or strategic consumers. We examine which strategy is the most profitable and what the pricing implications are. As pricing results under bundling are
already known, our contribution on the pricing side is focused on RPP and its contrasts vis-à-vis bundling. Our model extension compares RPP and MB with price skimming in a multi-product context.

5.1. Theoretical and Managerial Implications

On the optimal strategy, if the market consists only of strategic consumers, MB trumps RPP. Such consumers correctly anticipate the second stage discount from RPP and can stagger their purchase of one of the products so as to dent the seller’s advantage. But even with strategic consumers only, RPP does better than pure components (PC). This is because under RPP a buyer of two products still has to purchase one product at regular price to avail the discount on the other. And excluding very low marginal costs, RPP is also more profitable than pure bundling (PB). While PB has the image in the popular press as a vehicle for enhancing demand, it actually suffers from the opposite problem even when marginal costs are moderate or high. This weakness of PB makes RPP better.

The shortcomings of MB and its variants relative to RPP are manifested when at least “some” proportion of the market is myopic. With zero marginal costs, this threshold is \( \approx 50\% \); that is, if more than half of the market is myopic, RPP is the optimal strategy. But the threshold is sensitive to the level of marginal costs. Higher marginal costs significantly expand the domain of optimality of RPP. The disadvantage of MB under RPP is tied primarily to the bundle. When costs are high, MB becomes de facto PC.

For a practitioner seeking to implement RPP, we offer the following pricing takeaways:

- The levels of prices and discounts are particularly critical when marginal costs are not high.
  
  For costly products, the leeway to offer a sizable second stage discount is limited.
- As the proportion of myopic consumers increases, the RPP prices should be lower and the discounts higher. That is the price structure should emphasize greater market penetration. This is because, unlike strategic consumers, myopic consumers do not anticipate or cannot wait for the latter discounts, and the profits from the sale of full-priced products are not cannibalized.

- The concern that strategic consumers, who correctly anticipate future discounts, will delay their purchases is mitigated in the context of RPP. The reasoning is that even those consumers who seek to avail the lower prices in the second stage have to purchase one product at full price to get the discount.

While the implications for Amazon.com from the above conclusions are immediate, we have additional conjectures for the online giant. Specifically, with fad or impulse-driven products (plausibly, new technological gadgets or DVD releases) for which myopia takes the form of impatience, the role of RPP is likely to be significant. RPP’s role is arguably enhanced for “perishables” such as textbooks: students typically must have the textbooks when a semester begins, and so the delay embedded in RPP is attractive only to some students. Between ebooks and hardcopies (wherein the ebooks have significantly lower marginal costs), RPP discounts should be sharply higher for the electronic format. Nevertheless, the choice between RPP and MB is trickier with ebooks: a careful assessment of the proportion of myopic buyers is needed with this low marginal cost category.

Our extension encompassing multi-product price skimming (i.e., PC implemented in two stages) reveals that the latter is a potent strategy, especially when the proportion of myopic consumers is high (approaching one). While all three strategies – MB, RPP and skimming – have
their distinct domains of optimality, the proportion of consumers exiting the market in the second stage under skimming has a key bearing on the results.

5.2. Limitations and Future Research Directions

The model that we have set up and analyzed makes several restricting assumptions. Rooted in them are opportunities for further research. The monopoly model could be extended to include competition. Studies such as Matutes and Regibeau (1992) contain the bundling results under competition to compare RPP against. Our model assumes a two-product case, based on precedents in the bundling literature. It would be interesting to extend the results to the case where the seller has more products. While we compared MB and RPP with price skimming, there are other theoretical benchmarks within the domain of inter-temporal price discrimination. For example, a seller may choose to offer inter-temporal pure bundling (i.e., pure bundling in periods 1 and 2). Furthermore, our model rests on the assumptions of linear demand for the individual products and independently distributed reservation prices. Future work could examine the impact of correlated reservation prices on the MB vs. RPP decision under a Gaussian demand (as in Schmalensee 1994).

In conclusion, the potential avenues are rich and diverse. We urge more studies on the topic of price discrimination in a multi-product setting.
Appendix A

Demand and Profits under Mixed Bundling

\[
\max_{P,P_{12}} (P_{12} - 2c)D_{12} + 2(P - c)D \\
\text{s.t.} \\
D = (1 - P)(P_{12} - P) \\
D_{12} = (1 + P - P_{12})^2 - (2P - P_{12})^2 / 2 \\
P < P_{12} < 2P
\]

Appendix B

Proof of First Period Price and Second Period Discount under RPP (Equation 7)

We show how the maximization problem (4) is reduced to its simpler form in equation (7) and below. For this, we make use of the necessary condition in Equation (6), rewriting it as

\[
(1 - P)^2 + \delta^2 / 2 = \frac{2(1 - P)(P - c) - 4\delta(1 - P)}{(1 - \alpha)}.
\]  \hspace{1cm} (A1)

The simplifications are shown below.

\[
\max_{P \in [c,1]} 2\alpha(1 - P)[(P - c) + (P - \delta - c)\delta] \\
+ 2(1 - \alpha)[(P - c)
\left(P(1 - P) + \frac{(1 - P)^2}{2} + \frac{\delta^2}{4}\right) + (P - \delta - c)
\left(\frac{(1 - P)^2 + 2\delta(1 - P) + \delta^2 / 2}{2}\right)]
\Rightarrow \max_{P \in [c,1]} 2\alpha(1 - P)(P - c) + 2\alpha(1 - P)(P - \delta - c)\delta + 2(1 - \alpha)(P - c)\alpha(1 - \alpha)(P - c)P(1 - P)
\]

\[+ 2(P - c)(1 - P)(P - c - 2\delta) + 2(1 - P)(P - \delta - c)(P - c - 2\delta) + 2(1 - \alpha)(P - \delta - c)\delta(1 - P)
\Rightarrow \max_{P \in [c,1]} 2(1 - P)\{\alpha(P - c) + (P - \delta - c)\delta + (1 - \alpha)(P - c)P + (P - c - 2\delta)(2P - \delta - 2c)\}
\Rightarrow \max_{P \in [c,1]} 2\alpha(1 - P)^2(P - c) + 2(1 - P)((P - c)(2P - 2\delta - c) + (P - c - \delta)^2)\]
Proof of Result 2

Part (i): With $\alpha=1$ we had $\delta = (P - c)/2$ from Equation (6) and the maximization problem is given by Equation (1), which can be written as:

$$\max_{p \in [0,1]} 2(P-c)(1-P) + \frac{1}{2}(P-c)^2(1-P). \quad (A2)$$

The first term is the PC profit, so the RPP profit is at least as high as the PC profit and identical to it if $P = c = 1$ wherein no profit is attainable. But, from the first order condition, $P > c$ and the second term is positive because we assumed $c \in [0,1)$. Hence RPP strictly dominates.

Part (ii): For the problem in (A2), the necessary and sufficient condition for optimality is,

$$1 - 2P + c - \frac{(P-c)^2}{4} + \frac{(P-c)(1-P)}{2} = 0. \quad (A3)$$

This is a quadratic equation in price and has a unique solution given by

$$P = \frac{1+c}{2} + \frac{\sqrt{(9-c)^2 + 3(1-c)^2}}{2(9-c)} \quad (A4).$$

Proof of Result 3

While comparisons with PB and PC versus RPP are numerical, the superiority of MB over RPP can be proved: Let the optimal RPP solution be characterized by component price $p$ and discount $\delta$. We can replicate this optimal solution using MB by setting product price $P$ and bundle price $2P-\delta$. If we can show that the reverse is not the case, i.e., that RPP cannot replicate every MB12 solution, then we are done. Consider an MB solution of $P_{12}$ for the bundle and $P$ for each component where $P_{12} < P+C$ for component marginal cost $C$. For RPP to implement this solution it must set initial prices at $P$ and customers must have expectations that the discount in the next stage will be $2P-\delta$. If we can show that the reverse is not the case, i.e., that RPP cannot replicate every MB12 solution, then we are done. Consider an MB solution of $P_{12}$ for the bundle and $P$ for each component where $P_{12} < P+C$ for component marginal cost $C$. For RPP to implement this solution it must set initial prices at $P$ and customers must have expectations that the discount in the next stage will be $2P-P_{12}$. However, in the second stage, if the seller gives a discount of $2P-P_{12}$ it means that it is charging $P_{12}-P$ for a product that costs $C$. It will not do so (since $C > P_{12}-P$) and
customers in the first period can have no rational expectation that it will do so. Thus, this set of prices cannot be implemented under RPP.

Proof of Result 4:

For the maximization problem in Equation (8),

$$
\max_{P, \delta} \pi_i = (P_i - c_i)(1 - P_i)\alpha + (P_i - \delta_i - c_i)[\alpha\delta_\lambda + (1 - P_i + \delta_i)(1 - \alpha)]
$$

the objective function is globally concave and the necessary conditions, $\partial \pi_i / \partial P_i = 0$ and $\partial \pi_i / \partial \delta_i = 0$, for maximum yield:

$$
P_i = \frac{1 + c_i}{2} + \frac{\delta_i}{2} [\alpha\lambda + 2(1 - \alpha)], \quad \delta_i = \frac{\alpha\lambda(P_i - c_i) + (1 - \alpha)(2P_i - \delta_i - 1)}{2(1 + \alpha\lambda - \alpha)}, \quad (A5)
$$

Inserting rearrangements, $P_i - c_i = \frac{1 - c_i}{2} + \frac{\delta_i}{2}(\alpha\lambda + 2(1 - \alpha))$ and $2P_i - c_i - 1 = \delta_i(\alpha\lambda + 2(1 - \alpha))$, of the first condition into the second, we get $\delta_i = \frac{\alpha\lambda(1 - c_i)}{4(1 + \alpha\lambda - \alpha) - (\alpha\lambda + 2(1 - \alpha))^2}$ or,

$$
\delta_i = \frac{\lambda(1 - c_i)}{4 - \alpha(2 - \lambda)^2}.
$$

To obtain the expression for the optimal profit we insert the following rearrangements of the first necessary condition: $P_i - c_i = \frac{1 - c_i}{2} + \frac{\delta_i}{2}(\alpha\lambda + 2(1 - \alpha))$, $1 - P_i = \frac{1 - c_i}{2} - \frac{\delta_i}{2}(\alpha\lambda + 2(1 - \alpha))$, $P_i - c_i - \delta_i = \frac{1 - c_i}{2} + \alpha\delta_\lambda(\frac{\lambda}{2} - 1)$, and $1 - P_i + \delta_i = \frac{1 - c_i}{2} - \alpha\delta_\lambda(\frac{\lambda}{2} - 1)$, into the profit function

$$
\pi_i = (P_i - c_i)(1 - P_i)\alpha + \alpha\delta_\lambda(P_i - \delta_i - c_i) + (1 - \alpha)(1 - P_i + \delta_i)(P_i - \delta_i - c_i).
$$

After these insertions:

$$
\pi_i = \frac{\alpha(1 - c_i)^2}{4} - \frac{\alpha\delta_\lambda^2}{4}(\alpha\lambda + 2(1 - \alpha))^2 + \alpha\delta_\lambda\left(\frac{1 - c_i}{2} + \alpha\delta_\lambda\left(\frac{\lambda}{2} - 1\right)\right) + \frac{(1 - \alpha)(1 - c_i)^2}{4} - \frac{(1 - \alpha)\alpha^2\delta_\lambda^2(\lambda/2 - 1)^2}{4}.
$$

Collecting powers of $\delta_i$ we get,
\[ \pi_i = \frac{(1-c_i)^2}{4} + \frac{\alpha \lambda (1-c_i)}{2} \delta_i - \left( \frac{(\alpha \lambda + 2(1-\alpha))^2}{2} + \alpha \lambda (2-\lambda) + \frac{\alpha (1-\alpha)(2-\lambda)^2}{2} \right) \frac{\alpha \delta_i^2}{2}. \]

Using the expansion \( \frac{(\alpha \lambda + 2(1-\alpha))^2}{2} = 2 + \frac{\alpha^2 (2-\lambda)^2}{2} - 2\alpha (2-\lambda) \) we simplify to get,

\[ \pi_i = \frac{(1-c_i)^2}{4} + \frac{\alpha \lambda (1-c_i)}{2} \delta_i - \alpha \delta_i^2 \left( 1 - \frac{\alpha (2-\lambda)^2}{4} \right). \]

Now insert the solution \( \delta_i = \frac{\lambda (1-c_i)}{4 - \alpha (2-\lambda)^2} \) into this. We get the required expression:

\[ \pi_i = \frac{(1-c_i)^2}{4} \left( 1 + \frac{\alpha \lambda^2}{4 - \alpha (2-\lambda)^2} \right). \]  

(A6)
REFERENCES


