Probabilistic Selling vs. Markdown Selling:
Price Discrimination and Management of Demand Uncertainty in Retailing*

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Abstract

Markdown selling (i.e., price reductions over the course of the selling season) is a strategy to implement price discrimination and to manage market uncertainty that has been widely adopted by retailers. This paper explores the potential advantage of introducing an additional tool to the arsenal of retailers, probabilistic selling (i.e., offering consumers a choice to buy a product that can turn out to be any item from a predetermined set of distinct items). We show that both probabilistic and markdown selling strategies serve as price discrimination tools by offering buyers an option to purchase a “damaged” good (an uncertain product under the former and delayed consumption of a product under the latter). However, the two strategies segment markets based on different types of buyer heterogeneity: buyer preference strength under probabilistic selling and buyer patience under markdown selling. Our analytical model reveals that, compared with markdown selling, probabilistic selling can (1) improve margin management by increasing revenue from full-price sales and reducing the magnitude of discounts; and (2) improve inventory utilization by reducing stockouts and the amount of excess inventory. We identify the conditions required for probabilistic selling to be more profitable than markdown selling.

Keywords: probabilistic selling, pricing, demand uncertainty, markdowns, price discrimination
1. INTRODUCTION

In an effort to obtain the maximum profit across a diverse set of customers, retailers often offer price reductions over the course of the selling season. It is estimated that one-third of all goods are sold at marked-down prices (Friend and Walker 2001) and discounts due to markdowns by US retailers amount to $200B a year (Levy et al. 2004). Although costly, markdowns can be a valuable tool for improving profit margin management because they allow the retailer to price discriminate across time, i.e., sell the product at a high price early in the season to customers who value the product highly and are unwilling to wait, and at a discounted price later in the season to customers who are willing to delay their purchases (Nair 2007; Su 2007; Besbes and Lobel 2012). The markdown strategy can also enhance inventory management for retailers who are unable to accurately predict consumers’ demand for each particular product (Lazear 1986), e.g., by starting with a high price and reducing the price if units of the item remain unsold. Retailers are continually searching for more efficient ways to improve margin management and enhance inventory utilization.¹ In this paper, we consider one such alternate selling mechanism, namely probabilistic selling (PS), and show that there are situations in which this mechanism can be advantageous relative to traditional markdowns both in enhancing price discrimination and in overcoming the main problems associated with demand uncertainty, namely stockouts and excess inventory.

A probabilistic product is an offer involving the probability of obtaining any one of a set of multiple distinct items (Fay and Xie 2008). Probabilistic selling (PS) is a selling strategy under which the seller creates probabilistic goods using the seller’s distinct products or services and offers such goods to potential buyers as additional purchase choices. Notable examples of sellers of probabilistic products include priceline.com, lastminutetravel.com, and hotwire.com, websites where consumers can purchase travel services for which specific attributes of the service (e.g., the itinerary of the flight, the location of the hotel, or the identity of the car rental company) are not revealed until after payment. Recently, the idea of offering probabilistic goods has also been adopted by several online retailers (e.g., swimoutlet.com, agonswim.com, speedo.com, and kidsurplus.com) who offer discounted “grab bag” apparel and shoes, where patterns and styles are chosen randomly by the website.² As technological advances make it much more practical to implement PS both in online and brick-and-mortar shopping environments, more retailers can potentially benefit from adopting this novel selling strategy (Fay and Xie 2008). While the existing research on PS has significantly advanced our understanding of the fundamental drivers of PS and illustrates its general

¹ Previous research has focused on developing sophisticated dynamic markdown algorithms (e.g., Mantrala and Rao 2001; Sullivan 2005; Chung, Flynn, and Zhu. 2009; Bitran and Mondshein 1997), implementing inventory management systems (Khouja 1995; Friend and Walker 2001; Ross 1997), and identifying alternate ways to dispose of distressed goods, such as via off-price retailers and outlet stores (Coughlan and Soberman 2005; Levy and Weitz 2004, p. 56; Petruzzi and Monahan 2003) or online auctions (Wang, Gal-Or, and Chatterjee 2009; Wood et al. 2005).

² See an example at http://www.swimoutlet.com/product_p/1623.htm
applicability, it is important to extend the research to understand how this novel strategy may address some unique problems in the retailing industry and to explore whether PS can be a valuable alternative to offering late-season markdowns.

Most retailers strategically invest in inventory prior to the selling season, control the prices of their products over the entire selling season, and must account for how consumers time their purchases in response to these chosen prices. We introduce a model that incorporates each of these key characteristics. As shown in Table 1, among the current research on PS, ours is the only model that incorporates all of the following three key characteristics: (1) The seller optimally chooses its prices for the probabilistic goods and the specified goods; (2) the seller optimally adjusts its inventory orders when introducing probabilistic goods; and (3) consumers strategically choose when to purchase in order to maximize their expected surplus. By incorporating these three critical factors, we are able to develop the theory and implications of PS for the retailing industry. In particular, our model enables us to compare discounting on the basis of time (high initial price and a discounted price if the consumer delays her purchase) versus discounting on the basis of product opacity (i.e., setting a high price for each specified good and a discounted price if the consumer will purchase the probabilistic good). Thus, the paper’s primary contribution is that it is the first to examine the profit advantage of the PS strategy relative to the more commonly utilized strategy of marking down merchandise over time, i.e., the markdown (MD) selling strategy. We identify factors under which PS can be a more useful tool for retailers as they attempt to price discriminate across consumers. We find that PS and MD can be complementary strategies since, in some market settings, PS is a profitable form of price discrimination whereas MD is not, while, in other market settings, price discrimination is profitable via MD but not profitable via PS.

A second contribution of the paper is that, by introducing a model that allows a probabilistic good to cannibalize full-price sales, we can examine the factors that affect the extent of cannibalization by the probabilistic good and determine whether PS can remain advantageous in its presence. Most extant analytical research on PS utilizes a Hotelling model to account for consumer heterogeneity (Jiang 2007; Fay 2008; Jerath et al. 2010; Fay and Xie 2008, 2012). A feature of the Hotelling model is that all consumers have the same expected value for the probabilistic good. As a result, price can be set at this common expected value, thus eliminating consumer surplus for all buyers of the probabilistic good. Since the probabilistic good does not generate positive surplus, the seller does not have to worry about any consumers switching from a

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3 Several papers (Gallego and Phillips 2004, Mang et al. 2012, and Petrick et al. 2012) consider a seller who does not assign products to buyers of the probabilistic good until a time that is substantially later than the day of purchase. They refer to this business model as flexible selling rather than PS. However, consistent with Fay and Xie (2012), we consider these papers as part of the PS literature since delaying product assignment can be viewed as an alternative way of implementing the PS strategy.
higher-priced specified good to the lower-priced probabilistic good. However, cannibalization is a crucial concern under MD in the retailing industry because retailers are apprehensive that a discounted price at the end of the season may entice both low- and high-valuation buyers to delay their purchases (especially if the magnitude of the discount is very large). Thus, to provide an adequate comparison of PS with MD, the model must be capable of capturing the cannibalization effect under both strategies. Note that several empirical studies incorporate the cannibalization effect into their model estimations (Granados et al. 2008; Zouaoui and Rao 2009; Anderson and Xie 2012; Mang et al. 2012). However, since demand is modeled in reduced form in these papers, i.e., cross-price effects exist between the probabilistic good and the specified goods, these studies do not analyze the factors which affect the magnitude of this cannibalization effect or how cannibalization impacts the profitability of PS, as we do here.

The rest of this paper is organized as follows. In the next section, we use a lab experiment to illustrate the potential advantages of the PS and MD strategies relative to a No Discounting strategy. In Section 3, we illustrate how both PS and MD can enable a retailer to price discriminate and then compare the profitability of these two strategies. In Section 4, we extend the analytical model to allow for demand uncertainty and demonstrate that our key results continue to hold, and are even strengthened, in such markets. We conclude the paper by summarizing our results and suggesting areas for future research.

2. TWO ADVANTAGEOUS STRATEGIES: PROBABILISTIC SELLING AND MARKDOWNS

2.1. Motivation

For MD and PS to be viable selling strategies, they must be more advantageous than offering no discounts to consumers, i.e., the No Discounting (ND) strategy. We performed an experimental study to explore how consumers respond to these different types of discounts versus a situation in which no discounts are offered. One hundred and thirty-eight undergraduate business students participated in the web-based study in exchange for extra credit, purportedly to help researchers better understand the online purchase behavior of college students. The design was a 2 (Selling Strategy: MD, PS) x 2 (discount: 5% or 25%) between-subjects design with an ND control condition. Participants were randomly assigned to one of the five conditions. Five students were eliminated for failure to follow directions, which left 133 responses for analysis.

Participants were told that they would be shown offers for products (T-shirts) that had recently been for sale online, and then would be asked to answer the purchase questions. Next, the concept of probabilistic goods was explained to the participants, and they were told that the displayed offers may or may not include an option to purchase a probabilistic good. Participants were asked to make a purchase decision for T-shirt offers in Period 1 and then to rank their strength of preference (SOP) between the distinct T-Shirts on a scale of 0 to 50, where SOP is a measure of the difference between valuations for one’s preferred and one’s less-preferred product. Heterogeneity in the strength of preferences has been previously hypothesized as a key...
factor in determining the profitability of PS (Fay and Xie 2008). In the MD condition, full-priced items were offered in Period 1 and, if no purchase occurred in this period, marked-down items were offered in Period 2. In the PS condition, full-priced items as well as the probabilistic good were offered in Period 1, but there were no second-period offerings. In the ND control condition, both full-priced items were offered in Period 1 only. If participants chose to purchase a T-Shirt in Period 1, the shopping experience ended in all conditions.

2.2. Results

A one-way ANOVA for revenue generated by the purchase decisions of the participants was conducted across the five selling conditions. The results indicate a significant effect of selling condition (F(4,128)=4.79, p=.001). Specific contrasts indicate that the average revenue under ND ($5.86) was less than under the PS 5% condition (M_{PS5%} = $13.31, t(133)=4.00, p<.001), the PS 25% condition (M_{PS25%} = $12.16, t(133)=3.31, p=.001), the MD 5% condition (M_{MD5%} = $9.53, t(133)=1.95, p=.05) and the MD 25% condition (M_{MD25%} = $9.42, t(133)=1.95, p=.05). For the two PS conditions, SOP was significantly greater for those buying full-priced goods (M_{DS}=34.05) than for those who purchased a discounted good (M_{PS}=22.18; F(1,42) = 7.03, p=.01). No significant preference difference was present in the MD conditions between those who chose full price and those who chose a discounted good (p>.8).

Together these findings suggest that, for our sample, both MD and PS were effective at increasing revenue relative to ND, which raises the question of when each strategy works best. The significant effect of SOP is consistent with the arguments in the extant literature that heterogeneity in the strength of consumers’ preferences is a fundamental profit driver for PS. In the following sections, we use an analytical model to explore an environment, similar to our experiment, in which both PS and MD may enable a firm to effectively segment its customers. Specifically, we develop a model to explain the conditions under which PS will be more profitable than MD, and vice-versa.

3. PRICE DISCRIMINATION: PROBABILISTIC SELLING VERSUS MARKDOWNS

In this section, we introduce a formal mathematical model to explore whether, and under what conditions PS is more profitable than MD. We begin by focusing on how PS and MD enable price discrimination. In Section 4, we extend the model to incorporate demand uncertainty and capacity constraints. Our model allows for asymmetric product preferences, costly inventory, endogenous capacity constraints, heterogeneous discount rates, and a spectrum of different product valuations by consumers. Overall, we find that, although PS is not more profitable than MD in all scenarios, there exists a sufficiently broad range of situations in which PS is advantageous to warrant increased attention to this new tool.

3.1. Modeling Assumptions

**Seller Behavior.** Consider a retailer with two products, A and B, (e.g., a red T-shirt and a white T-shirt) and two possible selling periods (Periods 1 and 2). Prior to the first selling period, the retailer orders
$K_A$ units of product A and $K_B$ units of product B, where each unit costs $c$. The seller has three alternative selling strategies:

1. **No Discounting (ND)**, under which the prices offered do not change over time, i.e., the price of product A and the price of product B is $P^{ND}$ in both the first and second periods.
2. **Markdown Selling (MD)**, under which prices vary over time. Specifically, the price in the first period (of product A and of product B) is $P_1^{MD}$ and the price of each product is $P_2^{MD}$ in the second period.
3. **Probabilistic Selling (PS)**, under which the seller offers each product individually and also a probabilistic product that can turn out to be either product A or product B. Specifically, the price of product A and the price of product B is $P_1^{PS}$. In addition, the seller offers a probabilistic good in the first period (at a price of $P_0^{PS}$). After purchase, the firm immediately determines which product the buyer will receive, each of which is equally likely.\(^4\)

**Buyer behavior.** Each consumer purchases at most one product, choosing the purchase option that yields the highest net surplus. There are two types of consumers: $\theta = \{H, L\}$, where $\theta = H$ represents consumers with high product valuations and $\theta = L$ represents consumers with low product valuations. Each type makes up half of the total population and we normalize the size of each segment to one. Let $v_{\theta F}$ be consumer $\theta$’s value in Period 1 for her favored product and $v_{\theta U}$ be consumer $\theta$’s value for her less favored product. Thus, by definition, we have $v_{HF} > v_{LF}$, $v_{HF} > v_{HU}$ and $v_{LF} > v_{LU}$. To reduce notation, we normalize the valuations so that the maximum valuation is one: $v_{HF} = 1$. Furthermore, products are more highly valued if they are purchased in Period 1 rather than in Period 2. Valuations are lower in the second period due to consumer impatience, loss of perceived newness of the product, or loss of the opportunity to use the product during the first period. Specifically, in Period 2, a consumer’s favored product is valued at $d_{\theta} v_{\theta F}$ and her less-favored product is valued at $d_{\theta} v_{\theta U}$, where $d_{\theta}$ is naturally restricted to the parameter region $0 < d_{\theta} < 1$ for $\theta = \{H, L\}$. We allow the products to differ in their aggregate popularity. In particular, $\alpha$ of consumers ($\frac{1}{2} \leq \alpha \leq 1$) favor the “popular” product and $1 - \alpha$ of customers favor the “unpopular” product. Note that a specific consumer’s favored product may actually be the less popular one. Throughout this section, we assume product A is the popular product and that the seller and the buyers both know this. In section 4, we extend the model to allow for demand uncertainty.

\(^4\) Fay and Xie (2008) demonstrate that a seller typically finds an equal probability of assignment optimal under various demand conditions. Thus, the assumption of equal probability is commonly made (e.g., Fay and Xie 2010, Jiang 2007).
3.2. Three Strategies

No Discounting (ND). Under ND, the firm sells in both periods at a price $P^{ND}$. All sales will occur in the first period. The seller chooses the inventory orders ($K_A, K_B$) and price, $P^{ND}$, in order to maximize its profit. These optimal values and the resulting profit are:

$$K_A^{ND} = \begin{cases} 2\alpha & c \leq \hat{c} \\ \alpha & \hat{c} < c \leq 1 \\ 0 & c > 1 \end{cases} \quad K_B^{ND} = \begin{cases} 2(1-\alpha) & c \leq \hat{c} \\ (1-\alpha) & \hat{c} < c \leq 1 \\ 0 & c > 1 \end{cases} \quad P^{ND} = \begin{cases} v_{LF} & c \leq \hat{c} \\ 1 & \hat{c} < c \leq 1 \\ 0 & c > 1 \end{cases} \quad \Pi^{ND} = \begin{cases} 2(v_{LF} - c) & c \leq \hat{c} \\ 1 - c & \hat{c} < c \leq 1 \\ 0 & c > 1 \end{cases}$$ (1)

where $\hat{c} = 2v_{LF} - 1$

If costs are low ($c \leq \hat{c}$), the seller orders sufficient capacity to serve all consumers: $K_A^{ND} + K_B^{ND} = 2$. Price is set so that L-type consumers are willing to purchase. For moderate costs ($\hat{c} < c \leq 1$), the seller only orders enough capacity to serve the H-type consumers, $K_A^{ND} + K_B^{ND} = 1$, and the price is set so that H-type consumers are just willing to purchase. For higher costs, $c > 1$, it is impossible for the seller to earn a positive profit.

For the remainder of the paper, we assume $c \leq \hat{c}$, so that it is optimal to serve both consumer types, enabling us to focus on the role of price discrimination (since costs will be the same under all three strategies). For $c > \hat{c}$, total sales under either MD or PS will be higher than under ND, and thus the role of price discrimination is confounded with the role of market expansion. Furthermore, even at these higher costs, the magnitude of $c$ does not impact the focal comparison between MD and PS, because these two strategies require the same amount of inventory and thus incur the same costs.

Markdown Selling (MD). In MD, the seller offers each specified product in both the first and second periods (at prices of $P_1^{MD}$ and $P_2^{MD}$, respectively). Under this strategy, each H-type consumer buys her favorite product in the first period and each L-type consumer buys her favorite product in the second period.\(^5\)

To meet demand, the seller needs $K_A^{MD} = 2\alpha$ and $K_B^{MD} = 2(1-\alpha)$. The prices $P_1^{MD}$ and $P_2^{MD}$ induce such a purchasing pattern only if the following incentive compatibility and participation constraints are met:

$$[P1]: \quad 1 - P_1^{MD} \geq 0 \quad [IC1]: \quad 1 - P_1^{MD} - (d_H - P_2^{MD}) \geq 0$$

$$[P2]: \quad d_L v_{LF} - P_2^{MD} \geq 0 \quad [IC2]: \quad d_L v_{LF} - P_2^{MD} - (v_{LF} - P_1^{MD}) \geq 0$$ (2)

The seller chooses prices ($P_1^{MD}, P_2^{MD}$) to maximize its total profit given the constraints in (2).\(^7\)

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\(^5\) Alternative strategies in which H-types wait to purchase until the second period and L-types either purchase in the first period or also wait to purchase until the second period would yield strictly less profit.

\(^6\) One could envision a scenario in which L-type consumers also purchase in the first period. However, our focus is on identifying scenarios in which MD is strictly more profitable than ND. If L-types also purchase in the first period, no markdown sales occur and thus MD and ND would be identical.

\(^7\) At the optimal solution, constraint [P2] must be binding. If $P_2^{MD}$ was set such that [P2] did not hold with equality, then it would be possible for the seller to raise this second-period price and still sell to L-type consumers (and possibly even
\[ P_{1}^{MD} = 1 - \text{Max} \left[ d_{H} - d_{L} v_{LF}, 0 \right] \]  
\[ P_{2}^{MD} = d_{L} v_{LF} \]

These prices result in a profit of:
\[ \Pi^{MD} = 1 - \text{Max} \left[ d_{H} - d_{L} v_{LF}, 0 \right] + d_{L} v_{LF} - 2c \]  

**Probabilistic Selling (PS).** Under PS, in addition to selling each specified product in the first period (each at a price of \( P_{1}^{PS} \)), the seller also offers a probabilistic good in the first period (at a price of \( P_{o}^{PS} \)). After purchase, the firm immediately determines which product the buyer will consume (each of which is equally likely). Under this strategy, each H-type consumer buys her favored product in the first period and each L-type consumer buys the probabilistic good (also in the first period).\(^8\) To meet demand, the seller selects
\[ K_{A}^{MD} = \alpha + \frac{1}{2} = \frac{2\alpha + 1}{2} \quad \text{and} \quad K_{o}^{MD} = (1 - \alpha) + \frac{1}{2} = \frac{3 - 2\alpha}{2} . \]

The expected value of the probabilistic good to consumer \( \theta \) is \( v_{b_{o}} = \frac{(v_{\theta F} + v_{\theta U})}{2} \). The prices \( P_{1}^{PS} \) and \( P_{o}^{PS} \) induce such a purchasing pattern only if the following incentive compatibility and participation constraints are met:

\begin{align*}
[P1'] : & \quad 1 - P_{1}^{PS} \geq 0 \\
[IC1'] : & \quad 1 - P_{1}^{PS} - \left( \frac{1 + v_{HU}}{2} - P_{o}^{PS} \right) \geq 0 \\
[P2'] : & \quad \frac{v_{LU} + v_{LF}}{2} - P_{o}^{PS} \geq 0 \\
[IC2'] : & \quad \frac{v_{LU} + v_{LF}}{2} - P_{o}^{PS} - \left( v_{LF} - P_{1}^{PS} \right) \geq 0
\end{align*}

(5)

The seller chooses prices (\( P_{1}^{PS} \) and \( P_{o}^{PS} \)) to maximize its total profit given the constraints in (5):\(^9\)

\[ P_{1}^{PS} = 1 - \text{Max} \left[ \frac{1 + v_{HU}}{2} - \frac{v_{LF} + v_{LU}}{2}, 0 \right] = \text{Min} \left[ \frac{1 - v_{HU} + v_{LF} + v_{LU}}{2}, 1 \right] \]

\[ P_{o}^{PS} = \frac{v_{LF} + v_{LU}}{2} \]

These prices result in a profit of:
\[ \Pi^{PS} = \text{Min} \left[ \frac{1 - v_{HU} + v_{LF} + v_{LU}}{2}, 1 \right] + \frac{v_{LF} + v_{LU}}{2} - 2c \]

(7)

sell to the H-types at a higher first-period price), thus increasing its profit. Similarly, either [P1] or [IC1] must bind.

If \( P_{1}^{MD} \) was set such that neither of these constraints held with equality, then it would be possible for the seller to raise this first-period price and still sell to H-type consumers, thus increasing its profit.

\(^8\) An alternative strategy in which H-types also buy the probabilistic good would yield strictly less profit since the firm can charge a higher price for a consumer’s favored good than it can for the probabilistic good.

\(^9\) At the optimal solution, constraint [P2'] must be binding. If \( P_{o}^{PS} \) was set such that [P2'] did not hold with equality, then it would be possible for the seller to raise the price of the probabilistic product and still sell it to L-type consumers, thus increasing its profit. Similarly, either [P1'] or [IC1'] must bind. If \( P_{1}^{PS} \) was set such that neither of these constraints held with equality, then it would be possible for the seller to raise the price of the specified goods and still sell to H-type consumers, thus increasing its profit.
3.3. Comparison of Profit

Lemma 1 summarizes the conditions under which MD and PS, respectively, are more profitable than ND. Proofs of the Lemmas, Corollaries, and Propositions are given in the Web Appendix.

Lemma 1 (Conditions Required for Price Discrimination)

Compared with the No Discounting strategy, which earns the seller the same profit from both H- and L-type consumers,

a) Markdown Selling allows the seller to benefit from price discrimination if the increase in profit from H-type consumers (who buy in the first period) is larger than the decrease in profit from L-type consumers (who buy in the second period).

b) Probabilistic Selling allows the seller to benefit from price discrimination if the increase in profit from H-type consumers (who buy the specified goods) is larger than the decrease in profit from the L-type consumers (who buy probabilistic good).

Mathematically, 
\[
\Pi_{PS} > \Pi_{ND} \text{ if } \Delta_{Price to H-Types}^{PS} - \Delta_{Price to L-Types}^{PS} > 0
\]
\[
\Pi_{MD} > \Pi_{ND} \text{ if } \Delta_{Price to H-Types}^{MD} - \Delta_{Price to L-Types}^{MD} > 0
\]

where \(\Delta_{Price to H-Types}^{PS} = P_1^{PS} - P_1^{ND}\), \(\Delta_{Price to L-Types}^{PS} = P_1^{ND} - P_0^{PS}\), \(\Delta_{Price to H-Types}^{MD} = P_1^{MD} - P_1^{ND}\), and these prices are given in Equations (1), (3), and (6), respectively.

Two important corollaries to Lemma 1 are:

Corollary 1: A necessary condition for the inequality in Lemma 1a) to be satisfied is \(d_L > d_H\), i.e., H-types are less patient than L-types.

Corollary 2: A necessary condition for the inequality in Lemma 1b) to be satisfied is \(v_{LF} - v_{LU} < 1 - v_{HU}\), i.e., H-types have stronger product preferences than L-types.

These two corollaries indicate that “damaging” the good (either by delaying consumption or pairing it with one’s less-favored good) must create greater disutility for the high- than for the low-value consumers in order for each respective pricing strategy to be profitable. Under MD, the discounted second-period product must be relatively attractive to the low-value consumers (so that the second-period price is not too low), but also relatively unattractive to the high-value consumers (so that they are willing to pay a premium to purchase in the earlier period). A necessary condition to achieve a net advantage from MD is that high-value consumers must be less willing to wait than low-value consumers. Su (2007) and Besbes and Lobel (2012) also point out that inter-temporal price discrimination is predicated on there being an inverse relationship between consumers’ patience and their valuations. Similarly, under PS, the probabilistic good must be relatively more appealing to the low-value consumers so that the discount for the probabilistic good does not have to be too large and, thus, a relatively high price can be obtained for the specified goods.

Furthermore, although Lemma 1 indicates that both MD and PS can be used to segment the market, the basis of this segmentation is very different in each case: MD is based on buyer heterogeneity in their discount for...
time (Corollary 1) and PS is based on buyer heterogeneity in their product preference strength (Corollary 2). Such differences create the potential for retailers to benefit from PS, as summarized by the following proposition.

**Proposition 1: (Profit Advantage of Probabilistic Selling)**

(a) PS expands the scenarios under which the seller can benefit from price discrimination, i.e., there is a subset of parameters such that PS is more profitable than ND, but MD is not.

(b) PS is more profitable than MD if it can create the following two advantages (or if one of these advantages is sufficiently large):

1. **A More Valuable “Damaged” Good**, i.e., L-type consumers are willing to pay more for the probabilistic product in the first period than for their preferred product in the second period.

2. **A Lessened Cannibalization Threat**, i.e., H-type consumers demand less surplus to forgo purchasing the discounted product under PS than under MD.

Formally, PS earns the retailer a higher profit if the following condition holds:

\[
\Pi_{PS} - \Pi_{MD} > 0
\]

\[
= \Delta_{\text{Value to L-Types}} + \Delta_{\text{Surplus to H-Types}}
\]

(c) The profit advantage of PS relative to MD \((\Pi_{PS} - \Pi_{MD})\) increases as

1. L-types’ valuations for the probabilistic good increase (i.e., a larger \(V_{LU}\))

2. L-types become more impatient (i.e., a lower \(d_L\))

3. H-types’ valuations for the probabilistic good decrease (i.e., a smaller \(V_{HU}\))

4. H-types become more patient (i.e., a higher \(d_H\))

Figure 1 illustrates the main results from Proposition 1. In Figure 1a), we show the impact of \(v_{LU}\) and \(d_L\) on the relative profit of PS, MD, and ND (holding \(v_{LF}, v_{HU}, \) and \(d_H\) constant). Figure 1b) shows the impact of \(v_{HU}\) and \(d_H\) on the relative profit, holding \(v_{LF}, v_{LU}, \) and \(d_L\) constant). The unshaded regions indicate parameter values for which neither MD nor PS can outperform ND. In the polka-dotted regions, PS is more profitable than both MD and ND. In shaded regions, MD is more profitable than both PS and ND.

Consistent with Corollary 1, MD is not a useful price discrimination tool if either L-type consumers are too impatient (i.e., \(d_L < 2/3\) in Figure 1a) or H-type consumers are too patient (i.e., \(d_H > 5/8\) in Figure 1b).

However, even with such parameters, L-type consumers may have sufficiently weak preferences (i.e., \(v_{LU} > \frac{1}{2}\) in Figure 1a) and/or H-types may have sufficiently strong preferences (i.e., \(v_{HU} < \frac{1}{2}\) in Figure 1b) so that PS is more profitable than ND. Thus, PS expands the range of market settings under which the seller can price discriminate, i.e., in the regions denoted “Only PS is advantageous to ND,” price discrimination on the
basis of time is not profitable, but price discrimination on the basis of consumer preference strength is profitable. In contrast, in the regions denoted “Only MD is advantageous to ND,” price discrimination is profitable if it is done on the basis of time but not on the basis of consumer preference strength. Thus, PS and MD can be viewed as complementary strategies, with one strategy often being profitable in markets where the other strategy would not be beneficial.

In markets where both MD and PS outperform ND, the relative advantage of the two price discrimination mechanisms depends on two effects: (1) Difference in Value to L-types, i.e., how valuations for the low-value consumers differ across the two price discrimination mechanisms, and (2) Difference in Surplus to H-types, i.e., how the amount of surplus that must be allocated to the high-value consumers differs between MD and PS.\(^\text{10}\) First consider the Value-to-L-types effect. Under both MD and PS, low-value consumers purchase a product that is less valuable to them than their preferred product in the first period and the seller charges a price that is just low enough to induce them to make this purchase. Thus, the difference in revenue per L-type customer between MD and PS equals the difference between how much such a customer values a probabilistic good in Period 1 and how much she values her preferred good in Period 2. The relative advantage of PS is greater the weaker the SOPs of low-value consumers and the less patient they are (Proposition 1(c)). On the other hand, PS is unable to create an advantage (relative to MD) in Value to L-Types if low-value customers are patient but “picky” about which product they consume.

We now turn to the Surplus to H-types. Under both MD and PS, the seller must set its first-period price for the specified goods such that the high-value consumers will purchase their preferred products in Period 1. Specifically, the seller must reduce the price below these consumers’ willingness-to-pay so they receive at least as much surplus as they would receive if they purchased the discounted product. This maximum obtainable price depends critically upon how much the H-type consumers value the alternate purchase offering and also on the price of this alternate purchase option. Specifically, as indicated in Proposition 1(c), the PS advantage increases as the SOP of low-value consumers weakens (so that \(P^\text{PS}_o\) is higher), as low-value consumers become less patient (so that \(P^\text{MD}_2\) is lower), as the SOP of high-value consumers become stronger (so that purchasing the probabilistic good would yield less surplus), and as the latter become more patient (so that waiting to purchase until the second period would yield more surplus). On the other hand, PS fails to generate an advantage in Surplus to H-types if low-value customers are patient and picky, while high-value customers are impatient and are not picky. In short, whether or not PS generates an advantage in Surplus to H-types depends on whether it is easier for the seller to prevent high-value

\(^{10}\) In a Hotelling model, which is most often employed in the extant literature (e.g., Jiang 2007; Fay and Xie 2008,2010; and Jerath et al. 2010), the Surplus to H-types effect would be missing since, in that model, no consumer obtains a positive surplus from purchasing the probabilistic good and, thus, no extra incentive is needed to induce strong-preferenced consumers to buy their preferred good. Therefore, the current model has the advantage of allowing us to analyze the cannibalization effect of PS.
consumers from opting for the (discounted) probabilistic good or to prevent them from waiting until the second period to purchase their preferred products.

As shown in the equation in Proposition 1(b), PS is preferable to MD only if the sum of these two effects is positive. If PS creates a more valuable “damaged” product for low-value consumers and a lower (or no change in) required surplus for high-value consumers, then PS is clearly more profitable than MD. This occurs in the region labeled “A” in Figure 1b. On the other hand, neither advantage is present in region “C” of Figure 1a. Here, MD is more profitable than PS. If the two effects go in different directions, the larger effect determines which pricing discrimination mechanism is more profitable. Thus, even if PS has one disadvantage, this strategy can still be more profitable than MD if its advantage is greater than its disadvantage. In the regions labeled “B1,” PS creates a sufficiently large Value-to-L-types effect to offset the negative Surplus-to-H-types effect. However, in the regions labeled “B2,” the negative Surplus-to-H-types effect is of a greater magnitude, and thus MD is more profitable than PS.

3.4. Summary

In sum, for a price discrimination mechanism (e.g., PS or MD) to improve profit, it must create a new purchase option that is attractive to low-value consumers (thus enabling the firm to earn significant revenue from the new purchase option), but is not attractive to high-value consumers (so that high margins can be maintained for the original products). Whether varying prices over time (via the MD strategy) or creating a probabilistic product (via the PS strategy) is more advantageous, depends crucially upon the heterogeneity in consumers’ degree of patience and the strength of their preferences. Thus, in some markets we would expect PS to be more profitable than MD and in other markets for the reverse to be true (depending on whether or not the condition from Proposition 1(b) holds).

4. MODEL EXTENSION: DEMAND UNCERTAINTY

Since retailers often cannot predict which products will be more popular, inventory is depleted asymmetrically, with popular products selling faster and unpopular products selling more slowly than expected. Unpopular items that remain after the primary selling season are often severely marked down. In this section, we incorporate demand uncertainty into the base model from Section 3 to examine whether PS remains a viable strategy (relative to MD) in such market settings and to garner additional insights into how demand uncertainty impacts the tradeoff between the two strategies. Specifically, at the time inventory orders are made and when first-period prices are chosen, the seller does not know whether product A or product B will be the more popular good. We assume that the seller knows the value of $\alpha$, i.e., that one good will be more popular than the other, but not whether $\alpha$ will apply to product A or product B. Instead, the seller

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11 Note, we assume that the low- and the high-value segments are of equal size. With asymmetric segment sizes, these two effects would need to be weighted according to each segment’s size.
believes each product is equally likely to be the popular one. In period 2, the seller learns which product is popular and can adjust its prices accordingly.

In this section, we derive the optimal inventory orders that occur under MD and PS and make the simplifying assumptions that \( c < \min\{d_L v_{LF}, (v_{LF} + v_{LU})/2\}, d_H = d_L/2, \) and \( v_{LU} = v_{HU} = 0 \). These conditions guarantee that MD and PS are advantageous in the absence of demand uncertainty, i.e., the conditions given in Corollary 1 and Corollary 2 both hold, and that the L-type’s value of both the probabilistic good under PS and the delayed product under MD exceed the cost of acquiring a unit of a product in order to meet this demand. Lemma 2 gives the optimal solution under PS when demand uncertainty is present:

**Lemma 2 (PS with Demand Uncertainty)**

If \( d_H = d_L/2, v_{LU} = v_{HU} = 0 \) and \( c < \frac{v_{LF}}{2} \), in the presence of demand uncertainty,

(a) The firm will purchase \( \frac{2\alpha + 1}{2} \) units of both product A and product B.

(b) The resulting profit is \( \Pi^{PS, DU} = v_{LF} + \frac{1}{2} - (2\alpha + 1)c \).

Lemma 2 shows that, if costs are sufficiently low, the inventory order under PS will enable the firm to sell to all customers (of both types). Demand uncertainty does not impact the prices that the firm sets or the amount of sales. However, the firm must order \( \frac{2\alpha + 1}{2} \) units of both product A and product B, whereas without demand uncertainty, the firm would order units of the \( \frac{2\alpha + 1}{2} \) popular good and \( \frac{3 - 2\alpha}{2} \left( \frac{2\alpha + 1}{2} \right) \) units of the unpopular good. Thus, demand uncertainty leads to excess inventory for the unpopular good and reduces profit.

Proposition 2 compares profit and inventory utilization under PS to that which occurs under MD when demand uncertainty is present.

**Proposition 2 (Demand Uncertainty with Costly Inventory)**

Probabilistic Selling addresses demand uncertainty more efficiently than does Markdown Selling. Specifically,

(a) For sufficiently low costs \( \left( c < \max\left[ \alpha, \frac{2\alpha}{3} d_L v_{LF} \right] \right) \), demand uncertainty reduces the profit under Markdown Selling more than under Probabilistic Selling.

(b) Relative to Markdown Selling, Probabilistic Selling achieves a higher level of inventory utilization and reduced wastage for all \( c < \frac{d_L v_{LF}}{2} \).

(c) Total sales are (weakly) higher under Probabilistic Selling than under Markdown Selling.

In order to understand Proposition 2, it is instructive to examine how products with asymmetric demand are allocated in the absence of demand uncertainty. Under MD, H-type consumers purchase their preferred good (where \( \alpha \) prefer the popular good) in the first period and L-type consumers purchase their
preferred good (also where $\alpha$ prefer the popular good) in the second period. Thus, across the two selling periods, $2\alpha$ units of the popular good and $2(1-\alpha)$ units of the unpopular good are sold. Under PS, in the first period, H-type consumers purchase their preferred good (where $\alpha$ prefer the popular good) and L-type consumers will purchase the probabilistic good (where a fraction $1/2$ will be assigned to the popular good). Thus, in total, $\alpha + \frac{1}{2} = \frac{1 + 2\alpha}{2}$ units of the popular good and $1 - \alpha + \frac{1}{2} = \frac{3 - 2\alpha}{2}$ units of the unpopular good are consumed. In the absence of demand uncertainty, under both PS and MD, the firm orders inventory to exactly meet the demand for each product, which involves purchasing more units of the popular product. It is critical to note that total consumption of the popular good and the difference in consumption between the popular and the unpopular good are both higher under MD than under PS.

When demand uncertainty exists, the firm does not know which product will be more popular and thus inventory orders cannot be tied to a product’s popularity. Mathematically, this is equivalent to imposing a constraint, $K_A^s = K_B^s, s \in \{\text{ND, MD, PS}\}$ on the maximization problems in Section 3. It is intuitive that, as Proposition 2(a) states, this constraint will have a larger detrimental impact on MD since, without such a restriction, the inventory orders would be more asymmetric under MD than under PS.

Under PS, as long as costs are not too large, Lemma 2 shows that the firm’s best response to demand uncertainty is to order $K_{A,DU}^{PS} = K_{B,DU}^{PS} = \frac{2\alpha + 1}{2}$ units. Some units of the unpopular good will go unsold. Thus, even though the firm generates the same revenue as in the absence of demand uncertainty, higher costs are incurred (due to a larger total amount of inventory).

Under MD, one of two options will be optimal. First, if costs are very low $c \leq \frac{d_LV_{LF}}{2}$, the firm will order $K_{A,DU}^{MD} = K_{B,DU}^{MD} = 2\alpha$ units so that sales are the same as under demand uncertainty. Since the firm sells to all consumers and at the same prices as when there was no demand uncertainty, the latter has no effect on the revenue earned under MD. However, costs increase due to the higher inventory order, thus leading to unsold units remaining and lower profit. Figure 2a graphs the number of unsold units under MD and shows the magnitude of the profit decrease that demand uncertainty inflicts on the MD strategy ($\Delta_{MD,DU}$) as a function of demand asymmetry ($\alpha$). Notice that, as $\alpha$ increases, the seller faces more severe demand uncertainty. Two results are apparent: (1) MD results in more unsold units than does PS (since inventory orders are higher under MD); and (2) demand uncertainty reduces profit more for MD than for PS. As demand uncertainty grows, the magnitude of both of these effects increases.

Second, if costs are higher $c > \frac{d_LV_{LF}}{2}$, the retailer will order less than $2\alpha$ units of each good. This
scenario is illustrated in Figure 2b. While ordering less inventory reduces the number of unsold units, this also creates stockouts, as some or all of the L-type consumers would be unable to purchase in the second period. As stated by Proposition 2(c), total sales would be higher under PS since, in this case, all consumers receive one unit of one product, whereas under MD some consumers do not purchase anything. This reduction in revenue makes the profit decrease from demand uncertainty larger under MD than under PS.

It is important to recognize that, whichever of these two options is adopted under MD, PS offers the seller a more effective tool for addressing demand uncertainty. Relative to PS, MD either requires a larger investment in inventory (as in option one) or fewer total sales (as in option two). In sum, not only does PS remain a viable strategy for retailers who face demand uncertainty, the relative advantage of PS over MD becomes even stronger when demand uncertainty is present.

5. CONCLUDING COMMENTS

5.1. The Benefits of Probabilistic Selling

Markdowns are commonplace in retailing and can result in painful repercussions, such as very low (or even negative) margins and a consumer mentality of delaying purchases in anticipation of a big sale. In this paper, we show that probabilistic selling (PS) can be preferred over a typical markdown (MD) strategy. In particular, we illustrate that introducing probabilistic products can be beneficial because they (1) allow the seller to obtain higher prices from customers with strong preferences for one product over alternatives (because probabilistic products may present less of a cannibalization threat to full-price sales than do traditional time-dependent markdowns); (2) reduce the size of necessary discounts (because consumers may be willing to pay more for a randomly selected product at the beginning of the season than for their preferred product late in the season); and (3) more effectively address the negative consequences of demand uncertainty and limited capacity by helping a firm prevent stockouts and reduce excess inventory. We find that, while both probabilistic products and markdowns can be used to segment consumers, PS is most effective when low-value consumers have weak preferences and high-value consumers have strong preferences. In contrast, markdown pricing relies on specific differences in consumers’ willingness to delay purchase until a later date. The markdown pricing strategy is most effective when low-value consumers are relatively patient and high-value consumers are relatively impatient. Because the MD and PS strategies rely on distinctly different mechanisms to segment the market, PS is a valuable additional tool for retailers, especially those who operate in markets where markdowns are ineffective (e.g., non-fashion products or products unaffected by seasonality so that consumer arrival times are not negatively correlated with product valuations or price sensitivity).

5.2. Limitations and Future Research

While markdown pricing strategies are ubiquitous, the PS strategy is still nascent. More research is needed to fully flesh out how to optimally implement this selling strategy. For example, it is unclear how PS
impacts which products (and how many) a seller should stock. Other research has considered a version of a probabilistic good (which they call a “flexible good”) in which the seller does not assign products to buyers until after demand uncertainty is resolved (e.g., Gallego and Phillips 2004; Post 2010; Mang et al. 2012). As refinements to PS are made, such as determining the optimal product mix and the optimal timing of product assignments, the profit obtainable from the strategy should increase and, thus, the market settings in which PS is advantageous to traditional markdowns should be enlarged beyond the regions identified in this paper, which utilizes a base case (and a rather conservative) definition of PS.

The paper employs a stylized model and thus has several limitations. First, in our model, we assume no inventory holding costs and that first- and second-period profits are equally weighted. If holding costs and discounting of future cash flow were incorporated, PS would have the additional benefit over MD of shifting sales forward in time. This advantage would be especially important to sellers who desire rapid turnover or incur significant inventory holding costs. Second, there may be circumstances in practice in which it is beneficial to offer both temporal-based discounts and discounted probabilistic products. Third, our stylized model leads to the firm charging the same price for each product (both in the first and second periods). In addition, we assume that inventory orders can only be placed prior to the selling season. We believe our assumptions are quite realistic for many retailers since there are many cases in retailing where markdowns are symmetric and mid-period inventory acquisition is not possible. However, in practice, some retailers do offer different-sized markdowns as they better learn the demand for each particular product and others are able to replenish inventory throughout the selling season. Relaxing these (and other) assumptions of the model could be a valuable direction for future research.

Additional research questions that remain unresolved include: What are the ramifications of PS on suppliers (e.g., their market power and effects on supply chain dynamics)? Can PS be effective when the seller is uncertain about total category demand rather than the relative popularity of each specific item? Do some consumers prefer the probabilistic good because (rather than in spite of the fact that) it offers a gamble? Does PS alter consumer preferences (e.g., dilute brand- or store loyalty)?

REFERENCES


Table 1

Related Literature and Distinguishing Characteristics of Current Paper

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*“AM” = Analytical Modeling; “E” = Empirical; “LE” = Lab Experiments

b In Fay and Xie (2010), the two selling periods are the advanced period (prior to consumers learning their true valuations) and the spot period. In the current paper, consumers know their valuations in both periods. Thus, the major difference between the two periods is the time delay rather than differences in the information available to consumers.
The Advantage of Price Discrimination

a) $v_{LF} = \frac{3}{4}$; $v_{HU} = d_H = \frac{1}{2}$

b) $v_{LF} = d_L = \frac{3}{4}$; $v_{LU} = \frac{1}{2}$

The patterns of the regions in Figures 1a and 1b represent which strategy is most advantageous for the retailer. In the shaded regions, MD is the most profitable strategy. In the polka-dotted regions, PS is the most profitable strategy. In the unshaded regions, ND is the most profitable strategy. In the regions labeled A, B1, B2, and C, both PS and MD are more profitable than ND. In region A, PS generates more revenue than MD from both the L-types and the H-Types. In region C, PS generates less revenue from both consumer types. In regions B1 and B2, PS generates higher revenue from the L-types but lower revenue from the H-types. The net effect is that PS is more profitable than MD in region B1, but less profitable in region B2.

Note: The maximum value of $v_{LU}$ in Figure 1a is $\frac{3}{4}$ since, by definition, $v_{LU} < v_{LF} = \frac{3}{4}$. 
Figure 2

Impact of Demand Uncertainty on PS and MD

a) Low $c \left( c \leq \frac{d_L v_{LF}}{2} \right)$

b) Moderate $c \left( \frac{d_L v_{LF}}{2} < c < \frac{d_L v_{LF}}{2} \right)$

The graphs are drawn assuming $v_{LF} = d_L = \frac{3}{4}$. For Figure 2a, which is drawn for $c = .2$, total sales equals 2 for both MD and PS. Figure 2b is drawn assuming $c = .3$. 