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Price, quality, or portfolio adjustment?

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• In a game-theoretic framework I analyze a brand manufacturer’s response to market entry.

• I consider price, quality or portfolio adjustment as basic options for the manufacturer.

• Depending on the situation I show which alternative is optimal.

• Situational factors include launching costs, entry uncertainty, or technological progress.
How to protect your premium product from low-price competitors: Price, quality, or portfolio adjustment?

Peter-J. Jost†

Abstract

In a game-theoretic framework, I analyze how a brand manufacturer can thwart new entrants into its market. Three strategic options are considered, a price adjustment of the premium product, a quality adjustment of the premium product and a portfolio adjustment of adding a fighter brand. In a basic setup, I show that the incumbent’s best response to entry is to choose a portfolio adjustment. If, however, the incumbent is uncertain about whether the rival firm will enter the market, a price adjustment of the premium product might be the better alternative if launching the fighter brand is associated with costs. Moreover, if technological progress improves the efficiency of product development, a combined quality and portfolio adjustment might be the best alternative for the incumbent.
1 Introduction

Brands are one of the few strategic assets available to a company that can provide a long-term sustainable competitive advantage. Wrigley’s, Coca-Cola or Gillette are examples for such brands that have remained market leaders over more than 80 years. These brand manufacturers enjoy higher-than-average margins because consumers are willing to pay high prices for their products. However, keeping the price level high gives rise to increasing competition from competitors entering the market with aggressive pricing. Generics, private labels or "clones" imitating the brand leaders are such low-price competitors. They enter markets where patents protecting the leader's technologies have run out, where the market volume, market growth or the brand’s profits are attractive, where large parts of the market with price-conscious consumers have not been served, or where the leader’s production is so labour expensive that foreign manufacturers have strong cost-advantages.

What should a brand manufacturer do to counter such attacks? The marketing literature on brand management suggests several reactions, see, for example, Aaker (2004, p. 230ff) or Keller (2008, p.224ff). Besides not to react at all there are three main types of basic responses to such low-price competitors:

- **Price adjustment**: One of the traditional reaction of a brand manufacturer is to lower the price of its premium product significantly. In the consumer good industry major brands usually cut prices to compete with private labels. For example, StarKist reduced the prices of its tuna to only five cents above the price of private labels. As a consequence, the share of private labels in this category sliced in half from 20 percent to 10 percent, see Keller (2008, p.224).

- **Quality adjustment**: To stay ahead of low-price competitors brand manufacturers may also emphasize innovation to improve existing brand products. Using this response requires a repositioning of the premium product. H.J. Heinz, for example, has retained more than 50 percent market share in the ketchup category for years by aggressive packing and product developments, see Keller (2008, p.224).

- **Portfolio adjustment**: A third option which has not been used widely so far, is to enlarge the product portfolio. In this case the brand manufacturer adds a second, lower positioned product, a so called "fighter brand", to the existing higher positioned brand product. For example, Philip Morris has used Chesterfield as a fighter brand to flank its brand Marlboro, see Quelch and Harding (1996, p.106).

Of course, these general responses to fight against new entrants could also be combined and modified. An example is Procter & Gambler with its leading diaper brand Pampers, see Berry and Schiller (1994). As the market share of private labels grew P&G repositioned its number three brand, Luvs, as a fighter brand. Despite its efforts to ensure that Luvs offers considerably less value
than Pampers, Luv stole sales from the premium brand. Only after a quality adjustment of the premium brand in form of a new, thinner diaper did Pampers recover.

This shows like many other case studies in the literature that neither a price nor a quality nor a portfolio adjustment is always successful. In fact, for each of these responses there exist several counterexamples demonstrating that this particular instrument failed for the company considered: Philip Morris’ decision, for example, to defend the market position of its brand Marlboro in 1992 by offering a 20 percent price cut per pack resulted in a 15 percent decrease in total revenues, see Hoch (1996). Or United Airlines, after launching its fighter brand Ted in 2004 to fight against discount airlines, ceased operations in 2009 after years of failure, see Ritson (2009). It is for this reason, that some researchers advise to avoid fighter brands at all, see Jain and Tucker (1997), while other researchers recommend exactly the opposite, e.g. Ritson (2009), or to use other responses instead, for example Steiner (2004, p.118), for whom innovation is "one of the strongest competitive weapon".

In this paper, I propose a model that highlights the costs and benefits of these options and evaluates the best response of a brand manufacturer in a game-theoretic framework. In particular, I consider a vertically differentiated market in which an incumbent firm can offer products of different qualities over two periods. In the first period the incumbent acts as monopolist whereas the market becomes a duopoly in the second period. This two-period framework allows me to analyze not only the short-run, one period consequences of entry into this market, but also the long-run effects potential entry has on the monopolistic behavior of the incumbent firm. To concentrate on the incumbent’s optimal response to entry I assume that consumers are uniformly distributed according to their willingness to pay for quality. This assumption implies that the market has one heterogeneous market segment only. In particular, the monopolistic firm never finds it optimal to produce a second brand without the threat of competition as a consequence of a decrease in its marginal revenues in the market. Within this framework, I then analyze the three above mentioned different basic responses of the incumbent firm to entry.

In the first scenario, the incumbent adjusts the price of its premium product in response to entry. This price reduction implies that the price-quality ratio of its premium product shrinks so that the market demand for its product is higher in the second period than in the monopoly period. On the other hand, however, a price cut implies that contributions are lost from those customers that previously had been willing to pay the higher price for the premium product.

In a second scenario, I consider the case in which the incumbent adjusts the quality of its premium product. Such a quality adjustment can go into two distinct directions. One possibility for the incumbent is to decrease its product quality. However, this implies that price competition becomes tougher and, in turn, leads to less profits. The other possibility is to upgrade its product quality. Although this direction of quality adjustment softens price competition and thus implies higher profits, improving product quality comes with costs for the incumbent. Since the incumbent lost its monopoly position, its marginal
benefits from quality improvement are lower in the second than in the first period.

In a third scenario, I assume that the incumbent firm extends its product portfolio by offering a fighter brand. The challenge here is to position the second product such that it serves two purposes: On the one hand, it should fend off the entrant in order to directly compete with the premium product. On the other hand, the introduction of the second brand should not lead to cannibalization in the sense that current consumers of the premium product switch to buy the incumbent’s lower quality product although they would have never switched to the low-price entrant’s product. To reduce this negative effect of cannibalization the incumbent then tries to position the quality level of its second product as low as possible to soften price competition. This, however, irretrievably leads to a situation in which the competitor would launch its product in the middle market segment, thus implying more price competition in the market. Trading off these effects I show how the incumbent can optimally position its second product such that the upper half of the entire market is still covered by its premium product.

Finally, I compare the three possible responses of the incumbent and analyze under which circumstances which option is more beneficial. In the basic model it turns out that a portfolio adjustment is always the best alternative. If, however, launching a second product implies a large enough cost for the incumbent, a price adjustment becomes the more valuable the greater the incumbent’s uncertainty is about whether the rival firm will actually enter the market or not. Moreover a quality adjustment becomes beneficial if the incumbent’s investments’ efficiency improves from Period 1 to Period 2. In particular, the introduction of a fighter brand accompanied by a quality improvement of its premium brand is then the best option for the incumbent firm as a response to entry.

The present paper is organized as follows. The next section presents the basic model of monopoly in a vertically differentiated market. In Section 3 entry into this market is considered in a basic model and the three strategic options of the incumbent to thwart the entrant are analyzed and compared. Section 4 then extends this basic model in three directions and considers the effect of imitation costs by the entrant, launching costs by the incumbent and the possibility of technological progress on equilibrium behavior and the incumbent’s optimal response. Section 5 then concludes with some final remarks. Most proofs of the results are presented in the Appendix.

2 Monopoly in a Vertically Differentiated Market

As a benchmark I briefly study in this section a model of vertical product differentiation under monopoly, see Gabszewicz and Thisse (1979), Shaked and Sutton (1982) and Choi and Shin (1992). First, the preferences of consumers are described and the supply side is introduced. I then analyze the monopoly
case in the absence of entry which is later compared with the entry game.

2.1 The basic framework

Consider a market in which an incumbent firm $I$ can offer products in different qualities over two periods. Assume that consumers can be described by a parameter $\theta$ which is uniformly distributed on the interval $[0, \overline{\theta}]$ with unit density, where $\overline{\theta} \geq 1$. The parameter $\theta_i$ of consumer $i$ can be interpreted as her willingness to pay for a product. Each consumer buys not more than one unit of the product per period of the qualities available in the market place. Her net surplus when buying a product with quality $q$ at price $p$ then is

$$u_i(q, p) = q\theta_i - p.$$  \hfill (1)

Consumers who do not purchase receive zero utility. I assume that (1) holds for all consumers in both periods.

On the supply side, the incumbent introduces its products in the beginning of Period 1 to the market. To offer a variety of product qualities $q_1, \ldots, q_n$ with $q_1 > \ldots > q_n > 0$ for $n \geq 1$, positive development costs are required. I assume that the product with the highest quality determines total development costs. In particular, the incumbent has to incur development costs $c_I(q)$ to produce the highest quality $q = q_1$ with

$$c_I(q) = \frac{1}{2}\gamma_I q^2.$$ \hfill (2)

$\gamma_I > 0$ is a parameter that reflects, for example, the efficiency of the incumbent’s investments. When choosing its other product qualities $q_2, \ldots, q_n$ the incumbent then has no further development costs such that it can costlessly develop products of lower quality. Moreover, I assume that the variable costs of production are independent of quality and equal to zero.

The incumbent offers its product qualities at monopoly prices $p_{1I}, \ldots, p_{nI}$ in both periods $t = 1, 2$. If $x_{1i}$ denotes the resulting demand of product $(q_{1I}, p_{1I})$, $i = 1, \ldots, n$, the incumbents monopoly gross profits over both periods are

$$\pi_M = \sum_{i=1}^{n} p_{1I} x_{1i}^1 + \delta \sum_{i=1}^{n} p_{1I}^2 x_{1i}^2 - c_I(q_{1I}),$$

where profits in Period 2 are discounted by the factor $\delta \in [0, 1]$.

2.2 Product differentiation in the monopoly case

As a benchmark consider briefly this monopoly case. So suppose that there is no potential entrant and the incumbent acts as a monopolist over both periods. Of course, monopoly prices are identical in both periods as well as demand.

Consider the case in which the monopolist offers only one product $(q_M, p_M)$ and let $\theta_M \in [0, \overline{\theta}]$ be the marginal consumer who is served by the monopolist. Then $\theta_M$ is given by the condition $q_M \theta_M - p_M = 0$, that is, $\theta_M = p_M / q_M$. 

\[ \]
Hence, all consumers with a higher willingness to pay than $\theta_M$ buy $(q_M, p_M)$ and demand is $x_M = (\overline{\theta} - \theta_M)$. The monopoly price then maximizes $p_M (\overline{\theta} - \theta_M)$ and is characterized by the condition that the marginal gain from serving an additional consumer is equal to the loss of all other consumers. Using the optimal monopoly price as a function of quality $q_M$, the incumbent then maximizes profits.

**Proposition 1** In the absence of entry, the monopolist offers the following price-quality combination in both periods:

$$q_M^* = \frac{\overline{\theta} (1 + \delta)}{4\gamma_I}, \quad p_M^* = \frac{\overline{\theta} (1 + \delta)}{8\gamma_I}.$$  

Moreover, the incumbent never has an interest to produce more than this product quality.

The optimal monopoly price-quality relation always splits the market equally, $\theta_M = \overline{\theta}/2$: Only those consumers whose willingness to pay this price is higher than the one of the medium consumer will buy the product. As a consequence, the monopolist’s revenues are increasing in product quality due to a higher monopoly price, and, in turn, since quality is costly, the optimal product design balances marginal revenues and marginal development costs.

Since a high quality product only captures the upper part of the market, it seems beneficial for the monopolist to introduce a second lower quality product. However, although the monopolist can enlarge the overall demand in the sense that some consumers who didn’t buy a product before will now buy the lower quality product, the cannibalization effect outweighs this demand effect: In fact, some consumers now switch from the high quality product to the low quality product. Since the marginal gain from selling the high quality product is always greater than the marginal gain from selling the low quality product, the monopolist has an incentive to increase the low quality product price up to the point at which the price-quality relations of the two products are identical. But then nobody would demand the low quality product and, consequently, the monopolist would not introduce a second product at all.1

### 3 Entry into a Vertically Differentiated Market

In this section I extend the model of vertical product differentiation in Section 2 and consider entry in Period 2. According to the discussion in the introductory section, one can distinguish between three different responses of the incumbent firm to entry, see Figure 1: First, the incumbent adjusts its price setting behavior; second, the incumbent adjusts also the quality of its premium product; and third, the incumbent firm extends its product portfolio by offering a fighter brand.

1Note that this result depends on the assumption that marginal costs of production are unaffected by the level of quality. Once higher quality has a higher marginal cost, the monopolist might find it advantageous to market more than one product quality.
In the following Section 3.1, I first describe the basic framework of entry in Period 2 assuming that the incumbent firm does not adjust its initial product. This case of price adjustment as a response to entry is analyzed in Section 3.2. I then consider the incumbent’s option to adjust the quality level of its initial product for Period 2, the case of a quality adjustment. In Section 3.4 I then assume that the incumbent introduces a second product, the case of portfolio adjustment. These three different responses of the incumbent firm to entry are then compared in Section 3.5.

3.1 The basic framework

I extend the previous model of vertical differentiation as follows: In the beginning of Period 1, the incumbent introduces a product with quality $q_I > 0$ in the market and remains monopolist during this period. In the beginning of Period 2, a second firm, called the entrant, enters the market. The entrant then competes for consumers by offering a product with quality $q_E$ with $q_E > 0$. When choosing its product quality I assume that the entrant’s cost function captures two stylized facts relevant in business practice. First, an entrant usually has the possibility to imitate the incumbent’s products, and, therefore, to obtain a certain quality level $q_E \leq q_I$ with lower development costs. I assume here that there is full imitation, that is, the entrant has free access to the incumbent’s technology and, hence, no development costs for a lower quality level. And second, the entrant as quality follower usually has a lower level of efficiency than the incumbent to develop a higher level of quality $q_E > q_I$. Both assumptions guarantee that the incumbent is always the high quality firm and the entrant
produces a lower quality as follower.

The incumbent observes the entrant’s quality choice \(q_E\) and both firms then set prices \(p_I^2\) and \(p_E^2\) conditional on \(q_I\) and \(q_E\). This choice then determines firms’ demands \(x_I^2\) and \(x_E^2\) in the second period. Since production is costless the entrant’s profits in period 2 are

\[
\pi_E = p_E^2 x_E^2.
\]

The total profits of the incumbent over both periods then are

\[
\pi_I = \pi_I^1 + \delta \pi_I^2 - c_I(q_I) = p_I^1 x_I^1 + \delta p_I^2 x_I^2 - c_I(q_I).
\]

In sum, I consider a two-period model with the following sequence of events:

**Stage 1a** The incumbent sets a quality level \(q_I\) and charges a price \(p_I^1\) for the product in the first period.

**Stage 1b** Consumers choose whether to purchase the product or not.

**Stage 2a** A second firm chooses a product of quality \(q_E\) and enters the market.

**Stage 2b** Having observed the product qualities offered in the market, the two firms compete by simultaneously choosing prices \(p_I^2\), resp. \(p_E^2\), in the product market.

**Stage 2c** Finally, a consumer will buy from the firm that offers the best price-quality combination or she doesn’t buy at all.

Whereas this game structure assumes that the incumbent firm foresees potential entry in Period 2 when choosing its product quality in Period 1, an alternative scenario is possible in which the incumbent’s premium quality brand is fixed and the game starts in Period 2. Since the price-quality ratios as well as the products’ demands do not change compared to the general two-period model I focus in the following on the long-term perspective and include the first period into the analysis.

### 3.2 Option 1: Price adjustment

To analyze firms’ optimal behavior, I use backward induction and start with the consumers’ consumption decision in Stage 2c. For a given quality level \(q_{I1}\) of the incumbent firm suppose that the second firm entered the market and offered a quality choice \(q_{E1} < q_{I1}\). Then the price setting behavior of the two firms in Stage 2b as well as the entrant’s quality decision in Stage 2a differ in no way from the model of Choi and Shin (1992) and lead to the well known result that \(q_{E1}^* (q_{I1}) = \frac{1}{4} q_{I1}\). When introducing product quality \(q_{I1}\) in Stage 1a, the incumbent then maximizes total profits from monopoly and competition over both periods, taking into account the entrant’s best response \(q_{E1}^* (q_{I1})\):

\[
\pi_{I1} = \frac{1}{4} \theta^2 q_{I1} + \delta \theta^2 \frac{7}{48} q_{I1} - \frac{1}{2} \gamma_I q_{I1}^2.
\]
From the first-order condition one can then immediately derive the optimal quality level $q^*_I$:

**Proposition 2** Under price adjustment the incumbent responds to entry with lowering its product price under the first period monopoly price. Its product quality is below the monopoly quality but still higher than the entrant’s one.

**Proof.** See Appendix.

Several remarks are worth noting: First, under entry the incumbent firm still offers the high quality product and leaves the lower part of the market to the entrant. However, since the incumbent foresees entry in Period 2, it chooses a lower product quality than in the monopoly case to defend its position:

$$q^*_M > q^*_I = \frac{\theta^2 (12 + 7\delta)}{48\gamma_I} > q^*_E = \frac{\theta^2 (12 + 7\delta)}{84\gamma_I}$$

Product prices are ordered according to these qualities with the following properties: In the first period, the incumbent covers the upper part of the market as in the monopoly case. Price competition in the second period, however, implies the incumbent firm uses price cuts to defend the rival’s entry in Period 2. In turn, the price-quality ratio shrinks so that more consumers with a lower willingness to pay now buy the high quality product:

$$\frac{p^*_M}{q^*_M} > \frac{p^*_I}{q^*_I} > \frac{p^*_E}{q^*_E}$$

This also reflects market demand which is higher for product $q^*_I$ in Period 2 than in the monopoly case,

$$x^*_M = \frac{1}{2} x^*_I < x^*_E = \frac{7}{12} x^*_I$$

Overall, the incumbent’s profit are lower than its monopoly profits but higher than the entrant’s ones.

### 3.3 Option 2: Quality adjustment

Besides a price adjustment the incumbent could also consider an adjustment of its product quality. Two directions are possible: The incumbent could either decrease product quality or, alternatively, upgrade its product.

Consider the first alternative and suppose the incumbent lowers its product quality from $q^*_I$ to $q^*_I$ before the second firm enters the market. From the model of Choi and Shin (1992) it is well known that the entrant positions its product in a low quality range by setting $q^*_E = \frac{4}{7} q^*_I$. This, however, implies that the equilibrium prices are increasing in the product quality the incumbent offers,

$$p^*_I = \frac{1}{4} \gamma q^*_I, \text{ and } p^*_E = \frac{1}{14} \gamma q^*_I.$$
Hence, price competition becomes tougher the more the incumbent adjusts its high product quality downwards and, in turn, leads to less profits. In sum, the incumbent has no incentive to adjust the quality of its premium product downwards.

However, this argumentation suggests, that it might be beneficial for the incumbent to adjust the quality of its premium brand upwards as a response to potential entry. Concerning price competition this implies higher prices and, in turn, higher profits for the incumbent. To discuss this in more detail suppose that the incumbent firm can upgrade its product $q_I$ before the second firm actually enters the market. Of course, improving product quality comes with positive costs for the incumbent. Using the cost function (2) for the development of product quality I assume in the following that a quality improvement to $q_I > q_{I2}$ implies costs

$$c_I(q_{I2}) = \frac{1}{2} \gamma_I (q_{I2}^2 - q_{I2}^2).$$

Hence, the incumbent’s development costs $\gamma_I q_{I2}/2$ from the first period are sunk and any later quality improvement in Period 2 is as expensive as it would have been in Period 1. This implies that the incumbent’s marginal costs of quality improvement are identical to the ones in Period 2, $c_I'(q_{I2}) = \gamma_I q_{I2}$. On the other hand, however, the incumbent’s profits in Period 2 are lower than the ones in Period 1 due to competition,

$$\pi_{I2}^1(q_I) = \frac{1}{4} \theta_I q_I > \pi_{I2}^2(q_I) = \frac{7}{48} \theta_I q_I.$$

This not only implies that marginal benefits from quality improvement are lower in the second than in the first period, but also implies that the incumbent has no incentives to improve the initial quality level of its product.

**Proposition 3** In the basic model, it is never optimal for the incumbent to respond to entry with quality adjustment.

### 3.4 Option 3: Portfolio adjustment

A third option for the incumbent is to adjust its product portfolio as a response to entry. Under this option, the incumbent firm has the possibility to introduce a second product to thwart the new entrant. Of course, the previous discussion suggests that this second product has to be of lower quality than the initial product $q_I$. Indeed, positioning a second product higher in quality than the existing product cannot be optimal for the incumbent due to additional marginal costs but reduced marginal benefits in Period 2 compared to Period 1. As a

\[\text{In the absence of entry costs, this assumption is not restrictive. Indeed, knowing that the second firm always enters the market, it is always beneficial for the incumbent to improve the quality of its product ex ante before entry occurs than ex post after entry occurred. This is because in the first case the incumbent has the possibility to influence the entrant’s choice of product specification whereas the incumbent would only react to entry in the second case.} \]
consequence, I term the initial product as the high quality brand \( q_{IH} \) and the second, new product as its lower quality product \( q_{IL} \). The sequence of events in this modified interaction then is as follows:

**Stage 1a** The incumbent sets a quality level \( q_{IH} \) and charges a price \( p^1_{IH} \) for the product in the first period.

**Stage 1b** Consumers choose whether to purchase the product or not.

**Stage 1c** The incumbent has the possibility to credibly introduce a second product quality \( q_{IL} \) with \( q_{IL} < q_{IH} \).

**Stage 2a** A second firm enters the market and offers a product of quality \( q_{E3} \).

**Stage 2b** Having observed the product qualities offered in the market, the two firms compete by simultaneously choosing prices \( p^2_{IH}, p^2_{IL} \) resp. \( p^2_{E3} \), in the product market.

**Stage 2c** Finally, a consumer will buy from the firm that offers the best price-quality combination for her or she doesn’t buy at all.

This game has the following solution:

**Proposition 4** When responding to entry with portfolio adjustment the incumbent positions its fighter brand such that its premium product still covers the upper part of the market while the entrant launches a lower quality product.

**Proof.** See Appendix.

In equilibrium, the incumbent uses its second product as a firewall in order to weaken the market position of the entrant, that is

\[
q^*_{IH} = \frac{7^2(12 + (12 - 5\beta)\delta)}{4871} > q^*_{IL} = \beta q^*_{IH} > q^*_{E3} = \frac{4}{7} \beta q^*_{IH},
\]

with \( \beta = 0.54806 \). By positioning its second product in its quality below the one of the top product the incumbent fends off the rival firm in order to directly compete with its premium product. Of course, this introduction of a second product has drawbacks for the incumbent’s overall profits because it still competes with its premium product. Such cannibalization implies that current consumers of the premium product switch to buy the new second product although they would have never switched to the rival’s low-price product. To reduce this negative effect of cannibalization the incumbent tries to position the quality level of its second product as low as possible to soften price competition with the rival’s product, and in turn, to soften price competition between its own products. However, if the quality level of the second product was too low, it would lose its purpose as a firewall and the competitor would launch its product in the middle market segment. Although this would eliminate any cannibalization, such a positioning would lead to more price competition in the market. As a result, this not only reduces the demand for the incumbent’s second but also
for its premium product. Using the fighter brand as a firewall, however, allows
the incumbent to still cover the upper half of the entire market by optimally
choosing its product prices, as in the first period,

\[ x_{1H}^1 = x_{1H}^2 = \frac{\theta}{2}. \]

3.5 The optimal response

If one compares the incumbent’s price adjustment with its portfolio adjustment,
the latter one has two advantages: First, the incumbent can set a higher quality
product in Period 1, that is, \( q_{1H}^* > q_{11}^* \), see Figure 2. This is because its launch
of a second brand gives the incumbent more flexibility to adjust its premium
product towards its monopoly quality \( q_{M}^* \). And second, the entrant can be forced
to reduce the quality of its product introduction. This follows from the fact,
that the second product has a lower quality than the premium product, that
is, \( q_{L}^* < q_{M}^* \) and that the entrant’s best response is always \( 4/7 \) of the lowest
quality offered in the market.

Figure 2: Price-quality ratios in Period 2 under price (■) and portfolio
adjustment (▲)

The fundamental difference between a price adjustment and a portfolio ad-
justment response can also be seen by looking at the price-quality ratios for
these two options: Whereas with the price adjustment, Option 1, the incum-
bent reduces the price-quality ratio to attract more consumers with a lower
willingness to pay, the incumbent can adjust its portfolio by launching a second
product, Option 3, to protect its high quality product, see Figure 2. In fact, the relationships

\[
\frac{p_M^*}{q_M^*} = \frac{1}{2} > \frac{p_{IH}^2}{q_{IH}^*} = \frac{1}{2} \gamma \left( 1 - \frac{\beta}{2} \right) > \frac{p_{II}^2}{q_{II}^*} = \frac{1}{4} \gamma > \frac{p_{IL}^2}{q_{IL}^*} = \frac{1}{4} \gamma \beta > \frac{p_{E3}^2}{q_{E3}^*} = \frac{1}{8} \gamma
\]

show that because its second product still has a better price-quality ratio than the entrant’s product the incumbent can adjust the price-quality ratio of its premium product towards its monopoly product.

If one compares the profitability of the portfolio and the price adjustment response it is then not surprising that in the basic model the first is always better than the latter. The opposite is true for the entrant.

Proposition 5 In the basic model, the incumbent’s best response to entry is always to choose a portfolio adjustment.

Proof. See Appendix. ■

As a side remark, these findings also shed light on the question how a firm’s brand portfolio should be optimally designed to retain profit while maintaining brand equity, see e.g. Aaker (1991), Aaker and Joachimsthaler (2000) and the empirical studies by Randall et al. (1998) or Nijssen (1999). Using the price premium as a measure for brand equity, Figure 2 and the discussion above indicate that introducing a fighter brand can indeed preserve brand equity of the incumbent’s premium product. The recommendation to improve the quality of the existing premium product, however, cannot be justified in the basic model because if such a quality adjustment is profitable, it should have been introduced in the first period. In addition, the analysis shows that the performance impacts of a brand portfolio strategy not only depend on firm-specific aspects of brand portfolio strategies such as the number of brands or the degree of intraportfolio competition within a firm’s portfolio, see e.g. Reddy et al. (1994), Cohen et al. (1997), Andrews and Low (1998), Nijssen (1999), Putsis and Bayus (2001), Brodley (2003) or Morgan and Rego (2009), but also on the competitive positioning of a firm’s brands relative to its rivals’ products: the negative impact of intraportfolio competition might outweigh the profit loss from competitive pressures on premium brands.

4 Extensions

I now extend the basic framework of Section 3 in three directions: First, I relax the assumption of costless imitation by the entrant and discuss how this affects equilibrium behavior. Second, I assume that the incumbent incurs positive costs when launching a second product and analyze the incumbent’s optimal response in presence of entry uncertainty. Third, I introduce the possibility of technological progress and analyze when it is optimal to introduce a fighter brand and simultaneously increase the quality of the premium brand.
4.1 Imitation costs and quality adjustments

In the basic framework I assumed that the entrant can costlessly imitate the incumbent’s quality. In a more general setup, suppose that the capability of the entrant to imitate is limited, for example, by the intensity of imitation, the size of spillovers, or the degree of intellectual property rights violation. Let $\mu$ represent this degree of imitation, $\mu \in [0, 1]$, such that the higher the value of $\mu$, the easier imitation is: For $\mu = 0$, no imitation is possible, whereas $\mu = 1$ corresponds to the case of full imitation. When choosing a quality $q_E$, the entrant then has imitation costs

$$c_E(q_I, q_E) = \begin{cases} \frac{1}{2} \gamma_E (1 - \mu) q_I^2 & \text{if } q_E \leq q_I \\ \frac{1}{2} \gamma_E (q_I^2 - \mu q_E^2) & \text{if } q_E > q_I \end{cases}$$

where $\gamma_E$ is the entrant’s investment efficiency to develop a certain level of quality, see also Kovac and Zigic (2006) and Pepall (1997). Since the entrant usually is less efficient than the incumbent to produce a certain quality level, $\gamma_E > \gamma_I$. The entrant’s profits in period 2 then read as

$$\pi_E = p_E x_E - c_E(q_I, q_E).$$

As in the basic model, it is never optimal for the entrant to produce a higher level of quality than the incumbent. This results from its lower level of efficiency. Moreover, when choosing the optimal product quality $q_E$ the entrant now has to take into account the additional marginal imitation costs $\gamma_E (1 - \mu) q_E$. The following proposition summarizes how equilibrium behavior will change:

**Proposition 6** When it is costly for the entrant to imitate the quality of the incumbent:

1. The lower its entry quality will be,

2. The higher are the qualities of products offered by the incumbent independent of whether the incumbent reacts by price or by portfolio adjustment and,

3. The quality of the incumbent’s premium product increases towards its monopoly product, the higher are the entrant’s costs of imitation.

**Proof.** See Appendix. ■

4.2 Launching costs and entry uncertainty

The discussion in Section 3.4 suggests that a portfolio adjustment is always better for the incumbent firm than a price response. Of course, this conclusion depends crucially on the costs the incumbent has to bear when introducing a second product. If these costs are sufficiently low, the previous result still holds and the incumbent will always launch a new product to influence the entrant’s quality decision. However, if the launching costs are sufficiently high, such a
portfolio adjustment will not be beneficial any more. Instead, the incumbent uses a price adjustment to influence the rival’s entry decision.

This argumentation suggests that only in case of high launching costs a price adjustment is better than a portfolio adjustment. This, however, neglects the fact, that the latter option of the incumbent is a long-run commitment to potential entry whereas a price adjustment is not. Indeed, the portfolio adjustment option is only beneficial for the incumbent if it is taken before entry - otherwise it would not influence the entrant’s quality choice any more. On the other hand, however, a price adjustment is a short-run decision in the sense that the incumbent reacts to the competition after entry actually occurred. To analyze this in more detail, suppose that the incumbent is uncertain about entry. This might happen, for example, in a situation in which the potential entrant has to bear positive entry costs. These costs might incur for advertising expenditures to inform consumers about the entrant’s product or for investments in transportation channels. In case the incumbent is uncertain about these entry costs, let $\mu \in [0,1]$ be the priori probability that a potential competitor does not enter the market and $(1 - \mu)$ be the probability that entry occurs with certainty. Then the next proposition shows that the trade-off between a price and a portfolio adjustment depends crucially on the probability of entry as well as on the incumbent’s costs to launch a second product.

**Proposition 7** When it is costly to launch a second product, the incumbent’s optimal reaction to entry is:

- Price adjustment as in Proposition 2, if launching costs are sufficiently high, \( F_L \geq \hat{F}_L(\mu) \);
- Portfolio adjustment as in Proposition 4, otherwise.

**Proof.** See Appendix.

Calculation shows that the critical value $\hat{F}_L(\mu)$ can be written as

\[
\hat{F}_L(\mu) = \frac{5 \delta (1 - \beta) \gamma_1}{4608 \gamma_1} (1 - \mu) (24 + \delta (19 + 5 \mu (1 - \beta))).
\]

Then $\hat{F}_L(\mu)$ is increasing in the probability of entry. Hence, if the probability that the potential entrant actually enters the market is arbitrarily low, a price adjustment might be the better option even if the incumbent’s launching costs when introducing a second product are low. Of course, investing in such a situation in a product launch imposes costs which might be worthless in case the entrant actually does not enter. Here, instead of making such a risky long-term decision implying sunk costs $F_L$, it might be more beneficial to wait until the entrant has actually entered and then use a price adjustment as a short-term tactical decision to defend the premium product.

Note that the incumbent never has an incentive to withdraw its second product even in case entry did not occur and independent of possible exit costs.
The reason is obvious: Even with two products the incumbent can still earn monopoly profits. In fact, if the incumbent sets the price $p_{IL}^*$ such that the price-quality ratio of both products is identical, that is,

$$p_{IL}^* = q_{IL}^* \frac{p_{IH}^*}{q_{IH}^*},$$

no consumer will buy the low-quality product.

### 4.3 Development costs and technological progress

The discussion in Section 3.4 also suggests that an investment in quality improvement can never be a best response for the incumbent firm: If such an investment is profitable as a response to entry, it should have been done in the first period where the incumbent’s monopoly power gave maximal profits from such an investment.

This conclusion, of course, relies crucially on the assumption that the degree of the incumbent’s investments’ efficiency is constant over both periods. Suppose now that the degree of the efficiency increased from Period 1 to Period 2. Higher efficiency in product development could, for example, be the result of overall technological progress or firm-specific learning cost advantages. Let $\lambda < 1$ be the parameter that reflects this increase in efficiency from $\gamma_I$ to $\lambda \gamma_I$. Using the previous results the incumbent then chooses its quality improvement to maximize second period profits for a given initial quality level $q_{I2}$,

$$\pi_{I2}^* = \frac{7}{48} \theta q_{I2} - \frac{1}{2} \gamma_I (\lambda q_{I2}^2 - q_{I2}^2).$$

The first-order condition implies that the incumbent optimally chooses $q_{I2}^* = \frac{7}{48} \lambda \gamma_I \theta$. Using its monopoly pricing in Period 1 the incumbent then introduces an initial quality level $q_{I2}$ to maximize overall profits

$$\pi_{I2} = \frac{1}{4} q_{I2}^2 + \frac{49}{4608 \lambda \gamma_I} \theta^2 \gamma_I - \frac{1}{2} (1 - \delta) \gamma_I q_{I2}^2.$$

In the optimum, $q_{I2}^* = \frac{1}{4(1-\delta)\gamma_I} \theta^2$ which is lower than $q_{I2}$ as long as

$$\lambda \leq \lambda = \frac{7}{12} (1 - \delta).$$

**Proposition 8** When the efficiency of product development increases, the incumbent’s optimal reaction to entry is:

- **Quality adjustment of the premium product, if efficiency increase is sufficiently high, $(\lambda \leq \lambda)$**, 
- **Portfolio adjustment as in Proposition 4, otherwise.**
Proof. See Appendix. □

Hence, if the improvement in product development is sufficiently strong, an upgrade in product quality is beneficial for the incumbent. In this case, the price-quality ratio of the entrant’s product increases compared to the case of portfolio adjustment whereas the price-quality ratio of the incumbent’s premium brand remains unchanged:

$$\frac{q_{I2}^*}{p_{I2}^*} = \frac{1}{2} \bar{\gamma} > \frac{q_{I2}^*}{p_{I2}^*} = \frac{1}{4} \bar{\gamma} > \frac{q_{E2}^*}{p_{E2}^*} = \frac{7}{32} \bar{\gamma} > \frac{1}{8} \bar{\gamma}$$

Of course, the incumbent then also has the possibility to simultaneously use a quality improvement of its premium brand and the introduction of a fighter brand. The next proposition shows that this is indeed the optimal combined response to entry even if the efficiency of product development is not that strong, that is $\lambda \leq \hat{\lambda}'$, with

$$\hat{\lambda}' = \frac{(12 - 5\beta)(1 - \delta)}{12} > \hat{\lambda}.$$

**Proposition 9** When the efficiency increase of product development is sufficiently high, $(\lambda \leq \hat{\lambda}')$, and the incumbent can simultaneously react with several strategic options, a combined portfolio and quality adjustment is optimal.

As in Section 3.3, the incumbent uses its fighter brand as a firewall in Period 2. Due to the improvement in efficiency, however, the incumbent not only upgrades its premium product to $q_{IH}^*$ but also introduces its fighter brand on a higher quality level $q_{IL}^*$:

$$q_{IH}^* = \frac{(12 - 5\beta)}{48\lambda \gamma^I} > q_{IH}^*$$

and

$$q_{IL}^* = \beta q_{IH}^* > q_{IL}^*$$

However, compared to the case in which the incumbent only chooses a portfolio adjustment without a quality improvement, the price-quality ratios of all three products remain unchanged

$$\frac{p_{IH}^*}{q_{IH}^*} = \frac{1}{2} \bar{\gamma} \left(1 - \frac{\beta}{2}\right) > \frac{1}{4} \bar{\gamma} > \frac{p_{E2}^*}{q_{E23}^*} = \frac{1}{8} \bar{\gamma}$$

as well as the equilibrium demand for these products,

$$x_{IH}^* = \frac{\bar{\gamma}}{2}, x_{IL}^* = \frac{1}{12} \bar{\gamma}, x_{E23}^* = \frac{7}{24} \bar{\gamma}.$$

As a consequence, not only the incumbent earns higher profits from a quality improvement at the high end together with the introduction of a fighter brand, but that also the entrant is better off. This, of course, results from the fact, that the quality improvement of the premium product not only leads to a higher quality of the fighter brand, but, in turn, also improves the entrant’s product quality. Since marginal gains from selling a product are higher the higher its quality, the entrant benefits from the incumbent’s quality improvement as well.
5 Conclusion

Throughout the analysis I assumed that competition on the sales stage between the incumbent and the entrant is in prices and that both firms can adjust their price setting behavior in the short-run. How the nature of competition between firms might influence the results of this paper shows the following example, see Ritson (2009): In 2003 before the patent of its blockbuster drug Zocor in Germany expired, Merck decided to launch a second brand called Zocor MSD. To avoid too much cannibalization and to preserve customer loyalty with its premium product, Merck introduced its second product four months before the patent expiration of Zocor and priced it slightly underneath the original premium brand. Once generics entered the market, the new product’s price dropped to 90% of Zocor’s. But this price cut was insufficient to seriously compete with the generics that invaded the market. More than 30 of these generic competitors, accustomed to competing on price, divided almost the entire generic market among themselves. Merck’s desire to protect its premium brand failed. Even more important, when Merck realized that it had set the wrong initial price, it was incapable of quick course correction. This example suggests that the mode of competition on the sales stage - price or quantity - as well as the mode of competition on the differentiation stage - vertical or horizontal - is crucial for the success of a second product launch to fight against low-price competition. In particular, the trade-off between a price or portfolio adjustment might crucially depend on the underlying mode of competition.

Another restrictive assumption of the previous analysis is the fact that consumers are totally aware of the differences in product qualities. In particular, I assumed that consumers know that the incumbent’s second product is simply a product of lower quality at a lower price than the high quality product. The following story, however, shows what happens if this assumption is not satisfied, see also Ritson (2009): In 1994 Kodak launched a second film to fend off its best seller Gold Plus film against its Japanese rival Fuji with its Fujicolor Super G film. This new film called Funtime had a lower quality compared to its own premium brand since Kodak manufactured Funtime with an older, less effective technology than Gold Plus. Moreover, it was sold at the same price as Fuji’s offering. Although, in principle, Funtime was created explicitly to win back customers that had switched to Fuji’s low-price alternative, two years later, in 1996, Kodak withdrew Funtime from the market. The reason was that most consumers were unaware of the quality differences between the two products of Kodak: they simply saw Funtime as a high quality Kodak film at a lower price. This seriously damaged Kodak’s reputation for high quality and Gold Plus sales were more damaged than Fuji’s. This example shows that the success of a portfolio adjustment depends crucially on how the incumbent firm brands its products. As in the case of Kodak, the incumbent can use umbrella branding so that the premium product as well as the second, lower quality product are labelled with a single brand name. Although umbrella branding might play an informational role in markets in which consumers are uncertain about product characteristics, see e.g. Cabral (2009), it leads to increased cannibalization of
the premium brand in the context of the present model. But then it might be beneficial for the incumbent to sell its two products under different names or, if this is not possible, to use price adjustment to protect its premium product from low-price competitors.

I also assumed in the previous analysis that customers follow the standard model of rational choice by evaluating prices and qualities of different products in a rational way. Of course, under bounded rationality different behavioral patterns are important for consumers’ choice, e.g. the compromise effect, see Geyskens et al. (2010), and may determine the success of the incumbent’s strategic options as well the entrant’s entry strategy.

6 Appendix

Proof of Proposition 1. Suppose the incumbent offers only one product \( (q_M, p_M) \). In the absence of entry, the incumbent then chooses a price \( p_M \) to maximize per period gross profits

\[
\pi_M = p_M \left( \bar{\theta} - \frac{p_M}{q_M} \right),
\]

for a given quality level \( q_M \). Hence

\[
p_M = \frac{1}{2} q_M \text{ and } x_M = \frac{1}{2} \bar{\eta}.
\]

Using this monopoly price the optimal level of quality then maximizes

\[
\pi_M = p_M x_M (1 + \delta) - c_I(q_M) = \frac{1}{4} \bar{\theta}^2 (1 + \delta) q_M - \frac{1}{2} \gamma_I q_M^2.
\]

The first-order condition reads as

\[
\frac{\partial}{\partial q_M} \pi_M = \frac{1}{4} \bar{\theta}^2 (1 + \delta) - \gamma_I q_M = 0,
\]

hence

\[
q_M^* = \frac{\bar{\theta}^2 (1 + \delta)}{4 \gamma_I}, \quad p_M^* = \frac{\bar{\theta}^2 (1 + \delta)}{8 \gamma_I}
\]

and monopoly profits are

\[
\pi_M^* = \frac{\bar{\theta}^4 (1 + \delta)^2}{32 \gamma_I}.
\]

To see the second part of the proposition, it is sufficient to show that the incumbent will never introduce two products. So suppose for the contrary that the incumbent would offer two product qualities \( q_{MH}, q_{ML} \). Without loss of generality, let \( q_{MH} > q_{ML} \). As before, in the absence of entry, the incumbent sets identical monopoly prices \( p_{MH} \) and \( p_{ML} \) in both periods to maximize profits

\[
\pi_M = p_{MH} x_{MH} + p_{ML} x_{ML} = p_{MH} (\bar{\eta} - \theta_{HL}) + p_{ML} (\theta_{HL} - \theta_{LO})
\]
where \( \theta_{HL} = (p_{ML} - p_{MH}) / (q_{ML} - q_{MH}) \) denotes the consumer who is indifferent between buying the high or low the quality product, and \( \theta_{LO} = p_{ML}/q_{ML} \) denotes the consumer who is indifferent between buying the low quality product and not buying at all. Profit maximization per period then leads to the following two first-order conditions

\[
\frac{\partial}{\partial p_{MH}} \pi_M = \frac{\theta (q_{MH} - q_{ML}) - 2(p_{MH} - p_{ML})}{q_{MH} - q_{ML}} = 0,
\]

\[
\frac{\partial}{\partial p_{ML}} \pi_M = \frac{2(p_{MH}q_{ML} - p_{ML}q_{MH})}{q_{ML}(q_{MH} - q_{ML})} = 0,
\]

and optimal monopoly prices can be computed as

\[ p_{MH} = \frac{1}{2} \theta q_{MH}, p_{ML} = \frac{1}{2} \theta q_{ML}. \]

Note that the second-order conditions \( \frac{\partial^2}{\partial p_{MH}^2} \pi_M = -\frac{2q_{MH}}{(q_{MH} - q_{ML})} < 0 \) and \( \frac{\partial^2}{\partial p_{ML}^2} \pi_M = -\frac{2q_{ML}(q_{MH} - q_{ML})}{(q_{MH} - q_{ML})} < 0 \) are both satisfied. This solution, however, implies zero demand for the low quality product, \( x_{ML} = 0 \) since \( \theta_{HL} = \theta_{LO} \). Moreover, the demand for the high quality product is \( x_{MH} = \theta/2 \), that is, the incumbent will not introduce a second product of lower quality.

Q.E.D. ■

Proof of Proposition 2. To solve the game use backward induction.

Stage 2c: Given qualities \( q_I > q_{E1} \) and prices \( p_{I1}^2 \) and \( p_{E1}^2 \), demand in Period 2 is

\[ x_{I1}^2 = \theta - \frac{p_{I1}^2}{q_{I1} - q_{E1}}, \quad x_{E1}^2 = \frac{p_{I1}^2 - p_{E1}^2}{q_{I1} - q_{E1}}. \]

Stage 2b: Given qualities \( q_I > q_{E1} \), the entrant maximizes profits

\[ \pi_{E1} = p_{E1}^2 x_{E1}^2 = p_{E1} \left( \frac{p_{I1}^2 - p_{E1}^2}{q_{I1} - q_{E1}} \right) \]

with respect to its product price \( p_{E1}^2 \). This leads to the first-order condition

\[ \frac{\partial \pi_{E1}}{\partial p_{E1}^2} = \frac{q_{E1}p_{I1}^2 - 2p_{E1}^2 q_{I1}}{q_{E1}(q_{I1} - q_{E1})} = 0, \]

hence, \( p_{E1}^2 = \frac{1}{2} \theta q_{E1} q_{I1} q_{E1} / q_{I1} \). Similarly, the incumbent maximizes its second period profits

\[ \pi_{I1}^2 = p_{I1}^2 x_{I1}^2 = p_{I1}^2 \left( \frac{\theta - \frac{p_{I1}^2 - p_{E1}^2}{q_{I1} - q_{E1}}}{q_{I1} - q_{E1}} \right) \]

with respect to its product price \( p_{I1}^2 \). The first-order condition then reads as

\[ \frac{\partial \pi_{I1}^2}{\partial p_{I1}^2} = \frac{\theta (q_{I1} - q_{E1}) + p_{E1}^2 - 2p_{I1}^2}{q_{I1} - q_{E1}} = 0, \]
hence, \( p_{I1}^2 = \frac{1}{2} p_{E1}^2 + \frac{1}{2} \theta (q_{I1} - q_{E1}) \). Hence, in a price equilibrium

\[
p_{I1}^2 = 2 \theta q_{E1} \frac{q_{I1} - q_{E1}}{4q_{I1} - q_{E1}}, \quad p_{E1}^2 = \theta q_{E1} \frac{q_{I1} - q_{E1}}{3q_{I1} - q_{E1}}
\]

**Stage 2a:** Given qualities \( q_{I1} \), the entrant chooses \( q_{E1} \) to maximize

\[
\pi_{E1} = \theta^2 q_{I1} q_{E1} (q_{I1} - q_{E1})
\]

\[
\frac{\partial}{\partial q_{E1}} \pi_{E1} = \theta^2 \left( \frac{4q_{I1} - 7q_{E1}}{(4q_{I1} - q_{E1})^3} \right) = 0
\]

if the entrant optimally chooses \( q_{E1}^* (q_{I1}) = \frac{4}{7} q_{I1} \) which results in profits \( \pi_{E1} (q_{I1}) = \frac{\pi}{48} q_{I1} \).

**Stage 1a:** Using the results from Proposition 1 for Period 1 the incumbent then maximizes total profits from monopoly and competition over both periods

\[
\pi_{I1} = \frac{1}{4} \theta q_{I1} + \theta^2 \frac{7}{48} q_{I1} - \frac{1}{2} \theta q_{I1}^2
\]

taking into account the entrant’s best response \( q_{E1}^* (q_{I1}) \). The optimal product quality \( q_{I1}^* \) then satisfies the first-order condition

\[
\frac{\partial}{\partial q_{I1}} \pi_{I1} = \frac{1}{4} \theta + \theta^2 \frac{7}{48} - \gamma_1 q_{I1} = 0
\]

hence

\[
q_{I1} = \frac{\theta (12 + 7 \delta)}{48 \gamma_1}
\]

Inserting this into the entrant’s best quality response gives

\[
q_{E1}^* = \frac{\theta (12 + 7 \delta)}{84 \gamma_1}
\]

Equilibrium prices then are

\[
p_{I1}^* = \frac{\theta^3 (12 + 7 \delta)}{96 \gamma_1}, \quad p_{I2}^* = \frac{\theta^4 (12 + 7 \delta)}{192 \gamma_1}, \quad p_{E1}^* = \frac{\theta (12 + 7 \delta)}{672 \gamma_1},
\]

resulting in equilibrium profits

\[
\pi_{I1}^* = \frac{\theta (12 + 7 \delta)^2}{4608 \gamma_1}, \quad \pi_{E1}^* = \frac{\theta^4 (12 + 7 \delta)}{2304 \gamma_1}
\]

Q.E.D. ■

**Proof of Proposition 4.** In solving the game, I distinguish two cases: either \( q_{IH} > q_{IL} > q_{E3} \) (Case 1) or \( q_{IH} > q_{E3} > q_{IL} \) (Case 2):
Stage 2c/b - Case 1: Given \(q_{1H} > q_{1L} > q_{E3}\) and prices \(p_{1H}^2, p_{1L}^2, p_{E3}^2\), the entrant maximizes profits

\[
\pi_{E3} = p_{E3}^2x_{E3}^2 = p_{E3}^2 \left( \frac{p_{1L}^2 - p_{E3}^2}{q_{1L} - q_{E3}} - \frac{p_{E3}^2}{q_{E3}} \right)
\]

with respect to its product price \(p_{E3}^2\). This leads to the first-order condition

\[
\frac{\partial \pi_{E3}}{\partial p_{E3}^2} = \frac{2p_{E3}^2q_{1L} - q_{E3}p_{1L}^2}{q_{E3}(q_{E3} - q_{1L})} = 0,
\]

hence \(p_{E3}^2 = \frac{1}{2}p_{1L}^2\). Similarly, the incumbent maximizes its overall profits in Period 2,

\[
\pi_{I3} = p_{1H}^2x_{I3}^2 + p_{1L}^2x_{I3}^2
\]

\[
= p_{1H}^2 \left( \frac{\bar{\varrho} - \frac{p_{1H}^2 - p_{1L}^2}{q_{1H} - q_{1L}}}{2} + p_{1L}^2 \left( \frac{p_{1H}^2 - p_{1L}^2}{q_{1H} - q_{1L}} + \frac{p_{1L}^2 - p_{E3}^2}{q_{1L} - q_{E3}} \right) \right)
\]

with respect to its prices \(p_{1L}^2\) and \(p_{1H}^2\). The first-order conditions then are

\[
\frac{\partial \pi_{I3}}{\partial p_{1H}^2} = \frac{\bar{\varrho}(q_{1H} - q_{1L}) - 2(p_{1H}^2 - p_{1L}^2)}{q_{1H} - q_{1L}} = 0,
\]

\[
\frac{\partial \pi_{I3}}{\partial p_{1L}^2} = \frac{2p_{1H}^2(q_{E3} - q_{1L}) + 2p_{1L}^2(q_{1H} - q_{E3}) - p_{E3}^2(q_{1H} - q_{1L})}{(q_{1H} - q_{1L})(q_{E3} - q_{1L})} = 0.
\]

The best response functions are obtained from these conditions and read as:

\[
p_{1H}^2 = p_{1L}^2 + \frac{\bar{\varrho}}{2}(q_{1H} - q_{1L})
\]

\[
p_{1L}^2 = \frac{p_{E3}^2 q_{1H} - q_{1L}}{2q_{1H} - q_{E3}} + \frac{p_{1L}^2 q_{1L} - q_{E3}}{q_{1L} - q_{E3}}
\]

Solving for \(p_{1H}^2, p_{1L}^2,\) and \(p_{E3}^2\) then gives equilibrium prices

\[
p_{1H}^2 = \frac{1}{2}q_{1H}(q_{1L} - q_{E3}) + 3q_{1L}(q_{1H} - q_{E3}),
\]

\[
p_{1L}^2 = \frac{2q_{1L}(q_{1L} - q_{E3})}{4q_{1L} - q_{E3}},
\]

\[
p_{E3}^2 = \frac{q_{E3}(q_{1L} - q_{E3})}{4q_{1L} - q_{E3}}.
\]

Of course, \(p_{1H}^2 > p_{1L}^2\) since \((q_{1H} - q_{1L})(q_{1L} - q_{E3}) + 3q_{1L}(q_{1H} - q_{E3}) > 0\) and \(p_{1L}^2 > p_{E3}^2\) since \(q_{1L} + (q_{1L} - q_{E3}) > 0\). Substituting these equilibrium prices into the demand function gives the equilibrium demand

\[
x_{1H}^2 = \frac{\bar{\varrho}}{2}, \quad x_{1L}^2 = \frac{\bar{\varrho}q_{E3}}{2(4q_{1L} - q_{E3})}, \quad x_{E}^2 = \frac{\bar{\varrho}q_{1L}}{4q_{1L} - q_{E3}}.
\]
Stage 2c/b - Case 2: Given $q_{IH} > q_{E3} > q_{IL}$ and prices $p^2_{IH}$, $p^2_{E3}$, $p^2_{IL}$, demand now is

\[ x^2_{IH} = \theta - \frac{p^2_{IH} - p^2_{E3}}{q_{IH} - q_{E3}} \]

\[ x^2_{E3} = \frac{p^2_{IH} - p^2_{E3}}{q_{IH} - q_{E3}} \]

\[ x^2_{IL} = \frac{p^2_{E3} - p^2_{IL}}{q_{E3} - q_{IL}} \]

The entrant’s profit function

\[ \pi_{E3} = p^2_{E3} x^2_{E3} = p^2_{E3} \left( \frac{p^2_{IH} - p^2_{E3}}{q_{IH} - q_{E3}} - \frac{p^2_{E3} - p^2_{IL}}{q_{E3} - q_{IL}} \right) \]

is maximized with respect to its product price $p^2_{E3}$ if the first-order condition is satisfied:

\[ \frac{\partial \pi_{E3}}{\partial p^2_{E3}} = \frac{p^2_{IH} (q_{E3} - q_{IL}) + p^2_{IL} (q_{IH} - q_{E3}) - 2p^2_{E3} (q_{IH} - q_{IL})}{(q_{IH} - q_{E3}) (q_{E3} - q_{IL})} = 0 \]

The best response function of the entrant then is

\[ p^2_{E3} = \frac{1}{2} p^2_{IH} \frac{q_{E3} - q_{IL}}{q_{IH} - q_{IL}} + \frac{1}{2} p^2_{IL} \frac{q_{IH} - q_{E3}}{q_{IH} - q_{IL}}. \]

The incumbent’s maximization of profits

\[ \pi^2_{I3} = p^2_{IH} x^2_{IH} + p^2_{IL} x^2_{IL} = p^2_{IH} \left( \theta - \frac{p^2_{IH} - p^2_{E3}}{q_{IH} - q_{E3}} \right) + p^2_{IL} \left( \frac{p^2_{E3} - p^2_{IL}}{q_{E3} - q_{IL}} - \frac{p^2_{IH} - p^2_{E3}}{q_{IH} - q_{E3}} \right) \]

with respect to its prices $p^2_{IH}$ and $p^2_{IL}$ gives the first-order conditions

\[ \frac{\partial \pi^2_{I3}}{\partial p^2_{IH}} = p^2_{E3} - 2p^2_{IH} + \theta (q_{IH} - q_{E3}) \frac{q_{IH} - q_{E3}}{q_{IH} - q_{E3}} = 0, \]

\[ \frac{\partial \pi^2_{I3}}{\partial p^2_{IL}} = \frac{p^2_{E3} q_{IL} - 2q_{E3} p^2_{IL}}{q_{IL} (q_{E3} - q_{IL})} = 0. \]

which imply $p^2_{IH} = \frac{1}{2} p^2_{E3} + \frac{\theta}{2} (q_{IH} - q_{E3})$ and $p^2_{IL} = \frac{1}{2} p^2_{E3} - \frac{\theta}{2} (q_{IH} - q_{E3})$. Solving all three conditions for equilibrium prices yields

\[ p^2_{IH} = \frac{\theta (q_{IH} - q_{E3}) (q_{IH} (q_{E3} - q_{IL}) + 3q_{E3} (q_{IH} - q_{IL}))}{2 (q_{E3} (q_{IH} - q_{E3}) + q_{IH} (q_{E3} - q_{IL}) + 2q_{E3} (q_{IH} - q_{IL}))}, \]

\[ p^2_{E3} = \frac{p^2_{E3} (q_{IH} - q_{E3}) (q_{IH} - q_{E3})}{2 (q_{E3} (q_{IH} - q_{E3}) + q_{IH} (q_{E3} - q_{IL}) + 2q_{E3} (q_{IH} - q_{IL}))}, \]

\[ p^2_{IL} = \frac{q_{E3} (q_{IH} - q_{E3}) (q_{IH} - q_{E3})}{2 (q_{E3} (q_{IH} - q_{E3}) + q_{IH} (q_{E3} - q_{IL}) + 2q_{E3} (q_{IH} - q_{IL}))}. \]

$(q_{IH} - q_{E3}) (2q_{E3} (q_{IH} - q_{E3}) + q_{IH} (q_{E3} - q_{IL}) + q_{E3} (q_{IH} - q_{IL})) > 0$ implies that $p^2_{IH} > p^2_{E3}$ and $p^2_{E3} > p^2_{IL}$ since $q_{E3} + (q_{E3} - q_{IL}) > 0$. Substituting these
equilibrium prices into the demand function gives the equilibrium demand in Period 2

\[
x_{1H}^2 = \frac{\bar{\sigma}(3q_{E3}(q_{IH} - q_{IL}) + q_{IH}(q_{E3} - q_{IL}))}{2(q_{E3}(q_{IH} - q_{E3}) + q_{IH}(q_{E3} - q_{IL}) + 2q_{E3}(q_{IH} - q_{IL}))},
\]

\[
x_{E}^2 = \frac{\bar{\sigma}q_{E3}(q_{IH} - q_{IL})}{2(q_{E3}(q_{IH} - q_{E3}) + q_{IH}(q_{E3} - q_{IL}) + 2q_{E3}(q_{IH} - q_{IL}))},
\]

\[
x_{1L}^2 = \frac{\bar{\sigma}q_{E3}(q_{IH} - q_{E3})}{2(q_{E3}(q_{IH} - q_{E3}) + q_{IH}(q_{E3} - q_{IL}) + 2q_{E3}(q_{IH} - q_{IL}))}.
\]

**Stage 2a:** Given \(q_{IH} > q_{IL}\) the entrant can either choose \(q_{E3} = q_{EL}\) with \(q_{EL} < q_{IL}\) or \(q_{E3} = q_{EH}\) with \(q_{EH} \in [q_{IL}, q_{IH}]\). If the entrant chooses \(q_{EL} < q_{IL}\), then using the equilibrium prices above, the first-order condition for profit maximization gives

\[
\frac{\partial \pi_{E3}}{\partial q_{E3}} = \frac{\partial}{\partial q_{E3}} \left( \frac{\bar{\sigma}q_{E3}(q_{IL} - q_{E3})}{(4q_{IL} - q_{E3})^2} \right) = \frac{\bar{\sigma}q_{IL}(4q_{IL} - 7q_{E3})}{(4q_{IL} - q_{E3})^3} = 0
\]

and it optimally chooses \(q_{EL}^* = \frac{4}{7}q_{IL}\). Hence,

\[
\pi_{EL} = \frac{1}{48} \bar{\sigma} q_{IL}, \quad \pi_{IL}^2 = \frac{1}{48} \bar{\sigma}^2 (12q_{IH} - 5q_{IL}).
\]

If, on the other hand, \(q_{E3} \in [q_{IL}, q_{IH}]\), then using the equilibrium prices above, the entrant’s profits read as

\[
\pi_{EH} = \frac{\bar{\sigma}q_{E3}(q_{IH} - q_{IL})(q_{IH} - q_{E3})(q_{E3} - q_{IL})}{(q_{E3}(q_{IH} - q_{E3}) + q_{IH}(q_{E3} - q_{IL}) + 2q_{E3}(q_{IH} - q_{IL}))^2}.
\]

In this case, the first-order condition is

\[
\frac{\partial \pi_{EH}}{\partial q_{E3}} = \frac{\bar{\sigma}q_{E3}(q_{IH} - q_{IL}) A}{(q_{E3}(q_{IH} - q_{E3}) + q_{IH}(q_{E3} - q_{IL}) + 2q_{E3}(q_{IH} - q_{IL}))^3}
\]

with \(A = -7q_{E3}^2 + 5q_{E3}^2q_{IL} + 4q_{E3}^2q_{IH} + 4q_{E3}^2q_{E3q_{IL}} - 2q_{E3}^2q_{IL} - 3q_{E3}q_{IL}q_{IL} - 3q_{E3}q_{IL}q_{IL} + 2q_{IL}q_{IL}^2\). Note that for \(q_{EH} = q_{IH}\) it follows that \(A = -3q_{IH}(q_{IH} - q_{IL})^2 < 0\) whereas for \(q_{EH} = q_{IL}\) it follows \(A = 3q_{IL}(q_{IH} - q_{IL})^2 > 0\). Continuity then requires that the entrant optimally chooses \(q_{EH} \in (q_{IL}, q_{IH})\) such that \(A = 0\), that is,

\[
\frac{3q_{IH}q_{E3} - 4(q_{E3})^2 - E_{q_{IH}q_{IL} + 3q_{E3q_{IL}}}}{2(2q_{E3} - 4q_{IH} + 2q_{IL})}
= \frac{q_{E3}(q_{IH} - q_{E3})(q_{E3} - q_{IL})}{(2q_{E3}q_{IL} - 4q_{E3}q_{IH} + q_{IH}q_{IL} + (q_{E3})^2)}
\]

\(\text{(C1)}\)
Note that \( \frac{\partial^2 \pi_{EH}}{\partial q_{IL}^2} < 0 \) requires \( 6q_{IH}q_{IL}^3 (q_{IH} - q_{IL})^2 + q_{EH}^2 A > 0 \) which is always satisfied in the optimum. Let \( \pi^*_{IL}, \pi_{EH} \) denote the resulting profits. Then the entrant chooses \( q_{EH} = q_{EL} \) if \( \pi_{EL} > \pi_{EH} \) and otherwise \( q_{EH} = q_{EL} \).

**Stage 1c:** Note that \( \pi_{EL} \) is monotonically increasing in \( q_{IL} \), \( \frac{\partial \pi_{EL}}{\partial q_{IL}} = \frac{1}{18} \sqrt{q_{IH}} > 0 \), with \( \pi_{EL} = 0 \) for \( q_{IL} = 0 \) and \( \pi_{EL} = \frac{1}{48} \sqrt{q_{IL} q_{IH}} \) for \( q_{IL} = q_{IH} \). Moreover, \( \pi_{EH} \) is monotonically decreasing in \( q_{IL} \),

\[
\frac{\partial \pi_{EH}}{\partial q_{IL}} = -\frac{q^2}{q_{EH}} (q_{EH} - q_{EL})^2 \times \\
\frac{2q_{EH} (q_{IH} - q_{IL}) + q_{IL} (q_{IH} - q_{EL}) + q_{IL} (q_{IH} - q_{EL})}{(2q_{EH} (q_{IH} - q_{IL}) + q_{IH} (q_{IH} - q_{EL}) + q_{IL} (q_{IH} - q{EL}))^2} < 0
\]

with \( \pi_{EH} = 0 \) for \( q_{IL} = q_{IH} \) and \( \pi_{EH} = \frac{1}{48} \sqrt{q_{IL} q_{IH}} \) for \( q_{IL} = 0 \). Let \( q_{IL} \) be the critical value given by \( \pi_{EL} (q_{IL}) = \pi_{EH} (q_{IL}) \), that is,

\[
\frac{1}{48} q_{IL} = \frac{\sqrt{q_{EH}}}{q_{EH}} (q_{EH} - q_{EL})^2 \left( \frac{(q_{IH} - q_{IL}) (q_{IH} - q_{EL}) (q_{IH} - q_{EL})}{2q_{EH} q_{IL} - 4q_{IH} q_{EL} + q_{IL} (q_{EH} - q_{EL})^2} \right)^{\frac{1}{2}} \tag{C2}
\]

Then condition (C1) and (C2) uniquely determine the optimal pair \( (q_{IL}, q_{EH}) \) with \( q_{IL} < q_{EH} \). Using (C1) and solving for \( q_{IL} \) then gives a quadratic equation with two roots

\[
q_{IL} = \frac{q_{EH} \left( 5(q_{EH})^2 + 4q_{IH} q_{EL} - 3q_{IH} \pm (q_{IH} - q_{EL}) \sqrt{X} \right)}{4q_{EH}^2 + 6q_{IH} q_{EL} - 4q_{IH}}
\]

with \( X = 25(q_{EH})^2 + 34q_{IH} q_{EH} - 23q_{IH}^2 \). Note that in equilibrium \( X > 0 \) since otherwise (C2) would not have a solution. Moreover note that \( q_{IL-} \in (0, q_{EH}) \), but \( q_{IL-} \) is always strictly greater than \( q_{EH} \). To see the last claim note that for \( q_{EH} > \frac{1}{2} q_{IH} \) we have \( q_{IL-} < q_{EH} \) if \( 0 < 12(2q_{IH} - q_{IH}) (2q_{IH} + q_{EH}) \) and for \( q_{EH} < \frac{1}{2} q_{IH} \) we have \( q_{IL-} < q_{EH} \) if \( 0 > 12(2q_{IH} - q_{IH}) (2q_{IH} + q_{EH}) \) which is true in both cases. Hence, \( q_{IL} = q_{IL-} \). Substituting this \( q_{IL} \) in condition (C2) and rearranging terms then yields

\[
q_{IH} (q_{IH} - q_{EH})^2 \left( 3q_{IH} q_{EH}^2 + 2(q_{EH})^2 - 2q_{IH}^2 \right)^{\frac{3}{2}} \times \\
\left( 73q_{IH}^2 - 192q_{IH} q_{EH}^2 + 57q_{IH} (q_{EH})^2 + 89 (q_{EH})^3 \right) = 0
\]

Hence, the optimal \( q_{EH} \) is linear in \( q_{IH} \), \( q_{EH} = \alpha q_{IH} \) and solution of

\[
73 - 192\alpha + 57\alpha^2 + 89\alpha^3 = 0.
\]

The only solution with \( \alpha > 4/7 \) then is \( \alpha = 0.77738 \). The optimal \( q_{IL} \) then is

\[
q_{IL} = \frac{4\alpha^2 + 5\alpha^3 - 3\alpha - \alpha (1 - \alpha) \sqrt{25\alpha^2 + 34\alpha - 23}}{4\alpha^2 + 6\alpha - 4} q_{IH}
\]

\[
= \beta q_{IH} = 0.54806 q_{IH}.
\]
The incumbent then positions its second product with an optimal fighter brand quality slightly above $q_{IL}$ such that the entrant’s best response is $q_{E3} = \frac{3}{4} \theta q_{IL} = \frac{3}{4} \theta q_{IH}$. The relationships between marginal consumers then is

$$\theta_{HL} = \frac{1}{4} \theta (2 + \beta) > \theta_{LE} = \frac{1}{12} \theta (7 \beta - 2) > \theta_{E0} = \frac{1}{8} \theta$$

which follow from $\beta \in \left(\frac{2}{3}, 1\right)$. Moreover, the resulting profits are

$$\pi_{E3} = \frac{1}{48} \theta^2 \beta q_{IH}, \quad \pi_{I3}^2 = \frac{1}{48} \theta^2 (12 - 5 \beta) q_{IH}$$

Note that if the incumbent positioned its second product slightly below $q_{IL}$ such that entrant’s best response would be in the intermediate range, its resulting profits are strictly lower. Calculation shows that the incumbent’s profits are $\pi_1^2 = 0.076993 \theta^2 q_{IH}$ which is strictly lower than $\pi_1 = 0.19291 \theta^2 q_{IH}$ in case the entrant’s best response is in the lower quality area.

**Stage 1a:** Given the optimal behavior in Stage 1c, and the results from Proposition 1, the optimal product quality $q_{IH}$ is then given by

$$\frac{\partial}{\partial q_{IH}} \pi_{I3} = \frac{\partial}{\partial q_{IH}} \left( \frac{1}{4} \theta^2 q_{IH} + \frac{1}{48} \theta^2 (12 - 5 \beta) \delta q_{IH} - \frac{1}{2} \gamma_1 \delta q_{IH} \right) = 0$$

hence,

$$q_{IH} = \frac{\theta^2 (12 + (12 - 5 \beta) \delta)}{48 \gamma_1}$$

Inserting this into the optimal reaction functions then gives equilibrium qualities

$$q_{IL}^* = \frac{\theta^2 (12 + (12 - 5 \beta) \delta)}{48 \gamma_1} \beta, \quad q_{E3}^* = \frac{\theta^2 (12 + (12 - 5 \beta) \delta)}{84 \gamma_1} \beta,$$

equilibrium prices

$$p_{1H}^* = \frac{\theta^3 (12 + (12 - 5 \beta) \delta)}{96 \gamma_1}, \quad p_{IH}^* = \frac{\theta^3 (12 + (12 - 5 \beta) \delta) (2 - \beta)}{192 \gamma_1}$$

$$p_{IL}^* = \frac{\theta^3 (12 + (12 - 5 \beta) \delta) \beta^2}{192 \gamma_1}, \quad p_{E3}^* = \frac{\theta^3 (12 + (12 - 5 \beta) \delta) \beta}{672 \gamma_1},$$

and equilibrium profits

$$\pi_{I3}^* = \frac{\theta^4}{4608 \gamma_1} (12 + \delta (12 - 5 \beta))^2, \quad \pi_{E3}^* = \frac{\theta^4}{2304 \gamma_1} (12 + \delta (12 - 5 \beta)).$$

Q.E.D.

**Proof of Proposition 5.** The proposition follows directly from Propositions 3 and 4 and the fact that the incumbent’s profits $\pi_{I3}^*$ in case of a portfolio adjustment are higher than $\pi_{I1}^*$ in case of a price adjustment if

$$\frac{\theta^4}{4608 \gamma_1} (12 + \delta (12 - 5 \beta))^2 > \frac{\theta^4}{4608 \gamma_1} (12 + 7 \delta)^2$$

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which is satisfied since \((12 + 7\delta) > \beta (12 + 5\beta)\). To compare equilibrium quantities and prices in these two cases, let \(\alpha = \frac{12 + 7\delta}{12 + 12\delta}\) and \(\tilde{\alpha} = \frac{12 + (12 - 5\beta)\delta}{12 + 12\delta}\). Then \(\alpha < \tilde{\alpha}\) and

\[
q^*_1 = \alpha q^*_M, \quad p^*_2 = \frac{\alpha}{2} p^*_M \quad \text{and} \quad q^*_{E1} = \frac{4\alpha}{7} q^*_M, \quad p^*_{E1} = \frac{\alpha}{7} p^*_M
\]
in case of a price adjustment and

\[
q^*_{IH} = \tilde{\alpha} q^*_M, \quad q^*_{IL} = \tilde{\alpha} \beta q^*_M, \quad q^*_E = \frac{4}{7} \tilde{\alpha} \beta q^*_M \quad \text{and}
\]

\[
p^*_{IH} = \frac{2 - \beta}{2} p^*_M, \quad p^*_{IL} = \frac{\beta^2 \tilde{\alpha}}{4} p^*_M, \quad p^*_{E3} = \frac{\beta \tilde{\alpha}}{14} p^*_M
\]
in case of a portfolio adjustment.

Q.E.D.

**Proof of Proposition 6.** Consider, for example, the case of price adjustment, Option 1. The first-order condition for \(q^*_{E1}\) reads as

\[
\frac{\partial^2}{\partial \gamma^2} q^*_{E1} (4q^*_1 - 7q^*_{E1}) - \gamma_E (1 - \mu) q^*_{E1} = 0
\]

Using the envelope theorem it follows then that

\[
\frac{\partial q^*_{E1}}{\partial \gamma_E} = - \frac{\partial q^*_{E1}}{\partial \mu} (1 - \mu) \gamma_E \frac{q^*_E (1 - \mu) (4q^*_1 - q^*_{E1})^3}{4 \gamma_E^2 q^*_1 + 4 \gamma_E (1 - \mu) (q^*_1 - q^*_{E1}) (4q^*_1 - q^*_{E1})^2} < 0
\]

Taking this into account, the incumbent maximizes

\[
\pi^*_1 = \frac{1}{4} \theta^2 q^*_1 + \delta \theta^2 \frac{7}{48} q^*_1 - \frac{1}{2} \gamma (q^*_1)^2
\]

Since

\[
\frac{\partial}{\partial q^*_E} \pi^*_1 = - \theta^2 q^*_1 (q^*_1 - q^*_{E1}) (4q^*_1 - q^*_{E1})^2 < 0
\]

it follows that

\[
\frac{\partial q^*_1}{\partial \gamma_E} > 0 \quad \text{and} \quad \frac{\partial q^*_1}{\partial \mu} < 0
\]

Moreover,

\[
\frac{\partial}{\partial \gamma_E} \left( \frac{p^*_{E1}}{q^*_1} \right) = \frac{6\theta}{(4q^*_1 - q^*_{E1})^2} \left( q^*_E \frac{\partial q^*_1}{\partial \gamma_E} - \frac{\partial q^*_{E1}}{\partial \gamma_E} \right) > 0
\]

\[
\frac{\partial}{\partial \gamma_E} \left( \frac{p^*_{E3}}{q^*_{E1}} \right) = \frac{3\theta}{(4q^*_1 - q^*_{E1})^2} \left( q^*_E \frac{\partial q^*_{E1}}{\partial \gamma_E} - q^*_1 \frac{\partial q^*_1}{\partial \gamma_E} \right) > 0
\]
since $\frac{\partial q}{\partial \gamma_k} < 0$ and $\frac{\partial q}{\partial \gamma_k} > 0$. The proof for the case of a portfolio adjustment is similar.

Q.E.D.

**Proof of Proposition 7.**  The incumbent firm has two strategic options: either not to introduce a second product and use a price adjustment (Case 1) or to launch a fighter brand and use a portfolio adjustment (Case 2). Let $\mu \in [0, 1]$ be the a priori probability that the rival firm will never enter the market and $(1 - \mu)$ be the probability that entry occurs with certainty.

**Case 1:** Using the results of Proposition 1 in case the incumbent remains monopolist with probability $\mu$ and offers only its brand product $q_{I1}$, and using the results of Proposition 2 in case the entrant enters with probability $(1 - \mu)$ and the market becomes a duopoly with two products ($q_{E1}, q_{I1}$), expected equilibrium profits are

$$E\pi_{I1}(q_{I1}) = \mu \frac{\theta^2}{4} (1 + \delta) q_{I1} + (1 - \mu) \frac{\theta^2}{4} \left(1 + \frac{7}{12} \delta\right) q_{I1} - \frac{1}{2} \gamma_I (q_{I1})^2,$$

where it is already assumed that the entrant in case of entry chooses $q_{E1}^* = \frac{\theta}{3} q_{I1}$. Then the optimal quality of the premium product is given by $\frac{\partial \pi_{I1}}{\partial q_{I1}} E\pi_{I1} = 0$, hence,

$$q_{I1}^* = \frac{\theta^2}{4 \gamma_I} \left(1 + \delta - \frac{5}{12} (1 - \mu) \delta\right),$$

and equilibrium expected profits read as

$$E\pi_{I1}^* = \frac{\theta^4}{4608 \gamma_I} (12 \delta - 5 (1 - \mu) \delta + 12)^2.$$

**Case 2:** Using the results of Proposition 1 in case the incumbent remains monopolist with two products ($q_{I1}, q_{I1H}$) with probability $\mu$, and using the results of Proposition 4 in case the entrant enters with probability $(1 - \mu)$ and the market becomes a duopoly with three products ($q_{E3}, q_{I1}, q_{I1H}$), expected equilibrium profits for the incumbent are

$$E\pi_{I3}(q_{I1}) = \mu \frac{\theta^2}{4} (1 + \delta) q_{I1} + (1 - \mu) \frac{\theta^2}{4} \left(1 + \delta - \frac{5 \beta}{12} \delta\right) q_{I1} - \frac{1}{2} \gamma_I q_{I1H} - F_L,$$

where the price equilibrium and the best responses $q_{E3}^* (q_{I1}) = \frac{\theta}{2} q_{I1}$ and $q_{I1}^* (q_{I1H}) = \beta q_{I1H}$ are already included. The first-order condition for profit maximization $\frac{\partial}{\partial q_{I1H}} E\pi_{I3} = 0$ then yields

$$q_{I1H}^* = \frac{\theta^2}{4 \gamma_I} \left(1 + \delta - \frac{5 \beta}{12} (1 - \mu) \delta\right),$$

and equilibrium expected profits read as

$$E\pi_{I3}^* (q_{I1H}) = \frac{\theta^4}{4608 \gamma_I} (12 \delta - 5 \beta (1 - \mu) \delta + 12)^2 - F_L.$$
Comparison with Case 1 then shows that $E\pi^*_I(q_{IH}) \geq E\pi^*_I(q_{IL})$ whenever

$$F_L \leq \frac{5\delta (1-\beta) \theta^4}{4608 \gamma_I} (1-\mu)(24+\delta (19+5\mu(1-\beta))) = \hat{F}_L.$$  

Note that the critical value such that the incumbent uses a price adjustment is decreasing in $\mu$ since

$$\frac{\partial}{\partial \mu} ((1-\mu)(24+\delta (19+5\mu(1-\beta)))) < 0.$$  

Q.E.D.  

**Proof of Proposition 8.** To prove this proposition it is sufficient to show that for $\lambda < 7(1-\delta)/12$ a quality adjustment actually leads to higher profits of the incumbent firm. In case of a quality improvement overall profits are

$$\pi^*_I = \frac{1}{2} \theta^4 \frac{49}{144} \delta + \frac{\lambda}{(1-\delta)}$$

whereas in case of a portfolio adjustment profits are

$$\pi^*_I = \frac{\theta^4}{4608 \gamma_I} (12 + \delta (12-5\beta))^2,$$

see Proposition 4. Then $\pi^*_I > \pi^*_I$ if

$$\lambda \left( - (12 - 5\beta) ((12 - 5\beta) \delta + 24) + ((12 - 5\beta) \delta + 12)^2 \right) + 49 (1-\delta) > 0.$$  

The left hand side of this inequality is monotone in $\lambda$ and positive for $\lambda = 0$. Moreover, the left hand side is positive for $\lambda = 7(1-\delta)/12$ since

$$\frac{7}{12} \left( (12 - 5\beta) ((12 - 5\beta) + 24) + ((12 - 5\beta) \delta + 12)^2 \right) + 49 = (\delta + 1.5484) (\delta + 0.043439) > 0.$$  

Q.E.D.  

**Proof of Proposition 9.** Using the proof of Proposition 4, Stage 1c, the incumbent positions its optimal fighter brand quality such that $\overline{q}_{IL} = \beta \overline{q}_{IH}$, whereas the entrant’s best response is $\overline{q}^*_{E23} = \frac{4}{\lambda} \overline{q}_{IL}$. The resulting profits are

$$\pi^*_E = \frac{1}{48} \theta^4 \overline{q}_{IH}, \quad \pi^*_E = \frac{1}{48} \theta^4 \overline{q}_{IH} - \frac{1}{2} \gamma_I (\lambda \overline{q}_{IH} - \overline{q}^*_{E23}).$$  

Maximizing $\pi^*_E$ with respect to $\overline{q}_{IH}$ then yields

$$\overline{q}_{IH} = \frac{(12 - 5\beta) \theta^4}{48 \lambda \gamma_I}.$$  

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The incumbent’s overall profits in Period 1 then are
\[
\pi_{123} = \frac{1}{4} \theta q_{123} + \delta \frac{(12 - 5\beta)^2}{4608\lambda\gamma_I} - \frac{1}{2} (1 - \delta) \gamma_I (q_{123})^2.
\]
These profits are maximized when the incumbent chooses
\[
q_{123} = \frac{1}{4 (1 - \delta)} \gamma_I \theta^2
\]
which is lower than the quality adjustment in Period 2, \( q_{123} < \overline{q}_{1H} \), if
\[
\lambda < \frac{(12 - 5\beta)}{12} (1 - \delta).
\]
Equilibrium prices can then be derived from the result of Stage 2c/b - Case 1 of Proposition 4 as
\[
p_{1H}^* = \frac{(12 - 5\beta) (2 - \beta)}{192\lambda\gamma_I} \theta^3, p_{1L}^* = \frac{(12 - 5\beta) \beta \theta^3}{192\lambda\gamma_I} \text{ and } p_{E23}^* = \frac{(12 - 5\beta) \beta \theta^3}{672\lambda\gamma_I}.
\]
Equilibrium profits then are
\[
\pi_{123}^* = \frac{1}{32} \theta^4 \left( \frac{(12 - 5\beta)^2}{144} \delta + \frac{\lambda}{(1 - \delta)} \right) \text{ and } \pi_{E23}^* = \frac{(12 - 5\beta) \beta \theta^3}{2304\lambda\gamma_I}.
\]
To show that \( \pi_{123}^* > \pi_{I3}^* \) holds for all \( \lambda \leq \lambda' \) it is sufficient to assume \( \lambda = \lambda' \) since \( \pi_{123}^* = \pi_{123}^* (\lambda) \) is decreasing in \( \lambda \). Calculation shows that \( \pi_{123}^* (\lambda') > \pi_{123}^* \) is equivalent to
\[
144 > 12 (1 - 2\delta)(12 - 5\beta) + (1 - \delta) \delta (12 - 5\beta)^2
\]
Since the left hand side of this inequality is increasing in \( \beta \), and for \( \beta = 0 \) always satisfied since
\[
1 > 1 - \delta^2 - \delta,
\]
it follows that \( \pi_{123}^* > \pi_{I3}^* \). Moreover,
\[
\pi_{E23}^* > \pi_{E3}^* = \frac{\theta^4 \beta}{2304\gamma_I} (12 + \delta (12 - 5\beta))
\]
iff
\[
\lambda < \frac{(12 - 5\beta)}{12 + \delta (12 - 5\beta)}
\]
which is always satisfied since \( \lambda < (12 - 5\beta) / 12 \) and
\[
\frac{(12 - 5\beta)}{12} (1 - \delta) < \frac{(12 - 5\beta)}{12 + \delta (12 - 5\beta)}
\]
iff \( -\delta (12 - 5\beta) - 5\beta < 0 \). Note also that \( q_{1H}^* > q_{1H}^* \) is also equivalent to
\( \lambda < (12 - 5\beta) / (12 + (12 - 5\beta) \delta) \).
Q.E.D.
7 Literature


