The Effect of Seasonality in the CPI on Indexed Bond Pricing and Inflation Expectations

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Abstract

Inflation expectations estimates are among the important indicators considered when formulating an inflation-targeting monetary policy. These estimates include primarily (1) inflation expectations for the next 12 months, calculated from the prices of CPI (Consumer Price Index)-indexed and unindexed government bonds, and (2) professional forecasters’ projections for the next 12 CPI figures. This study outlines how these two expectations estimates are affected by deviations caused by the seasonality inherent in the CPI. The first effect derives from the impact of seasonality on the pricing of CPI-indexed financial assets, including CPI-indexed bonds. The second effect derives from limited internalization of seasonal factors when forecasters' projections are formulated.

We find that seasonal factors have a strong effect on the pricing of CPI-indexed bonds and on inflation expectations. However, these factors are not similar to the factors published by the Central Bureau of Statistics (CBS), and are generally smaller in their absolute value. This finding indicates that bond market participants in Israel, as well as professional forecasters, do not use the full information inherent in seasonal fluctuations of the CPI when they formulate their projections and expectations. Fully taking into account the seasonal factors will improve the pricing of CPI-indexed bonds and the estimates of inflation expectations.
1. Background

The estimates of inflation expectations for the next 12 months, together with the forecasts of the next 12 consumer price index (CPI) figures, are among the most important indicators in the formulation of inflation targeting monetary policy. These forecasts and expectations are based on numerous sources: surveys, analysts’ forecasts and financial asset prices. Each of these sources has advantages and disadvantages in the estimation of expectations. However, the calculated expectations\(^1\), as well as the forecasts with a horizon of one year or 12 indices, are not affected by seasonal factors since these factors cancel out over a horizon of one year. Therefore, if we calculate, over time, the average of forecasts and inflation expectations with a horizon of one year and present the results according to month of the year, we can expect to obtain a horizontal (or very similar) line at the level of the average expected inflation over the course of the measurement period. If this is not the case and we obtain a curve that is not a horizontal line, then there is the possibility of seasonal bias in one-year expectations and the reasons for this need to be investigated. This includes determining whether the seasonal factors implicit in the monthly indexes influence the forecasts and expectations of inflation with a horizon of one year.

In general, rational expectations are based on all the information that is relevant and available when the expectations are formulated. The rational expectations assumption, which is common in many economic models, includes the claim that there is an unknown variable, which impacts inflation and will only become known in the future. Then, the expectation will be correct only on average but will not be biased in any systematic way.\(^2\) Thus, in accordance with this assumption, if the 12-month ahead expectations of inflation tend to be higher during certain months and lower during others, then they are not rational.

In this paper, I will describe two possible seasonal effects\(^3\) inherent in CPI-indexed bond pricing and in expectations and forecasts of inflation. The first is manifested when the terms to maturity are not full years. This is the case for most bond series at any given point in time. The second bias is a result of the only partial internalization of seasonal factors during the formulation of inflation expectations and forecasts. As a result of this bias, analysts are surprised by indexes that are high/low relative to their forecasts and accordingly they revise their forecasts of inflation for the next 12 months. In this paper,

\(^1\) On the assumption that the calculation of the real yield is done correctly and that it takes into consideration the effect of seasonality in the conversion of yields-to-maturity of CPI-indexed bonds into annual terms.

\(^2\) This method for including expectations in a model was proposed by John Muth (1961) and became widely known through its use by Robert Lucas in macroeconomic models.

\(^3\) These seasonal effects are caused by the seasonality implicit in the price index.
we will therefore examine whether seasonal factors are fully taken into account in the pricing of CPI-indexed bonds and in analysts’ forecasts, relative to the index actually published, and the biases in the estimated expectations of annual inflation.

The seasonal factors published by the Central Bureau of Statistics (CBS) for the CPI are characterized by large fluctuations, particularly in the months of January, April and September.\(^4\) It is important to mention that players in the bond market may be using internal models for calculating seasonal factors, which will differ from those of the CBS. Therefore, we do not explicitly know to what extent seasonal factors are indeed taken into account by the public and whether the factors being taken into account are similar to those published by the CBS. However, in this paper, the seasonal factors are estimated.

The paper is structured as follows. In the next subsection we survey the literature. Section 2 presents and explains how investors should consider in theory and in practice the seasonality implicit in the price index when pricing CPI-indexed bonds. In Section 3, we attempt to determine to what extent professional forecasts internalize this seasonality implicit in the price index. The conclusion and main findings are presented in Section 4.

1.1 Survey of the literature

Attempts to estimate the real yield curve are not found often in the professional literature, primarily because most bonds worldwide are not CPI-indexed and are not characterized by similar biases. Therefore, the literature has not given much attention to the biases present in the pricing of CPI-indexed bonds, which are also used to calculate breakeven inflation. However, in recent years, the empirical literature has begun to investigate such bonds and has found that the effect of seasonality on the pricing of CPI-indexed bonds is significant and should be taken into account in the calculation of their yield to maturity. Gapen (2003) found that the effect of seasonality on pricing varies both over time and over term to maturity. Thus, the effect of seasonality does not have a fixed structure over the course of the year, and furthermore is dependent on each bond’s term to maturity. He proposes a method for neutralizing seasonal indexation bias in the real yields calculated on TIPS bonds which is based on the seasonality factors published by the US Bureau of Labor Statistics. Ejsing, et al. (2007) claim that the effect of seasonality on bonds, and in particular relatively short-term bonds, is even more significant than the effect resulting from indexation lags. Canty (2009) claims that there is no consensus among investors and bond traders with regard to the correct way of taking seasonality into account and in most cases these effects are not sufficiently understood.

In contrast to the abovementioned studies, this paper goes beyond simply describing and explaining the pricing of CPI-indexed bonds and in addition estimates the seasonality

\(^4\) It should be mentioned that the seasonality factors in the Israeli CPI are smaller (in absolute terms) than those in the CPIs of Europe and the US. Therefore, the tests carried out in this paper are even more relevant for US and European data. Appendix 1 presents a comparison between the seasonality factors in Israel to those in other countries.
factors implicit in bond prices. This extension of the analysis is made possible by the unique data set—prices of Israeli government bonds, which are traded in the secondary market at a “dirty price” (which includes accrued interest and indexation). This is in contrast to most other studies which have been based on clean bond prices (which do not include accumulated interest and indexation). The paper also presents an analysis of the seasonality factors reflected in analysts’ forecasts of the CPI, which to the best of my knowledge has not previously been examined in the literature.

2. The pricing of CPI-indexed bonds

2.1 Calculating real annual yield

The real yield in the economy is an important indicator, since it used as a benchmark for financial asset investments and serves as a basis for the calculation of inflation expectations. The real yield can be calculated using CPI-indexed government bonds that bear a fixed interest rate. The cash flow from these bonds is indexed to the CPI and is not exposed to any inflation risk. However, these bonds are not fully indexed to the CPI, and therefore there is a built-in bias in the calculation of the real yield, a bias that arises from the lags in indexation. If the CBS could publish indices on a daily basis and the receipts from CPI-indexed bonds were indexed to the last known index, then this would constitute full indexation without any lag and the yield on these bonds would reflect the real yield in the economy over the life of the bond.

More generally, a bond’s yield to maturity reflects its expected cash flow from the day of the calculation until the day of maturity. Our goal is to use the price determined in trading in order to derive the yield to maturity estimated by traders. When the bond’s principal and annual receipts are fixed, such as in the case of fixed rate unindexed bonds, then the calculation of the yield is straightforward and does not require assumptions or estimates regarding information that has not yet been published. However, when the bonds are imperfectly indexed to the CPI the principal and annual receipts are not known (nominally). Then, a simple calculation of the bond’s yield will produce not just the real yield but also components of the nominal yield. Therefore, in order to calculate the real yield, assumptions must be made and estimates calculated that will make it possible to adjust the indexation terms, in order for the indexation to reflect the assessments of investors at the time of the trade. For further details, see Appendix 2.

The number of CPI-indexed bond series has declined significantly over the years, as part of the Ministry of Finance’s long term plan, and each series has become more liquid.

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5 The yield to maturity is the rate of interest that if used to discount the expected flow of income (interest payments and redemption value) yields a present value exactly equal to the market price of the bond. The calculation is based on the Makeham equation and this discount rate is known in the finance literature as the internal rate of return.

6 Perfect indexation is (theoretical) full indexation from the date of a bond’s purchase until the day of maturity.
Currently, there are 13 series of CPI-indexed government bonds with maturities of up to thirty years. Thus, there are only one or two traded series with a maturity of around one year. This leads to, among other things, a situation in which the calculation of the real yield for full years (and in particular the one year ahead yield) is based on only a small number of series whose maturities are distributed around the reference periods (i.e., full years from the day of the calculation). Similarly, the estimation of the zero yield curve, which is based on only a few bonds whose maturities are not necessarily full years, is likely to be biased due to the imprecise seasonal adjustment priced into CPI-indexed assets (see Ejsing, et al. (2007)).

In order to estimate the annual real yield over time and to accurately calculate inflation expectations, it is important to understand to what extent the market is efficient and sophisticated, and in what aspects the market does not behave like that. In an efficient market, the yields on CPI-indexed government bonds should reflect the factors that affect their future cash flows, which include, among others, seasonality factors. When not all the information that affects the cash flow of CPI-indexed bonds is taken into account, bond prices will be biased and the calculation of the real one-year yield and the derivation of inflation expectations will not be accurate. In Section 2.2, we will describe in detail the method commonly used to convert a bond’s yield to maturity into annual terms. In addition, we will describe the problems involved in applying this method to the real yields calculated for CPI-indexed bonds.

2.2 The effect of seasonality factors

It is not simple to calculate the real yields for a full-year period based on the real yields of traded bonds with differing maturities. The calculation requires a good understanding of the effect of the seasonality implicit in the CPI. Since the cash flows from CPI-indexed bonds are directly linked to the CPI, which is not seasonally adjusted, this must be taken into account when calculating the annual yield using yields whose periods are similar but not identical. Therefore, when calculating the real yield in the economy for full years (the yields to maturity on synthetic bonds, i.e., perfectly indexed to the CPI and without lags), it should be taken into account that inflation varies over the course of the year according to seasonal factors. If not, the calculation of the real yield will be carried out as if inflation is uniform over the course of the year. It is important to stress that the shorter the series, the larger will be the weight of seasonal factors in the pricing of the bond and

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7 It has been found that the investing public views the nominal interest rate as the basic interest rate while the real interest rate is determined from it and from the inflation expectations for that period (which are liable to vary significantly by month of the year, according to the seasonality factors). This phenomenon is known in the literature as money illusion, which is defined as the tendency to think of money/yields in nominal rather than real terms. In line with numerous empirical findings in the literature, this paper also found that the real yield is indeed affected by seasonality factors and that there is no such effect on the nominal yield. It is possible that this phenomenon exists as long as the central bank’s interest rate policy is not characterized by seasonal fluctuations.
therefore the yield to maturity will likely be even more biased if seasonality factors are not taken into account (see Figure 1).

We will illustrate the problem using an example. If we take a bond series that is indexed to 14 indexes and we wish to use it to calculate the real yield for one year (12 indexes), we need to first determine the extent to which seasonality is taken into account in the two additional indexes. If there are seasonality factors, they need to be adjusted for in the price of the bond and only then can the yield to maturity be converted into annual terms. Therefore, in order to calculate the real annual yield, an adjustment must be made to the price/yield of the bond according to these seasonality factors. The full details of this example, and others, can be found in Appendix 3. These examples assume that the seasonality factors published by the CBS are fully priced in. This assumption will be tested in Section 2.3.

Figure 1
The effect of seasonality in annual terms on the yield of a CPI-indexed government bond that matures on June 30, 2013
Daily data, 1/2008-2/2012

Figure 1 presents the effect of seasonality factors on the yield of a specific bond series over time (in terms of annual yield), where the seasonality factors published by the CBS are fully priced in. It can be seen that seasonality has a cyclical effect over the course of the calendar year and that this effect increases as the term to maturity shortens. As a

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8 This calculation is based on the difference between (1) the yield-to-maturity without taking into account seasonality factors and (2) the yield-to-maturity when fully taking into account the seasonality factors published by the CBS.
result of this effect, which is characteristic of yields to maturity on individual bond series, the calculation of the real yield for a horizon expressed in full years should adjust for the seasonality factors taken into account by the public. It is of interest to test the bias in the calculation of the real yield for each of the months of the year. The bias can differ for each month of the year since it is likely that at each date of calculation, bonds with different maturities are used. It is important to emphasize that the bias existing in the real yield is also found in the calculation of inflation expectations based on data from the capital market. Figure 2 presents the average bias for each of the months of the year.

**Figure 2**
The effect of the seasonality factors published by the CBS
Monthly averages*, 1/2008-3/2012

* An average for the whole period was calculated for each month of the year.

Figure 2 indicates that if the public indeed prices in the seasonality factors published by the CBS, then if the yield is calculated without adjusting for those seasonality factors the bias is likely to be particularly large during the first three months of the calendar year (which are characterized by high seasonal price indexes). Nonetheless, it is important to stress that we do not explicitly know to what extent seasonality factors are indeed taken into account and whether they are similar to the seasonality factors published by the CBS. In the next section, we will examine whether these factors indeed are fully taken into account by the public.

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9 In contrast to the real yield, the nominal yield is not influenced by seasonality factors (see footnote 6). Therefore, the effect of seasonality on CPI-indexed bonds will be manifested in the estimate of inflation expectations calculated as the break-even inflation rate.
account in bond prices. In Section 3, we will test whether analysts take these factors into account when making their forecasts of future indexes.

2.3 The extent to which seasonality factors are taken into account by investors

In order to take the effect of seasonality factors on the real annual yield into account, we need to determine to what extent these factors are taken into account (in other words, what is the level of awareness of seasonality factors among investors). In particular, the extent to which the seasonality factors taken into account by the public differ from the CBS factors should be examined. In order to examine this important issue, a number of calculations/methods are used to estimate the seasonality factors taken into account in the pricing of bonds:

1. Estimation of the extent to which the seasonality factors published by the CBS are taken into account in the pricing of bonds. The estimated parameter in this framework will range from zero to one. When the public does not take the CBS seasonality factors into account, the estimated parameter will take a value of zero; when the public fully takes into account the precise CBS seasonality factors, the estimated parameter will receive a value of one.
2. Estimation of 12 parameters that represent the seasonality factors (which are fixed over time) taken into account in the pricing of CPI-indexed bonds.

Section 2.4 will elaborate on the two estimations and will present the results and findings.

2.4 The main findings of the estimation

It is important to emphasize that the estimations are based on the assumption that seasonality factors are not correlated with the economic developments that influence the real annual yield. Therefore, the greater our success in neutralizing the effect of the seasonality factors taken into account by investors, the smaller will be the standard deviations of changes in real yields over time.\textsuperscript{10} Thus, the estimation of the seasonality factors is carried out through the minimization of the standard deviations of changes in yield. On the basis of this estimation method, it can be said that the volatility of the real yield—which is due not only to economic factors, but also to temporary biases\textsuperscript{11}—is larger. Therefore, the greater our success in neutralizing the seasonal bias, the smaller will be the volatility of the yield changes.

\textsuperscript{10} The calculation of the standard deviation of changes in yield, instead of the standard deviation of the yield itself, avoids dealing with the econometric problem of strong autocorrelation in the estimation variables.

\textsuperscript{11} Assuming that the temporary biases are not correlated with the economic factors at all.
1. Estimating the extent to which the CBS seasonality factors are taken into account in pricing

The first method assumes that the market only partially takes into account the CBS seasonality factors and the extent to which they are taken into account is estimated as being between zero and one. Within this framework, the extent ($\rho$) to which the sum of the CBS seasonality factors ($\sum S^n_i$) are taken into account in pricing (an example is presented in Figure 1) is estimated using a target function that minimizes the standard deviations (STD) of variation over time in the yield to maturity ($r^n_t$) for each of the series of CPI-indexed bonds ($n$). The minimization of the function is carried out through the search for a value of $\rho$, which represents the extent to which the CBS seasonality factors are taken into account. This is described by the following equation:

$$\rho = \text{MIN} \left[ \sum_{n=1}^{N} \text{STD} \left( d(r^n_t) - \rho \cdot d \left( \sum_{i=0}^{I} S^n_i \right) \right) \right]$$

At the minimal value (over the sample period of January 2005 to February 2012), $\rho$, the extent to which seasonality is taken into account in pricing, had a value of 80 percent. This finding indicates that investors partially take into account seasonality as published by the CBS.

**Figure 3**

The volatility of the changes in yields as a function of the extent to which seasonality factors are taken into account in bond prices

Volatility expressed in annual terms; daily data: 1/2008-2/2012

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12 The sum of seasonality factors, at each point in time and for each bond series, varies according to the bond’s number of months of indexation. Therefore, $n$ represents the series and $i$ represents the number of months of indexation, which can range from 0 (which represents an entire year) to 11.
2. Estimation of the 12 seasonality factors that are taken account in the pricing of CPI-indexed bonds

The second method assumes that the market believes that the seasonality factors remain fixed over time and that it fully takes them into account in the pricing of bonds. Within this framework, the seasonality factors \((S_i^*)\) which characterize each bond at each point in time are estimated by a different aggregate of factors \((\Sigma(S_i^*))\) for each bond (see footnote 10). This is done using a target function that minimizes the standard deviations (STD) of changes in the yield to maturity \((r^n_t)\) of each series of CPI-indexed bond \((n)\). The minimization of the function is carried out by searching for 12 values \((S_i^*)\) (whose sum equals zero) which represent the seasonality factors that are taken into account by investors in pricing CPI-indexed government bonds. This is described by the following equation:

\[
2. \quad \min_{S_i} \left[ \sum_{n=1}^{N} \text{STD} \left( d(r^n_t) - d \left( \sum_{i=0}^{12} S_i^* \right) \right) \right]
\]

The minimal value (over the sample period from January 2005 to February 2012) was found using the estimated factors presented in Column A in Table 1.

| Table 1: Seasonality factors published by the CBS and those estimated using equation 2 |
|---------------------------------|-----------------|-----------------|-----------------|
| Seasonality factors according to the estimated equation | Seasonality factors according to the CBS | Difference |  |
| Column A | Column B | C=A-B |  |
| January | -0.3 | -0.5 | 0.2 |
| February | -0.2 | -0.3 | 0.1 |
| March | 0.1 | 0.1 | 0.0 |
| April | 0.2 | 0.5 | -0.3 |
| May | 0.0 | 0.3 | -0.3 |
| June | 0.0 | 0.1 | -0.1 |
| July | 0.1 | 0.3 | -0.2 |
| August | 0.1 | 0.2 | -0.1 |
| September | -0.1 | -0.4 | 0.3 |
| October | 0.1 | 0.0 | 0.1 |
| November | 0.0 | -0.2 | 0.2 |
| December | 0.1 | -0.1 | 0.2 |
The estimated factors (equation 2) receive negative values only in January, February and September. According to the publications of the CBS, these months are characterized by strong negative seasonality. In contrast to the factors published by the CBS, the estimated factors for November and December are not negative, although their values are low in absolute terms. In addition, although the estimated seasonality factor for April is positive and statistically significant, it is still significantly smaller than the CBS seasonality factor for that month.

Although the standard deviation of the series of differences (between the estimated factors (A) and the CBS factors (B); see Table 1) is 0.2, which is large relative to the size of the seasonality factors, the correlation coefficient between the estimated factors and the CBS factors is high, 0.82. This shows a strong link between the two factors.

The estimation for the first and second equations indicates that the seasonality factors taken into account in the pricing of CPI-indexed bonds have similar characteristics to those of the CBS factors, although their absolute values are smaller by an appreciable and statistically significant amount. These findings indicate that the CPI-indexed bond market in Israel is not sophisticated and the yields on CPI-indexed government bonds do not fully reflect the seasonality factors, even though these affect the bond’s future cash flow. Therefore, when calculating estimated annual yields and inflation expectations, it is recommended that account be taken of the seasonality factors that are implicit in bond prices, which are not similar to those of the CBS. It is important to monitor the estimated seasonality factors from time to time since it can be assumed that over time—and in particular with the publishing of research in this area—investors in the bond market will improve their understanding of seasonality and will price CPI-indexed bonds more accurately by taking seasonality factors into account more fully.

3. Testing of analysts’ forecasts

3.1 12-month-ahead forecasts

This section tests the effect of seasonality factors on analysts’ forecasts. As mentioned, the calculated expectations\textsuperscript{13}, as well as the forecasts provided with a 12-month horizon or of 12-index ahead figures, are not influenced by seasonality factors since these factors cancel out for a one-year horizon. Therefore, if we calculate the average of the forecasts and the expectations of inflation over time and present the results according to months of the year, then we can expect to obtain a horizontal line (at least approximately) at the level of the average expected rate of inflation during the period of measurement. If this is not the case and we obtain a curve that differs from a horizontal line, then there exists the possibility of seasonal bias in 12-month expectations and the cause of this phenomenon

\textsuperscript{13} On the assumption that the calculation of the real yield is carried out correctly and taking into account the influence of seasonality on the conversion of the yields to maturity of CPI-indexed bonds into annual terms.
needs to be investigated. This includes determining whether the seasonality factors implicit in the monthly indices affect the 12-month forecasts and inflation expectations.

The effect of the seasonality factors on forecasts and inflation expectations can be determined by looking at analysts’ 12-index-ahead forecasts. This has two main advantages:

1. Length of the sample: The Bank of Israel has been regularly collecting data on analysts’ forecasts since December 2000.

2. Simplicity of the calculation: The analysts’ forecasts are straightforward, in contrast to the calculation of the expectations derived from capital market data (i.e., breakeven inflation). Capital market data include various assumptions regarding seasonality and there is the possibility (as we saw in Section 2) that these assumptions can affect the results of the current test.

The calculation in this section is based on the average of the forecasts provided in each of the months of the year (see Figure 4). It can be seen from Figure 4 that the average 12-index-ahead forecast of analysts changes over the course of the year according to a fixed pattern. Thus, the average forecast tends to be relatively low in the winter months and relatively high in the summer months.

Figure 4
Analysts’ forecasts of the next 12 indexes

*The average for the whole period was calculated for each of the months of the year.
A possible explanation of this phenomenon involves the effect of seasonality within the indices announced at the time the forecasts are made. From May, when a particularly high seasonal index is announced (for April), until August there are relatively high seasonal price indices. It may be that analysts are not fully aware of this seasonality and that they are essentially “surprised” by the high price indexes. As a result, they forecast inflation to be higher than it actually is. During the winter months, there is negative seasonality in the price indexes. Thus, it may be that analysts are “surprised” that the price indexes are lower than forecasted and as a result forecast inflation to be lower than it actually is.\(^{14}\) It should be stressed that the average of the analysts’ forecasts changes in a statistically significant manner over the course of the year according to the pattern described above (see Appendix 4).

If we examine the average surprises in the CPI over time relative to the forecasts provided by analysts prior to the announcement of the index for each of the 12 months of the year (forecast for the current month, the current month+1 and the current month+2) we can shed light on the phenomenon that can be seen in Figure 4. On the assumption that the analysts use all the relevant information available, including the seasonality factors published by the CBS, then we can expect that the surprises will not indicate any fixed biases. In other words, the series of surprises can be expected to behave like white noise.

However, the empirical findings point to the opposite conclusion (Figure 5); the average surprises over a long period, for each of the months of the year\(^{15}\), have fixed biases. For the months of April, June and July, the surprise is generally positive, i.e., the announced index is higher than the forecasts, while for the months of September, October and November the surprise is generally negative. These findings were statistically significant when using dummy variables that represent the months of the year (see Appendix 5).

\(^{14}\) There is a hidden assumption here that the surprises in the monthly index affect analysts’ 12-index-ahead forecasts in the same direction. See Appendix 6 for an empirical proof of this assumption.

\(^{15}\) The differences between the forecasts, which are published just prior to the announcement of the index; and the actual CPI indexes.
In order to analyze the forecasts of analysts for more distant indexes, we can use the following table of correlations:

**Table 2**

Table of correlations between the CPI, the current forecasts and seasonality factor

**10/2000-4/2012**

<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>Current</th>
<th>Current+1</th>
<th>Current+2</th>
<th>Seasonality factor</th>
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<td>0.76</td>
<td>0.47</td>
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<tr>
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<td>0.65</td>
<td>0.56</td>
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<tr>
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<td>1</td>
<td>0.78</td>
<td>0.58</td>
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<tr>
<td>Current+2</td>
<td></td>
<td></td>
<td>1</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>Seasonality factor</td>
<td></td>
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</tr>
</tbody>
</table>

*Forecast of the current index: forecast given for the next index to be announced. Forecast of current+1 index: forecast of the index to be announced in about another month. Forecast of current+2 index: forecast of the index to be announced in about two months (when the forecast is provided on August 14th, the forecast of the current index will be for the July index while the forecast of the current+2 index will be for the September index that covers the period 16/8 to 15/9.**
The correlation coefficient between the current CPI and the current forecast for the same CPI (0.87) is higher than the correlation coefficient between the CPI and earlier forecasts of that CPI and also higher than the correlation between the CPI and the seasonality factor. This finding indicates that the forecasts made just prior to the announcement of the index are highly correlated with the index, which is a logical and expected result. However, the longer the horizon of the forecast, the smaller becomes the correlation coefficient, such that the forecasts for current+2 are less correlated with the index than the seasonality factors. This implies that the analysts’ forecasts for relatively distant indexes are less correlated with the index and basically do not internalize all the information implicit in the published seasonality factors. Using equation 3, we will examine the extent to which the analysts’ forecasts do not take into account seasonality.

We estimate the actual CPI using the analysts’ forecasts immediately prior to (i=1), one month prior to (i=2), and two months prior to (i=3) the announcement of the CPI, and add the seasonality factors as an additional explanatory variable (Equation 3). Since the seasonality factors are significant in the estimation (Table 3), we can infer that the forecasts do not fully take into account the information implicit in the seasonality factors. The forecasts can be improved by adding the relative portion obtained from the estimation equation (λ) multiplied by the seasonality factors.

\[ cp_t = \alpha + \beta \times \text{forecast}_i + \lambda \times \text{seas}_i + \epsilon_i \]

According to theory, when the analysts’ forecasts take into account all available and relevant information, then any additional variable, and in particular if it is known to the public, cannot add any benefit as an explanatory variable. The results presented in Table 3 indicate that the seasonality factors can be added to Equation 3 as a significant explanatory variable and that they increase the equation’s explanatory power. Furthermore, when the estimation equation is used to test the forecasts of the current+2 index, i=3, the estimated coefficient of the average forecast (β) is lower than that of the seasonality factors (λ). This finding indicates that analysts’ forecasts succeed in explaining the actual CPI index to a lesser extent than the CBS seasonality factors.
The most important finding in this section is the apparent existence of an indirect effect for seasonality factors on 12-index-ahead inflation expectations, such that analysts revise their forecasts over the course of the year in a fixed manner. A possible explanation of this phenomenon is related to the under/overestimation of seasonality factors. Thus, analysts are surprised by CPI indexes that are higher/lower than their forecasts and accordingly they update their view of the inflation environment, which is reflected in their average 12-index-ahead forecasts.

4. Summary and conclusions

This paper describes two possible biases in the estimates of inflation expectations that are caused by the seasonality implicit in the CPI. The first is a result of the seasonal effect on the pricing of CPI-indexed bonds. The second is a result of the only partial internalization of seasonality factors during the formulation of expectations, in particular analysts’ forecasts. The assumption that inflation expectations with a horizon of one year are not affected by seasonality factors is rejected by the results of this study.

With regard to the effects of the real yield on the calculation, it is important to emphasize that because a bond’s maturity declines over time, the effect of seasonality factors on yield to maturity varies from one bond series to another and also over time. This situation makes it difficult to calculate real yields if the seasonality factors are not correctly identified. According to the estimation results presented in Section 2, market participants do not fully take into account the seasonality factors published by the CBS and the seasonality factors priced into these bonds are in general smaller in absolute value.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>i=1</th>
<th>i=2</th>
<th>i=3</th>
<th>i=3*</th>
</tr>
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<tbody>
<tr>
<td>α</td>
<td>0.009</td>
<td>0.03</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(0.42)</td>
<td>(0.86)</td>
<td>(2.8)</td>
<td>(2.5)</td>
</tr>
<tr>
<td>β</td>
<td>0.98</td>
<td>0.94</td>
<td>0.32</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>(15.2)</td>
<td>(9.5)</td>
<td>(1.67)</td>
<td>(1.25)</td>
</tr>
<tr>
<td>λ</td>
<td>0.23</td>
<td>0.32</td>
<td>0.72</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>(3.19)</td>
<td>(3.35)</td>
<td>(5.36)</td>
<td>(5.65)</td>
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<tr>
<td>δ</td>
<td></td>
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<td></td>
<td>0.37</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>(4.34)</td>
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<tr>
<td>R²</td>
<td>0.77</td>
<td>0.61</td>
<td>0.36</td>
<td>0.44</td>
</tr>
<tr>
<td>D.W.</td>
<td>1.64</td>
<td>1.4</td>
<td>1.3</td>
<td>1.99</td>
</tr>
</tbody>
</table>

* When added to an AR(1) estimation equation.
than the CBS factors. These findings suggest that the CPI-indexed bond market in Israel is not efficient or sophisticated and the yields on CPI-indexed government bonds do not fully reflect seasonality factors, even though these factors affect a bond’s future cash flow. Therefore, when calculating estimated annual real yields and expectations of inflation, one should take into account the seasonality factors implicit in bond prices—and that those factors are not necessarily identical to the factors published by the CBS. It is important to monitor the estimated seasonality factors since it can be assumed that over time—and particularly with the publication of new research in this area—investors in the bond market will improve their understanding of seasonality and will price CPI-indexed bonds more precisely.

With regard to the effect on analysts’ forecasts, it was found that the forecasts of upcoming indexes do not fully take into account the information implicit in the seasonality factors published by the CBS. Furthermore, the estimation of equation 3 indicates that the analysts’ forecasts can be improved by taking into account the seasonality factors.
References


Appendix 1

Seasonality factors in Israel, the US and Europe

Figure A1

Seasonality factors for Israel (published by CBS), Europe and the US

Figure A2

Standard deviations of the seasonality factors

It can be seen that the seasonality factors that characterize the CPI in Israel are similar to those in the US and Europe, though not identical (it is worth mentioning that the factors in the US and Europe are also not identical). It is important to note that the volatility of the seasonality factors is lower in Israel than in the rest of the world, despite the particularly high seasonality in April.
Appendix 2

Adjusting the indexation of a CPI-indexed bond

In order to adjust the indexation of an CPI-indexed bond, the period of each of the bond receipts (both principal and interest) needs to be divided into three sub-periods: (1) The period during which there has been a change in the price index (since the last known index and up until the current day of trading) and a new index has yet to be announced\(^\text{17}\), which will be denoted as period A. For this period, the previously announced price index needs to be adjusted to its forecasted present value, such that the payments on the bond will be indexed to the inflation to occur from today onward. (2) The period in which the bond provides full compensation for the change in prices, which will be denoted as period B. (3) The period near to the payment on the bond for which there is no compensation for the change in the price index\(^\text{18}\), which will be denoted as period C.

Since CPI-indexed government bonds are not perfectly indexed, appropriate estimates of inflation during periods A and C need to be created:

- During period A, use should be made of the estimated inflation that has occurred but has not yet been announced. For the calculations carried out by the Bank of Israel, this estimate is based on the average forecast of economic consulting companies and of the banks that announce their forecasts on a regular basis. However, it should be noted that in some cases when the next coupon is close to the date of payment and, according to the terms of the bond, there is no “compensation” for inflation that has not yet been announced by the day of payment. Therefore, it is not correct to adjust this coupon together with all the others. This situation arises when the calculation of the yield takes place during the period between 45 days prior to the payment of the next coupon and the ex-date.

- For period C (the period prior to the payment), for which there is no indexation to the price index, there are two ways to deal with the unindexed period:
  1. We can define compensation for the bond’s only partial indexation at maturity. Since a bondholder does not receive inflation compensation for

\(^{17}\) When calculating the real yield on a particular day, such as for example the 10\(^{th}\) of May, the principal and interest payments are priced on the basis of the known index announced on April 15\(^{th}\) and this reflects the average level of prices during March. However, it is reasonable to assume that investors, who determine the price of the bond on May 10\(^{th}\), also take into account the inflation up until that day, which has still not been announced, particularly if during this interim period there was a significant change in a factor that has an effect on prices (such as the exchange rate, a surprise change in the interest rate, energy prices, etc.).

\(^{18}\) Each payment on a bond is made according to the index announced on the 15\(^{th}\) of the month and this index represents the average level of prices during the month previous to the X-day. Therefore, the assumption here is that the bond does not provide indexation during a 45-day period and this lack of indexation should be taken into account when calculating the real yield. We would mention that this phenomenon is called the indexation lag and exists to an even greater extent in most CPI-indexed bonds traded in other countries.
period C, bondholders will compensate themselves through the price of the bond. Therefore the price of the bond should be adjusted in accordance with the estimate that represents this compensation. An estimate of that compensation is based on inflation that will prevail during the 45-day periods prior to the payment of the coupons (for up to 30 years). In practice, there are no clear estimators for that.

2. During period C, the bond can be defined as a nominal bond and its nominal value during this short period may be subtracted from the price of the bond. After this has been done, it is possible to use the adjusted price to calculate the real short-terms yield for the 45 days prior to the bond’s date of maturity. The only possibility of deriving the nominal interest rate that will prevail during a 45-day period in the future is by calculating the forward rate of interest based on the estimation of the yield curve for unindexed bonds.
Appendix 3

Calculating the yields to maturity on CPI-indexed bonds

Following are two examples of calculating the yields to maturity on CPI-indexed bonds:

a. On February 1, 2012, there is a traded bond series that matures on June 30, 2013. The indexation period starts in the middle of the indexation month of March 2012 and ends at the end of the indexation month of May 2013\(^\text{19}\), for a total of 14.5 months. In order to calculate the real annual yield, only the 12 indexes starting on the day of the calculation should be taken into account. Therefore, an indexation adjustment needs to be conducted for half of March 2013 and for the months of April and May 2013. According to the CBS, the total seasonality for this period is 0.85 percent. Therefore, on the assumption that the real yield obtained for the original period of the series is 3 percent (not in annual terms), the seasonality should be subtracted from the yield first, and in our example this gives a result of 2.15 percent. Following this adjustment, the yield can be converted into annual terms.

b. On February 1, 2012, there is a traded bond series that matures on September 30, 2012. The period of indexation starts in the middle of the indexation month March 2012 and ends at the end of the indexation month August 2012, for a total of 5.5 months. In order to calculate the real annual yield, the indexation needs to be extended up to 12 months and therefore an indexation adjustment should be conducted from September 2012 until the middle of March 2013. According to the CBS, the total seasonality for this period is -1.45. Therefore, on the assumption that the real yield obtained for the original period of the series is 2 percent (not in annual terms), the seasonality should first be added and in our example this gives a result of 0.55 percent. Following this adjustment, the yield can be converted into annual terms.

Using the CBS method (X12)

\[
\begin{array}{ll}
\text{January} & -0.5 \\
\text{February} & -0.3 \\
\text{March} & 0.1 \\
\text{April} & 0.5 \\
\text{May} & 0.3 \\
\text{June} & 0.1 \\
\text{July} & 0.3 \\
\text{August} & 0.2 \\
\text{September} & -0.4 \\
\text{October} & 0.0 \\
\text{November} & -0.2 \\
\text{December} & -0.1 \\
\end{array}
\]

\(^{19}\) An indexation month begins on the 16\textsuperscript{th} of the previous month and ends on the 15\textsuperscript{th} of the current month.
Appendix 4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1+D9+D10+D11+D12</td>
<td>-0.130526</td>
<td>0.053378</td>
<td>-2.445325</td>
<td>0.0158</td>
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<tr>
<td>D2+D8</td>
<td>0.174348</td>
<td>0.08403</td>
<td>2.074825</td>
<td>0.0399</td>
</tr>
<tr>
<td>D4</td>
<td>-0.018333</td>
<td>0.116334</td>
<td>-0.157592</td>
<td>0.875</td>
</tr>
<tr>
<td>D3+D5+D6+D7</td>
<td>0.072222</td>
<td>0.060075</td>
<td>1.202204</td>
<td>0.2314</td>
</tr>
</tbody>
</table>

R-squared          | 0.081158    | Mean dependent var | -0.00292  |
Adjusted R-squared | 0.060432    | S.D. dependent var  | 0.415753  |
S.E. of regression | 0.402994    | Akaike info criterion | 1.048974  |
Sum squared resid   | 21.59979    | Schwarz criterion   | 1.134229  |
Log likelihood      | -67.85472   | Durbin-Watson stat  | 1.909006  |

D(FORCAST12A)_{t} – the average of analysts’ 12-index-ahead forecasts, which were provided during the period after the announcement of the price index for month \(t-1\) and before the end of month \(t\).

D_{t} – A dummy variable for calendar month \(t\) (where \(t=1\ldots12\)).

Main findings:

The average of the analysts’ forecasts varies according to month of the year. It was found that there are statistically significant upward revisions of the analysts’ average forecast in February and August while during the months of September to January there are significant downward revisions.

It is important to mention that when examining the 12-index-ahead forecasts of individual analysts, it is found that each analyst revises his forecast according to a somewhat different pattern, though it is nonetheless consistent and statistically significant.
## Appendix 5

Dependent Variable: SURP  
Method: Least Squares  
Sample(adjusted): 2000:12 2012:03  
Included observations: 136 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.033606</td>
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<td>D2+D8+D5+D12</td>
<td>0.030435</td>
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<td>0.915646</td>
<td>0.3615</td>
</tr>
</tbody>
</table>

R-squared       | 0.16074     | Mean dependent var | 0.007353  |
Adjusted R-squared | 0.135114   | S.D. dependent var | 0.242405  |
S.E. of regression | 0.225435   | Akaike info criterion | -0.1055  |
Sum squared resid  | 6.657549   | Schwarz criterion | 0.001586  |
Log likelihood     | 12.17379   | Durbin-Watson stat | 1.784075  |

SURP – the surprise in the index; the gap between the change in the price index and the average of the forecasts.  
Di – A dummy variable for the calendar month i (where i=1…12).

Main findings:  
The surprises in the forecasts relative to the announced indexes are positive and statistically significant for the months of March, April, June and July while they are negative and statistically significant for the months of September and November. For each of the rest of the months of the year, there are no large and consistent surprises.
Appendix 6

Dependent Variable: D(FORCAST12A)
Method: Least Squares
Sample (adjusted): 2001M02 2012M04
Included observations: 135 after adjustments

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
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</thead>
<tbody>
<tr>
<td>SURP(-1)</td>
<td>0.48</td>
<td>0.141</td>
<td>3.39</td>
</tr>
<tr>
<td>SURP(-2)</td>
<td>0.26</td>
<td>0.141</td>
<td>1.82</td>
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<tr>
<td>C</td>
<td>0.00</td>
<td>0.034</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

R-squared        0.113262  Mean dependent var 0.003852
Adjusted R-squared 0.099826  S.D. dependent var 0.414178
S.E. of regression 0.392962  Akaike info criterion 0.991761
Sum squared resid 20.38328  Schwarz criterion 1.056323
Log likelihood   -63.9439  Hannan-Quinn criter. 1.017998
F-statistic       8.430067  Durbin-Watson stat 1.919876
Prob(F-statistic) 0.000359

D(FORCAST12A)\_i – the change in the average of the analysts’ 12-index-ahead forecasts which were made during the period after the announcement of the price index for month i-1 until the end of month i.

SURP – the surprise in the index; the gap between the change in the price index and the average of the forecasts.

Main findings:

The surprise in the index (which is announced with a lag of one month) affects the analysts’ 12-month-ahead forecasts. A price index that is higher than the forecasts (accumulated over two months) leads to an upward revision of the annual forecasts in the amount of 74 percent of the surprise.