Bailout Uncertainty, Leverage, Duration Mismatches, and Lehman’s Collapse

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Goals

- Model the exante impact of bailout policy on the volume of leverage, real investments, and the likelihood of a crisis
- Analyze the impact of an expost downward revision of perceptions about the likelihood of bailout on financial markets and the real economy (Lehman’s collapse)
- Analyze the impact of higher bailout uncertainty on the amplitude of booms and busts
- Analyze the impact of expansionary monetary policy on leverage and risk appetite
Outline

- Introduction
- Framework
- Optimizations
- General equilibrium
- Comparative statics
- Conclusions
## Chronology of CDS rate around Lehman’s collapse (September-October 2008)

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>CDS Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>13-14/9</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>15/9</td>
<td>Lehman files for chapter 11</td>
<td></td>
</tr>
<tr>
<td>16-17/9</td>
<td>Paulson suggests TARP to Congress</td>
<td>250</td>
</tr>
<tr>
<td>18-19/9</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>22-23/9</td>
<td>Paulson &amp; Bernanke address</td>
<td>450</td>
</tr>
<tr>
<td>24-25/9</td>
<td></td>
<td>350</td>
</tr>
<tr>
<td>29/9</td>
<td>Congress rejects Tarp proposal</td>
<td>Almost 450</td>
</tr>
<tr>
<td>3/10</td>
<td>Amended Tarp approved by Congress</td>
<td>150</td>
</tr>
<tr>
<td>5-10/10</td>
<td>Aftermath of approval</td>
<td></td>
</tr>
</tbody>
</table>
Motivation and mechanism

- Animal spirits (Akerlof Shiller) vs rational agents subject to uncertainty
- The impact of bailout uncertainty on leverage and the role of duration mismatches
- Extended example of a general equilibrium partially microfounded model of financial markets aimed at analyzing the interrelationships between financial markets
- Focus is on the shadow banking system in which long term credit is financed by means of short term liabilities
Recent work

- Meltzer and others, bailout uncertainty
- John Taylor (2009), the impact of low interest rates on the likelihood of a crisis
- Farhi-Tirole (2009), collective moral hazard
- Tirole (2010), illiquidity
- Sieczka, Sornette and Holyst (2010), bankruptcy cascades
Financial flows

Leverage, Duration Mismatches, and Lehman’s Collapse

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Introduction

Framework
- General
- Players
- Yields and defaults

Optimizations
- Borrower
- Financial intermediary
- Lender

General equilibrium
- Period 1
- Period 0

Comparative statics
- Period 1
- Period 0

Conclusions

Appendix

Bailout Uncertainty, Leverage, Duration Mismatches, and Lehman’s Collapse

Financial flows

- L (Lender)
- F (Financial intermediary)
- B (Borrower)
- Econ
- Idio

- Z_L
- Z_F
- \( r_L \)
- \( r_B \)
- \( r_f \) (1)
- \( (1-Z_L) \)
- \( (1-Z_F) \)

Government

\( R_A (q_A) \)

\( R_I (q_I) \)
Timeline

- 3 periods labeled: 0, 1 and 2
- All investment decisions are made in period 0
- Once chosen the project size cannot be adjusted
Financing - debt maturity

- Only short term loans are available
- Two periods projects are financed by two consecutive one period loans
- If internal resources and credit refinancing does not suffice a default occurs
- In case of default all equity vanishes
Players

There is a large number of each of the following 3 kinds of players:

- **Borrowers** (B) - entrepreneurs, manufacturers
- **Financial intermediaries** (F) - hedge funds, SIVs and conduits
- **Lenders** (L) - pension funds

**Mass:** $M_B, M_F$ and $M_L$

**Capital:** Each individual player (irrespective of type) possesses one unit of equity capital. For Fs this unit may be composed of capital and exogenous long term debt.
Borrower

- Each borrower invests in a single long term (two periods) investment project.
- The project size, \( x \), is selected to maximizes the borrower’s expected utility.
- The borrower’s utility function is:

\[
u(W_B) = \begin{cases} 
W_B & \text{if } W_B \geq 0 \\
-P & \text{if } W_B < 0, \ P > 0
\end{cases}
\]

- When needed \((x > 1)\) B borrows an additional amount, \( L_B \) from \( P \) at a gross (one plus) interest rate \( r_{Bt}, \ t = 1, 2 \).
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Introduction

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Players

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Financial intermediary

Lender

General equilibrium

Period 1

Period 0

Comparative statics

Period 1

Period 0

Conclusions

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Financial intermediary

- Borrows an amount $L_F$ from many lenders at $r_L$
- In each period $F$ lends a fraction $z_F$ of his resources $(1 + L_F)$ to two borrowers at a gross rate $r_B$
- The remainder, $1 - z_F$, is invested in a risk free asset whose gross rate, $r_f$, is set by the central bank
- $F$ chooses $L_F$ and $z_F$ so as to maximize his expected utility from profits, $W_F$, in each period
- $F$ possesses a quadratic utility function:

$$u(W_F) = W_F - \frac{b}{2} W_F^2, \quad W_F < \frac{1}{b}$$
Each lender invests a portion $z_L$ of his resources in a fully diversified portfolio of loans to Fs and a portion $1 - z_L$ in the risk free asset.

Each lender lends to a large number of Fs at $r_L$. As a consequence his risky portfolio is fully diversified.

Lenders are risk averse characterized by a mean-variance preferences (CARA preferences):

$$u(W_L) = -\frac{1}{\alpha} e^{-\alpha W_L}, \quad \alpha \geq 0$$
Borrower (project) - yield

- Returns on all projects are equally distributed
- A project’s gross return in each period is the sum of an aggregate and an individual shock

\[ \tilde{R} = \tilde{R}_A + \tilde{R}_I \]

- The aggregate (economy-wide shock) \( \tilde{R}_A \), is binomially distributed:

\[ \tilde{R}_A = \begin{cases} R_A & \text{w.p. } q_A \\ 0 & \text{w.p. } 1 - q_A \end{cases} \]

- The idiosyncratic shock, \( \tilde{R}_I \), is binomially distributed:

\[ \tilde{R}_I = \begin{cases} R_I & \text{w.p. } q_I \\ 0 & \text{w.p. } 1 - q_I \end{cases} \]
Borrower (project) - yield

- \( \tilde{R}_A \) and \( \tilde{R}_I \) are independent across periods and mutually independent within a period
- The idiosyncratic shock, \( \tilde{R}_I \), is independent across projects
- \( R_A < \mu_B < R_I \)
- Consequently the distribution of payoffs is ranked

\[
1 < R_A < \mu_B < R_I < R_A + R_I
\]

\[
\mu_B \equiv E\tilde{R}
\]
Borrower - default

- Can possibly default in either period 1 (illiquidity) or in period 2 (insolvency)
- A borrower who defaults loses his investment project
- When B defaults the F who lent to him loses the (gross) rate, $r_{Bt}, \ t = 1, 2$
- A borrower defaults in period 1 if he can’t refinance the project
- A borrower defaults in period 2 if his cash flow is smaller than the required debt service
The income from a portfolio consist of two loans $\tilde{r}_B$:

<table>
<thead>
<tr>
<th>Yield</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_B$</td>
<td>$q_A + (1 - q_A)q_I^2$</td>
</tr>
<tr>
<td>$\frac{1}{2}r_B$</td>
<td>$2(1 - q_A)(1 - q_I)q_I$</td>
</tr>
<tr>
<td>0</td>
<td>$(1 - q_A)(1 - q_I)^2$</td>
</tr>
</tbody>
</table>

The two borrowers are solvent
One borrower defaults and one is solvent
Both borrowers default
Financial intermediary - default

- May default in period 1 (in which case he disappears) or in period 2
- Defaults when at least one of his two borrowers defaults
- His creditors (L) lose the fraction of the (gross) rate, $r_L$, invested in that particular F provided there are no governmental bailouts.
Government bailout policy

- Government may repay the gross debt owed to lenders by defaulting Fs
- The probability that the debt service of a defaulting F is payed by government (a bailout) is $\pi$
- Likelihood of bailout is independent across Fs
- In case of bailout lenders receives the debt service $r_L$
The return to a lender on his (fully diversified) portfolio of loans is normally distributed with mean

\[ E(\{\tilde{r}_L\}) = (q_A + (1 - q_A)q^2_I + \pi(1 - q_A)(1 - q^2_I)) r_L \]

and variance

\[ Var(\{\tilde{r}_L\}) = q_A(1 - q_A)(1 - \pi)^2(1 - q^2_I)^2r^2_L \]

Variance of return on risky portfolio is the covariance between two loans.
Equilibrium’s interest rates

Proposition

In equilibrium with positive quantities the yields satisfy
\[ 0 < r_f < r_L < r_B \]
Summary

<table>
<thead>
<tr>
<th>Player</th>
<th>Index</th>
<th>Leverage</th>
<th>Mass</th>
<th>Income</th>
<th>Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrower</td>
<td>B</td>
<td>(L_B)</td>
<td>(M_B)</td>
<td>(\tilde{R})</td>
<td>(r_B)</td>
</tr>
<tr>
<td>FI</td>
<td>F</td>
<td>(L_F)</td>
<td>(M_F)</td>
<td>(\tilde{r}_B)</td>
<td>(r_L)</td>
</tr>
<tr>
<td>Lender</td>
<td>L</td>
<td>0</td>
<td>(M_L)</td>
<td>(\tilde{r}_L)</td>
<td>0</td>
</tr>
</tbody>
</table>

- \(\tilde{R} = \tilde{R}_A + \tilde{R}_I\)
- Probability of receiving \(R_A\) is \(q_A\)
- Probability of receiving \(R_I\) is \(q_I\)
- Probability of bailout is \(\pi\)
- Probability of receiving \(r_B\) is determined by \(q_A\) and \(q_I\)
- Probability of receiving \(r_L\) is determined by \(q_A\), \(q_I\) and \(\pi\)
Borrower’s optimal leverage

- Since Bs payoffs are discrete the probability of default is a step function of $L_B$
- The optimal leverage is

$$L_B^* \approx \frac{\mu_B + (R_A - 1)r_{B2}^e}{r_{B2}^e(1 + r_{B1}) - \mu_B - r_{B2}^e R_A}$$
Financial intermediary’s optimization

- F’s wealth is at the end of each period
  \[ \widetilde{W}_F = (1 + L_F) [z_f \tilde{r}_B + (1 - z_f) r_f] - r_L L_F \]

- The distribution of return from two loans, \( \tilde{r}_B \), is

<table>
<thead>
<tr>
<th>Return</th>
<th>Probability</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_B )</td>
<td>( \gamma^1_F )</td>
<td>( S )</td>
</tr>
<tr>
<td>( \frac{1}{2} r_B )</td>
<td>( \gamma^2_F )</td>
<td>( PD )</td>
</tr>
<tr>
<td>0</td>
<td>( 1 - \gamma^1_F - \gamma^2_F )</td>
<td>( D )</td>
</tr>
</tbody>
</table>
Financial intermediary’s optimization

Proposition

At an optimum with positive leverage $F$, invests all his resources in risky loans to $B$s and

$$L^*_F(r_B, r_L) = \frac{(\gamma^1_F + 0.5\gamma^2_F)r_B - r_L - b[\gamma^1_F r_B (r_B - r_L) + 0.5\gamma^2_F r_B (0.5r_B - r_L)]}{b[\gamma^1_F (r_B - r_L)^2 + \gamma^2_F (0.5r_B - r_L)^2 + (1 - \gamma^1_F - \gamma^2_F)r^2_L]}$$

where

$$\gamma^1_F = 1 - (1 - q_A)(1 - q^2_I)$$

$$\gamma^2_F = 2(1 - q_A)(1 - q_I)q_I$$
Financial intermediary’s optimization

Proposition

Provided the marginal utility from $W_F$ is positive at twice the value of $W_F$ in the full solvency state (which is the case when $b$ is sufficiently small):

$$\frac{\partial L^*_F}{\partial r_L} < 0, \quad \frac{\partial L^*_F}{\partial r_B} > 0.$$


Lender’s optimization

Proposition

Conditional on the probability of bailout, $\pi$, a lender’s optimal investment in a fully diversified risky portfolio of loans to Fs satisfies

$$z^*_L = \frac{E(\{\tilde{r}_L\}) - r_f}{\alpha \text{Var}(\{\tilde{r}_L\})}.$$ 

Proposition

1. Holding $r_L$ constant, a less generous bailout policy (lower $\pi$) induces a ”flight to safety” by lenders (lower $z^*_L$).

2. Provided $r_f > \frac{r_L}{2}$ and holding $\pi$ constant an increase in $r_L$ raise the appetite for risky loans to financial intermediaries (raises $z^*_L$).
General equilibrium in period 1

- General equilibrium of the financial system is characterized by clearing in two credit markets: the market for loans by Fs to Bs and the market for loans by Ls to Fs
- Given the aggregate demand for loans by borrowers and realized rates of return on real investments the market clearing conditions determine \( r_B \) and \( r_L \)
- In a state of aggregate expansion (E) borrowers’ rates of return are \( R_A + R_I \) or \( R_A \)
- In a state of aggregate contraction (C) borrowers’ rates of return are \( R_I \) or 0
General equilibrium in period 1

- Although equilibrium on financial markets varies depending on whether the economy is in state E or in state C in period 1 the market clearing conditions are qualitatively similar.
General equilibrium in period 1

Expansion

\[
(1 - q_I) M_B \left\{ (1 + L^*_B)(1 - R_A) + r_{B1}L^*_B \right\} = M_F \left\{ r_{B1} + L^*_F(r_{B2}, r_{L2}) \right\}
\]

\[
M_F L^*_F(r_{B2}, r_{L2}) = r_{L1} M_L \frac{E(\{\tilde{r}_{L2}\} | \pi_1) - r_f}{\alpha \text{Var}(\{\tilde{r}_{L2}\} | \pi_1)}.
\]
The left hand side (LHS) of the first equation is total demand for loans by non rationned Bs and the RHS is the supply of such loans by Fs that survive into period 1.

The LHS of the second equation is total demand for loans by Fs and the RHS is the supply of such loans by Ls.
General equilibrium in period 1

**Contraction**

\[ q_I M_B \{(1 + L_B^*)(1 - R_I) + r_{B1} L_B^* \} = q_I^2 M_F \{ r_{B1} + L_F(r_{B2}, r_{L2}) \} \]

\[
\left( \gamma_{LC}^1 + \frac{\gamma_{LC}^2}{2} \right) r_{L1} M_L \frac{E(\{\tilde{r}_{L2}\} | \pi_1) - r_{f2}}{\alpha Var(\{\tilde{r}_{L2}\} | \pi_1)},
\]

where \( \gamma_{LC}^1 \) and \( \gamma_{LC}^2 \) are functions of \( q_I \) and of the proportion of solvent Fs in period 1.

**Proposition**

*A decrease in the probability of bailout reduces \( \gamma_{LC}^1 + \frac{\gamma_{LC}^2}{2} \).*
General equilibrium in period 0

- There are 6 equilibrium conditions: Period’s 0 equilibrium conditions (2 equations), period’s 1 forecasted equilibrium condition for the case of expansion (2 equations) and period’s 1 forecasted equilibrium condition for the case of contraction (2 equations).
- There is one model consistent equation for expectations about $r_{B2}$
- Those 7 conditions determine the following 7 endogenous variables: $r_{B1}, r_{L1}, r_{B2}^E, r_{L2}^E, r_{B2}^C, r_{L2}^C, r_{B2}^e$
General equilibrium in period 0

\[ M_B L_B^* = M_B \frac{\mu_B + (R_A - 1)r_{B2}^e}{r_{B2}^e(1 + r_{B1}) - \mu_B - r_{B2}^e R_A} \]

\[ = M_F \{1 + L_F^*(r_{B1}, r_{L1})\} \]

\[ M_F L_F^*(r_{B1}, r_{L1}) = M_L \frac{E(\tilde{r}_{L1} \mid \pi_0) - r_{f1}}{\alpha \text{Var}(\{\tilde{r}_{L1}\} \mid \pi_0)}, \]
Period’s 0 forecasts of period 1 equilibrium conditions

In case of aggregate expansion ($\tilde{R}_A = R_A$)

$$
(1 - q_I)M_B \{(1 + L^*B)(1 - R_A) + r_B1L^*_B\} = M_F \{r_B1 + L^*_F(r^E_B2, r^E_L2)\}
$$

$$
M_FL^*_F(r^E_B2, r^E_L2) = r_L1M_L \frac{E(\{\tilde{r}_L2\} | \pi_0) - r^e_{f2}}{\alpha Var (\{\tilde{r}_L2\} | \pi_0)},
$$
Period’s 0 forecasts of period 1 equilibrium conditions

In case of aggregate contraction \((\tilde{R}_A = 0)\)

\[
q_I M_B \left\{ (1 + L_B^*)(1 - R_I) + r_{B1} L_B^* \right\} = q_I^2 M_F \left\{ r_{B1} + L_F (r_{B2}^C, r_{L2}^C) \right\}
\]

\[
\left( \gamma_L^{1C} + \frac{\gamma_L^{2C}}{2} \right) r_{L1} M_L \frac{E (\{\tilde{r}_{L2}\} | \pi_0) - r_{f2}^e}{\alpha Var (\{\tilde{r}_{L2}\} | \pi_0)} = q_I^2 M_F L_F (r_{B2}^C, r_{L2}^C)
\]
Period’s 0 Expectation about $r_{B2}$

$$r_{B2}^e = q_A r_{B2}^E + (1 - q_A) r_{B2}^C$$
Financial flows

\[ Z_L \quad r_L \quad Z_F \]

\[ L \quad F \quad B \]

\[ r_f(1) \quad (1-Z_L) \quad (1-Z_F) \]

\[ R_A(q_A) \quad R_I(q_I) \]

\[ Econ \quad Idio \]

\[ \text{Government} \]

\[ \text{Borrower} \quad \text{Financial intermediary} \quad \text{Lender} \]

\[ \text{General equilibrium} \]

\[ \text{Comparative statics} \]

\[ \text{Conclusions} \]

\[ \text{Appendix} \]
Proposition

An increase in pessimism about governmental bailout policy leads to

1. An increase in the cost of funds, $r_{L2}$, to Fs above what it had been expected to be as of period 0
2. An increase in the cost of funds, $r_{B2}$, to borrowers above $r_{B2}^e$ reducing the profits expected for period 2 and raising the probability of borrowers’ default in that period
3. When the decrease in $\pi$ is sufficiently large the consequent increase in $r_{B2}$, may induce a total drying up of credit to borrowers whose period’s 1 return is $R_A$ and to consequent defaults by those borrowers already in period 1
Downward revision in beliefs about bailout policy (period 1)

The proposition is consistent with the view that much of the financial market panic in the aftermath of Lehman’s collapse was due to a downward revision in perceptions about the likelihood that the US government will step in and use public funds to reduce creditors’ losses following defaults by financial intermediaries.
The exante (period 0) impact of a higher perceived bailout probability and moral hazard

Proposition

Provided the direct impact of $\pi_0$ on $z_{L1}^*$ is sufficiently large in comparison to the absolute value of the (negative) impact of $\pi_0$ on $r_{B2}^e$, and provided $b$ is sufficiently low and $r_{f1} > \frac{1}{2} r_{L1}$, a higher $\pi_0$ is associated with

- Lower levels of $r_{B1}$ and of $r_{L2}$,
- Overall larger levels of credit by lenders to financial intermediaries and by intermediaries to borrowers (larger values of $L_B^*$ and of $L_F^*$).
The ex ante (period 0) impact of temporary expansionary monetary policy

Proposition

Provided \( b \) is sufficiently low and \( r_{f1} > \frac{1}{2} r_{L1} \) a temporary decrease in \( r_f \) leads to a decrease in both \( r_{B1} \) and \( r_{L1} \) and to an increase in leverage within both the financial and the real sectors (both \( L^*_F \) and \( L^*_B \) go up).
The impact of higher bailout uncertainty on the amplitude of booms and busts

Bailout uncertainty within the context of the model is characterized by the extent to which perceptions change between periods, or more formally by $\pi_0 - \pi_1$ where $\pi_1$ is the perceived likelihood of bailout in period 1.

- The higher $\pi_0$ the higher are the levels of leverage assumed by Fs and Bs, and the volume of real investments
- The lower $\pi_1$ the higher are interest rates $r_{B2}$ and $r_{L2}$ and the higher the likelihood of default by Bs and Fs in period 2
The impact of higher bailout uncertainty on the amplitude of booms and busts

- When the decrease in $\pi_1$ is sufficiently large refinancing credit offered to borrowers completely dries out leading to default by both Bs and Fs already in period 1
- The larger bailout uncertainty as characterized by the difference $\pi_0 - \pi_1$, the larger are both the expansion of credit in period 0 and its contraction in period 1 implying that bailout uncertainty raises the amplitude of booms and busts
Conclusions

- A sufficiently large decrease in the perceived likelihood of bailout (lower $\pi$) leads to deleveraging in the shadow banking system and to an increase in the volume of bankruptcies. This is consistent with the dramatic decrease in SIV’s, conduits and hedge funds following Lehman’s collapse.
- Independently of its magnitude a decrease in the perceived likelihood of bailout induces a general increase in interest rates.
- Higher ex ante optimistic beliefs about the likelihood of bailout raise leverage in both the financial and the real sectors, lower interest rate and creates moral hazard.
Conclusions

- The higher bailout uncertainty the larger is the amplitude of booms and busts
- Expansionary monetary policy (lower $r_f$) raises leverage, lowers rates and moderates bankruptcies in the short run
- But in the longer run expansionary monetary policy, by encouraging the expansion of short term debt, raises the likelihood of a crisis
Future research

- Use model to investigate how does the existence of financial institutions between ultimate borrowers and lenders affect leverage

- An interesting extension of the model will involve allowing financial intermediaries to obtain funds by means of long term (2 periods) as well as short term (one period) loans
Borrower’s financial requirements

- Borrower’s financial requirements in period 1 are either zero (when realized return is $R_A + R_I$) or, otherwise,

$$FR = (1 + L_B)(1 - \tilde{R}_{B1}) + r_{B1}L_B$$

- If he survives to period 2 B’s terminal wealth is

$$W_B = (1 + L_B)\tilde{R}_{B2} - FRr_{B2}$$
Borrower’s solvency conditions

A borrower is solvent in period 1 iff he obtains the refinancing required to maintain the project till period 2:

\[(1 + L_B)\mu_B \geq FRr_{B2}\]

or equivalently

\[L_B \leq \frac{\mu_B + (\tilde{R}_{B1} - 1)r_{B2}}{-K(\tilde{R}_{B1}, \mu_B)} \equiv L_B^c(\tilde{R}_{B1}),\]

where

\[K(\tilde{R}_{B1}, \mu_B) \equiv \mu_B + r_{B2}\tilde{R}_{B1} - r_{B2}(1 + r_{B1}).\]
Borrower’s solvency conditions

A borrower is solvent in period 2 iff his cash flow is larger than the required debt service

$$(1 + L_B)\tilde{R}_{B2} \geq FRr_{B2}$$

or equivalently

$$W_B = A(\tilde{R}_{B1}, \tilde{R}_{B2}) + K(\tilde{R}_{B1}, \tilde{R}_{B2})L_B \geq 0,$$

where

$$A(\tilde{R}_{B1}, \tilde{R}_{B2}) \equiv \tilde{R}_{B2} + r_{B2}\tilde{R}_{B1}.$$
Borrower’s optimization

An optimal positive level of B’s leverage implies

\[ K(\mu_B, \mu_B) > 0. \]

Lemma

*Period’s 0 B’s optimal level of leverage must be one of the following*

\[
L_{B3} = L^c_B(R_A) - \varepsilon, \quad \varepsilon > 0 \text{ and infinitesimally small}
\]

\[
L_{B4} = L^c_B(0) - \varepsilon
\]

Lemma

*Under some additional conditions on K and W_B for some combinations of \( \tilde{R}_{B1} \) and of \( \tilde{R}_{B2} \) the expected utility of a representative borrower is larger when leverage is at \( L_{B3} \) than when leverage is at \( L_{B4} \), implying that \( L_{B3} \) is B’s optimal leverage. This is more likely to be the case, the larger \( R_A \) and, provided \( q_i > \frac{1}{2} \), the larger is \( q_i \).*
Financial intermediary’s solvency conditions

- F is solvent iff

\[ W_F(L_F) = (1 + L_F)\tilde{r}_B - r_L L_F = \tilde{r}_B + (\tilde{r}_B - r_L) L_F \geq 0 \]

- Since \( r_B > r_L \), F is solvent for any \( L_F \) when \( \tilde{r}_B = r_B \)
- In the other two cases F is solvent only if \( L_F \) is sufficiently small:

\[ L_F \leq \frac{0.5r_B}{r_L - 0.5r_B} \quad \text{when} \quad \tilde{r}_B = 0.5r_B \]

\[ L_F = 0 \quad \text{when} \quad \tilde{r}_B = 0 \]

- We focus on an equilibrium in which Fs’ risk aversion as characterized by \( b \) is such that

\[ L_F^* > \frac{0.5r_B}{r_L - 0.5r_B} \]