Soft Shareholder Activism

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ABSTRACT

This paper studies voice and exit as mechanisms of shareholder activism when obtaining costly formal control is not feasible. Different from the existing literature, voice is interpreted as a strategic transmission of information from an activist investor to an opportunistic manager. The analysis provides four main results. First, voice and exit exhibit complementarity. For this reason, exit can be an effective form of governance even when managers do not care about the short-term stock price. Second, transparency reduces the credibility of the activist’s voice and thereby harms shareholders’ welfare. Third, when the activist cannot voice herself, the possibility of exit and the introduction of managerial myopia may harm shareholders. By contrast, in the presence of voice, exit and managerial myopia unambiguously benefit shareholders. Last, voice is an effective channel of shareholder activism even when the activist is biased, and sometimes, even more influential than when the activist is unbiased.

Keywords: Shareholder Activism, Voice, Exit, Communication, Cheap-Talk, Transparency, Corporate Governance.

JEL Classification Numbers: D74, D82, D83, G34

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Introduction

Shareholder activism is an important channel of corporate governance. It is the act of disciplining an otherwise opportunistic management. Discipline can be achieved in several ways. Most notably, investors can seek formal control of the company by accumulating a significant number of voting shares or by winning board seats in contested director elections. With sufficient control, the activist can command the incumbent management to follow a particular strategy or simply replace it. Obtaining formal control, however, can be very costly. To the extent that activist investors are not fully reimbursed for these and other expenses, these tactics could be under-used from a social point of view. With the frustration of not being able to obtain control, the activist may lose trust in the company and exit by selling her holdings in the firm.

Formal control, however, is not necessary for an effective implementation of shareholder activism. Investors may send letters and call the management, suggesting ways to unlock what the activist sees as a hidden value. This can include the common activist goals of spinning off a division of the company or a share buyback, but also changes in corporate strategy and operational matters with consequences for the company’s long-term growth. To the extent that investors have useful insights, the company’s management may listen and follow their advice. Anecdotally, this informal communication, whether it is taking place behind the scenes or it appears on the public media, becomes increasingly common. “Companies have been under so much pressure. Now you can walk in the door and just have a conversation about what to do to make the company better,” says Harlan Zimmerman of Cevian, a large European activist hedge fund (Economist 12/02/2010). More systematically, McCahery, Sautner, and Starks (2011) conduct a survey of institutional investors’ views on corporate governance. They find that 55% of the investors indicate that they would be willing to engage in discussions with the firm’s executives, and conclude that behind-the-scenes shareholder activism may be more prevalent than previously thought.

This paper explores the various ways through which shareholders can exercise activism when

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1 These costs consist of legal fees and the fees of hiring proxy advisors and solicitors, corporate governance experts, investment banks, public relations and advertising firms. Moreover, gaining board seats makes the activist a corporate insider. This restricts her ability to minimize losses by selling shares.

2 “Ready, set, dough - Activist investors are limbering up to make trouble once more” Economist 12/02/2010.

3 These responses are consistent with recent evidence surrounding specific firms. As a recent example, following press reports about Yahoo Inc.’s dealings with potential buyers, the WSJ reported (“At Yahoo, Top Investor Frets”, 11/10/2011) that Capital Research and Management, P. Schoenfeld Asset Management LP, and Third Point LLC, all large shareholders of Yahoo, contacted the company’s board to express concerns over a possible transaction. Another example in the late 1980’s is when CalPERS was dissatisfied with the governance of Texaco, they negotiated with Texaco’s management to appoint a pro-shareholder candidate to its board of directors (Gillan and Starks (2007)). Last, Becht, Franks, Mayer, and Rossi (2009) provide evidence on behind the scenes communication as a form of shareholder activism of the Hermes UK Focus Fund.
obtaining formal control is not feasible or too costly. It focuses on two primary mechanisms: 
voice and exit. Voice is the attempt of investors to persuade their opportunistic management 
to follow certain strategies by communicating private information. The effectiveness of voice 
crucially depends on the credibility of the message and its publicity. Thus, different from the 
existing literature, in this paper voice is interpreted as literally the act of transmitting infor-
mation. Exit, on the other hand, is the activist’s decision to sell her stake in the firm when 
she feels disadvantaged by keeping it for the long run. By exiting, the activist signals dissat-
sisfaction which could depress the share price and pressure managers to be more accountable 
to shareholders. The implementation of voice and exit does not require formal control and 
ettains very little or no direct costs on the activist. I therefore refer to this type of activism as 
“soft”. In this respect, the objective of this paper is to study the conditions under which soft 
shareholder activism is an effective form of corporate governance.

In order to study this topic I develop a model of shareholder activism. In the model, 
the manager of a public firm has the formal authority over its long run investment policy. 
The manager is not perfectly informed about the investment opportunities, and due to the 
separation of ownership and control, his policy does not necessarily maximize the value of 
shareholders. This inefficiency creates room for shareholder activism. Specifically, among 
shareholders there is an unbiased activist investor with private information that complements 
the manager’s knowledge.4 The activist can voice her opinion by sending a private message 
to the manager, thereby advising him about the optimal course of investment. Information is 
non-verifiable and the activist does not incur any direct cost or benefit from communication. 
Formally, voice is modeled as “Cheap Talk” a la Crawford and Sobel (1982). In addition to 
voice, the activist can to exit by selling her entire stake in the firm. The activist exits in order 
to satisfy her liquidity needs or because of her assessment that the share is over-valued. Trade 
takes place before the long-term value of the firm is realized. While the activist’s motives cannot 
be distinguished by the market, prices are set fairly given the public information, including the 
activist’s decision to exit.

The first result of the paper characterizes the interaction between voice and exit and shows 
that these mechanisms exhibit complementarity. That is, the option to exit enhances the 
positive effect of voice on shareholders’ value. Importantly, when management is highly oppor-

4 The activist is unbiased in the sense that conditional on keeping her stake in the firm, the activist and other 
shareholders share the same objective. However, the activist may trade against other shareholders of the firm, 
and in this sense, may be conflicted with them.

5 There is a broad literature on how corporate insiders may learn value-relevant information from outsiders. 
Among many, Holmstrom and Tirole (1993) argue that stock prices provide information about the manager’s 
actions and are therefore useful for managerial incentive contracts. Levit and Maleño (2011) analyze nonbind-
ing voting for shareholder proposals and show that the information that is conveyed by voting outcome can 
affect corporate decision makers. Marquez and Yilmaz (2008) examine tender offers where shareholders have 
information about the firm value that the raider does not have. In Dow and Gorton (1997), Foucault and Gehrig 
(2008), and Goldstein and Guembel (2008), firms use information in stock prices to make investment decisions.
tunistic, voice is an idle mechanism of governance unless exit is possible. At the same time and
different from the existing literature, with voice, exit becomes a powerful form of shareholder
activism even if the manager has no direct utility from a higher short-term stock price. Instead,
the channel through which exit exercises discipline is by improving the ability of the activist to
credibility communicate with the opportunistic manager.

To understand this result, note that inevitably the activist will manipulate some of her
information in order to overcome the inherent conflict of interests between shareholders and
the manager. Worrying about its credibility, the opportunistic manager will often ignore the
activist’s advice. This dynamic limits the amount of information that can be revealed by the
activist in any equilibrium and ultimately harms shareholders. The option to exit, however,
enables the activist to dispose her holdings in the firm at times she believes, based on her
private information and communication with the manager, that the share is over-priced. As a
result, with the option to exit, the activist is less sensitive to the long-run performances of the
firm and is more willing to compromise with inefficient managerial decisions. This increases
the credibility of the activist’s voice in the eyes of the manager and allows for an informed
deliberation. With more information the manager can make better decisions and improve the
value of the firm. Overall, voice is more effective with exit than without it.

The analysis of voice and exit reveals several interesting observations. First, when multiple
equilibria exist, the most informative equilibrium is not necessarily the equilibrium that maxi-
mizes the value of the firm. Second, while the manager’s bias reduces the expected long-term
value of the firm, it can also reduce the likelihood the activist exits in equilibrium. Third,
even though more frequent liquidity shocks relax the adverse selection problem in the activist’s
decision to exit, it may actually reduce the overall likelihood the activist exits in equilibrium.

In practice, the activist does not have to voice herself secretly. Instead, the activist can
make the letters to management public, ensuring that other market participants are aware of
her demand. The second set of results relates to the role of transparency in shareholder ac-
tivism. Apart from the transparency of voice, another relevant dimension of transparency is the
observability of corporate decisions (for example, the long run investment strategy of the firm).
I show that these two kinds of transparency generate exactly the same set of equilibria and
hence are equivalent. Moreover, all else equal, voice is less effective as a mechanism of share-
holder activism with transparency than without it. Interestingly, under some circumstances,
transparency is so harmful that voice is less effective with exit than without it. In other words,
with transparency, voice and exit may exhibit substitution. Nevertheless, when transparency
of voice is not mandatory and the activist is allowed to choose between public and private (or
both) forms of communication, the (adverse) effect of transparency disappears.

Intuitively, as any other seller, the activist would like to get the highest price possible for
her shares when she decides to exit and sell her holdings. It turns out that with transparency
the activist cannot resist the temptation to send messages that inflate the short term price
of the company’s shares. Knowing that information will be manipulated, the activist loses credibility and her voice becomes less effective even when she does not exit. In this sense, lack of transparency is a commitment not to manipulate prices and hence it enhances welfare.

For various reasons, managers of public companies are sensitive to the short-term performances of their company’s stock price. For example, short-term compensation or the desire to demonstrate talent can introduce this sort of managerial myopia. I extend the model and study soft shareholder activism when the manager has direct utility from the short-term stock price.\(^6\) I show that when the activist cannot voice herself, the effects of exit and managerial myopia on shareholders’ welfare are ambiguous and can be negative. By contrast, when the activist can communicate with the manager, managerial myopia and exit unambiguously benefit shareholders. Intuitively, since exit depresses the short term price of the stock, the myopic manager tries to minimize the likelihood the activist exits. With voice, this objective translates to stronger incentives to follow the activist’s advice. The enhanced ability of the activist to influence the manager’s decisions increases the value of shareholders.

Finally, activist investors may not share the same objective as other shareholders of the firm. For example, hedge funds are often blamed from being opportunistic and pursuing short-term goals which are inconsistent with the long-term value of the firm. Mutual funds often have business ties with their portfolio companies. They administrate their corporate pension plans and therefore could be biased toward the management’s agenda.\(^7\) I study the effect of opportunistic activism on voice and exit, and show two results. First, since a biased activist may exit for reasons which are not necessarily related to low valuation of the firm, regardless of the direction or magnitude of the this bias, the adverse selection problem upon exit is less significant when the activist is biased than when she is unbiased. Second, when the manager’s bias is significant, voice is an effective mechanism of shareholder activism if and only if the activist is biased as well. The smallest bias of the activist under which voice remains effective is always at the same direction as the manager’s bias, but strictly smaller. In this sense, a biased activist functions as an intermediary of information between shareholders and the manager.

The paper proceeds as follows. The remainder of this section discusses the relationship to the existing literature. Section I presents the baseline model. Section II studies voice and exit in the context of soft shareholder activism. Section III analyzes the effect of transparency on shareholder activism. Section IV introduces managerial myopia. Section V extends the basic model to study opportunistic activism. Section V concludes. All omitted proofs are collected in the Appendix.

\(^6\)I also require that the activist observes the manager’s decision to approve the project. Without this additional assumption, it is shown that the set of equilibria is invariant to managerial myopia.

\(^7\)Anabtawi and Stout (2008) provide a comprehensive discussion of common conflicts of interest between activist investors and other shareholders. Examples include short-termist goals of certain types of activists, self dealing involving other companies in activists’ portfolios, and the expansion of labor rights desired by union fund managers.
Relation to the Literature

Traditional models of shareholder activism focus on the benefits of corrective actions through direct intervention (for example, Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Maug (1998), Kahn and Winton (1998), Bolton and von Thadden (1998), and Faure-Grimaud and Gromb (2004)). These studies share the idea that large shareholders are able to exercise *formal control* and thereby either force the company’s management to improve the value of the firm or do it themselves. By contrast, the present study emphasizes that even without formal control, investors can exercise *real control* by communicating soft information and thereby persuading the management of the company to make better decisions.

In this respect, closely related are studies by Levit and Malenko (2011) and Cohn and Rajan (2011). Levit and Malenko (2011) investigate non-binding voting for shareholder proposals as a mechanism through which shareholders of public companies can voice their opinions about governance and strategic related issues. They show that because of strategic voting, this mechanism often fails to aggregate and convey shareholder views when the interests of the manager and shareholders are not aligned. Instead, the presence of a biased activist investor who can launch a proxy fight to replace the incumbent management may enhance the advisory role of non-binding voting, but only if the activist herself is biased. Similar to Levit and Malenko (2011), the present paper shows that shareholders can persuade their management to take a value enhancing decision by communicating information. Different from their work, however, the present paper focuses on the interaction of voice and investors’ trading decision, and shows that communication can be an effective form of shareholder activism even when there is no binding threat of discipline in the background. Cohn and Rajan (2011) study a model in which a board arbitrates between an activist investor and a manager, and focus on the interaction between internal and external governance. Similar to the present work, in their model the blockholder makes a recommendation for a strategic change in the company that she cannot implement herself. However, different from the present study, issues with credible communication of information and trading are assumed away in their model.

The current study is also related to Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011), who point out that activism through “exit”, that is, by selling their shares, can be an effective form of governance in itself. Key for their result is that the manager has direct utility from the short term stock price, the channel through which exit matters. By contrast, the present study shows that even when managers are not myopic in that sense, exit is an important mechanism of governance since it enhances the ability of the activist to credibly communicate with the manager. Among these three papers, only Edmans and Manso (2011)---

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8In a related paper, Dasgupta and Piacentino (2011) focus on the delegated nature of blockholding and argue that when the activist fund manager has career concerns, exit fails to impose discipline on managers.
consider exit and voice simultaneously. However, similar to the traditional literature of voice, they assume that each blockholder has formal control and hence can directly influence the value of the firm, whether or not the manager agrees with this decision. Admati and Pfleiderer (2009) also demonstrate that transparency of corporate decisions does not necessarily improve welfare. However, while in their paper the welfare loss, if realized, is the consequence of a distorted exit strategy by the blockholder, in the present paper transparency provides the activist with incentives to inflate the short term share price and thereby reduces the credibility of her voice. So, the channel through which transparency is harmful is fundamentally different in this paper.

This paper also contributes to the literature on governance and liquidity. Bhide (1993) argues that because blockholders add value through voice, and voice and exit are mutually exclusive, liquidity is harmful as it allows a shareholder to leave rather than intervene. The present paper argues the opposite. It shows that liquidity complements voice. In this respect, Maug (1998) and Kahn and Winton (1998) demonstrate that liquidity facilitates block formation in the first place, as activist shareholders can buy additional shares at a price that does not incorporate the gains from intervention. Different from the present study, conditional on the size of the activist’s holdings, liquidity discourages intervention in those models. Faure-Grimaud and Gromb (2004) show that liquidity encourages intervention as it increases stock price informativeness, and if the activist is forced to sell prematurely due to a liquidity shock, the price she receives will partially reflect the gains from intervention. Thus, in their paper liquidity directly increases the expected gains from intervention. By contrast, in the present paper, liquidity alleviates the adverse selection problem when the activist trades, and thereby reduces the sensitivity of the activist to inefficient decisions made by the manager. This effectively reduces the disagreement between the activist and the manager. Thus, in the present paper, liquidity enhances the credibility of voice and thereby improves the value of the firm.

I Baseline Model

Consider a public firm whose long term value to shareholders depends on its business strategy as well as on the fundamentals. Denote the business strategy by \( a \in \{A, R\} \). I refer to decision \( a \) as an investment in a project and say that the project is approved when \( a = A \) and is rejected otherwise. The value of the firm to shareholders is given by

\[
v(\theta, a) = \theta \cdot (1_{\{a=A\}} - 1_{\{a=R\}})
\]

where \( \theta \in [\underline{\theta}, \bar{\theta}] \) is a random variable whose probability density function \( f \) is continuous and has full support. I assume that \( \underline{\theta} \in [-\infty, 0) \) and \( \bar{\theta} \in (0, \infty] \). Thus, from shareholders’ point of view, it is optimal to approve the project if and only if \( \theta \geq 0 \).
Shareholders own the cash flow rights of the firm, but the manager has the formal authority over the firm’s investment policy. The manager and shareholders have conflicting preferences with respect to the investment policy. Specifically, the manager’s preferences are represented by,

\[ u_M = v(\theta + \beta, a) \]  

(2)

where \( \beta \in (0, -\theta) \) is the non-pecuniary private benefit the manager obtains from investment in the project. It follows from (2) that the manager approves the project if and only if \( \theta \geq -\beta \). If the manager knows \( \theta \), he would inefficiently invest in the project when \( \theta \in [-\beta, 0] \). Thus, the larger is \( \beta \), the greater is the conflict of interests between shareholders and their manager.

The ownership structure of the firm consists of dispersed shareholders and an activist investor. Dispersed shareholders have no ability or incentives to discipline the incumbent management and hence remain passive. The focus of the analysis is on the ability of the activist investor to influence the manager’s decision over the project. The activist, however, does not have and cannot obtain formal control. Thus, the manager cannot be forced to take a particular decision. Presumably, the cost of initiating and executing a proxy contest or a hostile takeover is too high. Instead, I study the ability of the activist to communicate her own view to the manager and thereby persuade him toward one action or the other.

To emphasize this channel of communication, let us assume that at the outset the activist obtains private and perfect information about \( \theta \). The key assumption of the model is that the activist’s information is incremental to the manager’s information. Specifically, I assume that all other market participants, including the manager, are uninformed about \( \theta \). Before the manager makes his decision about the project, the activist sends him a private message \( m \in [\underline{\theta}, \overline{\theta}] \) about \( \theta \). Formally, the communication between the activist and the manager is modeled as “Cheap Talk”. The activist’s private information is non-verifiable and her recommendation \( m \) cannot be backed-up with hard information. Moreover, the content of \( m \) does not affect the activist’s payoff directly but only through its effect on the manager’s decision. Thus, there are no private benefits or costs from communication per-se. Denote by \( \mu(m|\theta) \in [0, 1] \) the probability that the activist sends a message \( m \) conditional on her private information about \( \theta \), and by \( a(m) \in \{A, R\} \) be the manager’s decision whether to accept the project conditional on observing message \( m \).\(^9\)

After voicing her opinion and communicating her message to the manager, the activist can trade with a competitive risk neutral market maker. Unless the activist is hit by a liquidity shock, she is free to choose whether to exit or keep her holdings in the firm. With probability \( \delta \in [0, 1] \), however, the activist is forced to sell her entire stake in firm in order to accommodate her liquidity needs. Denote the activist’s decision to sell her entire stake in firm by \( \sigma = 1 \), and

\(^9\)If the manager is indifferent it is assumed he approves the project with probability one.
her decision to keep it by $\sigma = 0$.\footnote{The activist cannot buy shares or commit to an exit strategy.} The analysis will demonstrate that the activist \textit{strategically exits} when based on her private information the firm is over-valued. The decision of the activist to exit is observed by the market maker. However, the market maker does not observe the message the activist sent the manager or the needs of the activist for liquidity. These are the private information of the activist. Moreover, at the time of trade, neither the activist nor the market maker know for sure what is the manager’s final decision.\footnote{Section III relaxes the assumptions that the manager’s decision is unobservable and the communication between the activist and the manager is private.} Based on all the available public information, the market maker sets the short term price of the firm’s share to be the expected value of the company. I denote this price by $p(\sigma)$. Overall, the activist’s preferences are given by,\footnote{To save on notation, whenever the activist is subject to a liquidity shock $\sigma = 1$.}

$$ u_A = \sigma p(\sigma) + (1 - \sigma) v(\theta, a) \quad (3) $$

To summarize, there are four periods in the model (see Figure 1 below). Initially, before the activist observes her liquidity needs but after she becomes informed about $\theta$, the activist communicates with the manager, trying to persuade him to follow her advice by sending a private message $m$. At period 1 the manager decides whether to approve the project, taking into account the activist’s message. The manager’s decision is unobservable by other market participants. At period 2 the activist observes her liquidity needs and based on her private information about $\theta$ and communication with the manager, she decides whether to sell her holdings in the firm. The market maker observes the order flow and determines the stock price accordingly. Finally, at period 3 the outcome of the project is realized and becomes public. All agents are risk neutral and preferences are common knowledge.

![Figure 1 - Timeline](image)

**Solution Concept**

A Perfect Bayesian Equilibrium in our analysis consists of four parts: the activist’s communication strategy $\mu(m|\theta) \in [0, 1]$, the manager’s decision on the project $a(m) \in \{A, R\}$, the
activist’s trading decision \( \sigma (\theta, m) \in \{0, 1\} \), and the market maker’s pricing decision \( p (\sigma) \in \mathbb{R} \). In any equilibrium the following must hold:

1. For any \( \sigma \), the market maker sets the price \( p \) equals to the expected value of the firm taking the activist’s communication strategy \( \mu (\cdot | \theta) \) and the manager’s project approval strategy \( a (\cdot) \) as given.

2. For any \( \theta \) and \( m \) the activist’s trading decision \( \sigma (\theta, m) \) maximizes the value of her holdings in the firm taking the manager’s approval strategy \( a (m) \) and the market maker’s pricing policy \( p (\cdot) \) as given.

3. For any message \( m \) the approval decision \( a (m) \) maximizes the manager’s expected utility, taking the activist’s communication strategy \( \mu (\cdot | \theta) \), trading strategy \( \sigma (\cdot, m) \), and the market maker’s pricing policy \( p (\cdot) \) as given.

4. For any \( \theta \), if message \( m \) is in the support of \( \mu (\cdot | \theta) \), then \( m \) maximizes the expected utility of the activist given the manager’s approval strategy \( a (\cdot) \), her own trading strategy \( \sigma (\theta, \cdot) \), and market maker’s pricing policy \( p (\cdot) \).

II Analysis

I solve the model backward and start with several observations. Suppose in equilibrium the activist strategically exits if and only if \( \theta \in \Upsilon \subseteq [\underline{\theta}, \overline{\theta}] \) and the manager accepts the project with probability one if \( \theta \in \Theta \subseteq [\underline{\theta}, \overline{\theta}] \) and otherwise he rejects it for sure. The market maker uses this information to price the shares of the company. In equilibrium, the value of the share conditional on the activist’s decision to exit satisfies,

\[
    p (\sigma, \Theta, \Upsilon) = \begin{cases} 
    E [v(\theta, 1_{\theta \in \Theta})] + \rho Pr[\theta \in \Theta] E [v(\theta, 1_{\theta \in \Theta}) | \theta \in \Upsilon] & \text{if } \sigma = 1 \\
    E [v(\theta, 1_{\theta \in \Theta}) | \theta \not\in \Upsilon] & \text{if } \sigma = 0 
    \end{cases}
\]

(4)

where \( \rho \equiv \frac{1-\delta}{\delta} \in (0, \infty) \). Note that \( \Upsilon \) and \( \Theta \) themselves may depend on the share price. The explicit formulation of the price when the activist does not exit plays no role in the analysis until managerial myopia in considered in Section IV. Thus, in what follows and whenever there is no risk of confusion, \( \sigma \) is omitted from the notation of price and \( p \) is simply the short term stock price conditional on the activist’s decision to exit. Since \( \Theta \) and \( \Upsilon \) uniquely determine the outcome of the game, I will often say that two equilibria are equivalent if they have identical sets \( \Theta \) and \( \Upsilon \).

Suppose the activist sends the manager message \( m \). Recall the manager’s decision to approve the project is unobservable. Therefore, from the manager’s point of view the probability of exit
and the short term share price are independent of his actual decision. They only depend on the (equilibrium) expectations of which action the manager eventually undertakes. Therefore, it follows from expression (2) that the manager approves the project if and only if

$$E_\mu [\theta|m] + \beta \geq 0$$  

where $E_\mu$ is the manager’s expectation of $\theta$ conditional on message $m$ and the activist’s communication strategy $\mu$.

**Lemma 1** In any equilibrium the manager accepts the project if and only if $E_\mu [\theta|m] + \beta \geq 0$.

Lemma 1 implies that as long as action $a$ is unobservable the short term stock price does not have a direct effect on the manager’s incentives to approve the project. This observation holds even if the manager has direct utility from the short term stock price, for example, through short term compensation. Nevertheless, I will show that the short term stock price may have an indirect effect on the manager’s incentives to approve the project through the channel of communication. This feature is one aspect by which this model departs from the existing literature.

In what follows we focus attention on equilibria in which a meaningful (but possibly noisy) communication between the activist and the manager is feasible. In these equilibria the manager incorporates the activist’s advice into his decision making and hence voice is a meaningful mechanism of shareholder activism. Formally,

**Definition 1** An equilibrium is “responsive” if and only if there exist $m_A \neq m_R$ and $\theta_A \neq \theta_R$ such that $\mu(m_A|\theta_A) > 0$, $\mu(m_R|\theta_R) > 0$, and $a(m_A) \neq a(m_R)$.

Note that an equivalent requirement to Definition 1 is the existence of messages $m_A \neq m_R$ and communication strategy $\mu$ such that

$$E_\mu [\theta|m_R] < -\beta \leq E_\mu [\theta|m_A]$$  

Thus, an equilibrium is responsive if and only if $\Theta \neq [\overline{\theta}, \overline{\theta}]$. In words, voice is an effective form of shareholder activism if and only if there are at least two different messages the activist sends the manager such that for one message, denoted by $m_A$, the manager responds by approving the project and for the other message, denoted by $m_R$, the manager responds by rejecting the project. In principle, an equilibrium can be informative yet non-responsive. This would be the case if despite the revelation of information the manager does not condition his decision on the activist’s message. Since the communication between the manager and the activist is private,
any information that does not affect the manager’s decision has no effect on the equilibrium outcome as well.

Denote by $V(\Theta, \Upsilon) \equiv \mathbb{E}[v(\theta, 1_{\theta \in \Theta})]$ the ex-ante long term value of shareholders.\(^{13}\) For example, in the first best equilibrium the manager accepts the project if and only if $\theta \geq 0$ ($\Theta^{FB} = [0, \bar{\theta}]$). Thus, $V(\Theta^{FB}, \Upsilon) \equiv \mathbb{E}[\theta]$. An equilibrium is more efficient if it generates a higher ex-ante value of the firm. When multiple of equilibria exist, we let $\overline{V}$ and $\overline{V}$ be shareholders’s ex-ante value under the most and least efficient equilibrium, respectively. Throughout the analysis the focus will be on the most efficient equilibrium.

**Definition 2**: Voice and exit are “complement” (“substitute”) mechanisms of governance if and only if $\overline{V}_{\text{Voice, Exit}} - \overline{V}_{\text{NoVoice, Exit}} > (<) \overline{V}_{\text{Voice, NoExit}} - \overline{V}_{\text{NoVoice, NoExit}}$.

Definition 2 will be helpful when I study the interaction between voice and exit. If the efficiency gains from the activist’s option of exit are higher when voice is allowed than when it is restricted, then voice and exit are complement mechanisms of governance. Otherwise, voice and exit substitute each other.

Finally, I restrict attention to a subset of equilibria of the game in which $\Theta = [\tau, \bar{\theta}]$ for some $\tau \in [\hat{\theta}, \overline{\theta}]$. I name equilibria within this subset as threshold equilibria and often say that the manager follows a threshold strategy $\tau$. Note that the efficiency of a threshold equilibrium decreases with $|\tau|$. The next result shows that in our search for efficiency the focus on threshold equilibria is without the loss of generality.

**Lemma 2**: For any responsive equilibrium there is a threshold equilibrium which is more efficient.

\section{A Benchmarks}

Let us consider two benchmarks. In one benchmark case the activist can exit and sell her holdings to the market maker, but she cannot voice her opinion to the manager. The other benchmark is one in which the activist can voice herself, but she cannot exit.

**Exit Without Voice**

To better understand the role and the effect of exit on communication, I start with the benchmark case in which the activist cannot communicate with the manager. According to Lemma 1, if communication is not allowed then regardless of the activist’s ability or incentives to exit,\(^{13}\) Note that conditional on $\Theta$, the set $\Upsilon$ has no effect on shareholders’ value.
the manager approves the project if and only if
\[ \beta \geq -\mathbb{E} [\theta] \]  
(7)

where \( \mathbb{E} [\cdot] \) is the unconditional expectations with respect to \( \theta \). If condition (7) holds (does not hold) then in equilibrium the manager approves (rejects) the project with probability one and \( \Theta = [\theta, \bar{\theta}] \) (\( \Theta = \varnothing \)). In that case, the activist strategically sells her shares in the firm if and only if \( \theta \leq p \) (\( \theta \geq -p \)) and hence \( \Upsilon = [\theta, p] \) (\( \Upsilon = [-p, \bar{\theta}] \)). The market maker is competitive and hence the price is set fairly and reflects the expected value of the share conditional on the sell order. Substituting the above \( \Theta \) and \( \Upsilon \) into expression (4) implies that the price is given by the solution of \( p = \varphi (\theta, p) \) when \( \beta \geq -\mathbb{E} [\theta] \) and by the solution of \( p = \varphi (-\theta, p) \) otherwise.

For any real function \( g (\cdot) \), the function \( \varphi (g (\theta), p) \) is given by
\[ \varphi (g (\theta), p) = \frac{\mathbb{E} [g (\theta)] + \rho \Pr [g (\theta) \leq p] \mathbb{E} [g (\theta) | g (\theta) \leq p]}{1 + \rho \Pr [g (\theta) \leq p]} \]  
(8)

and the expectations are taken with respect to \( \theta \). To ease the exposition, hereafter I assume that condition (7) holds. That is, without more information, the manager will follow his bias and approve the project.\(^{14}\) The following lemma summarizes the equilibrium when communication is not feasible, which corresponds to the non-responsive equilibrium of the game. As in any “cheap talk” game, this equilibrium always exists, even when the communication is allowed.

**Lemma 3 (Non-Responsive Equilibrium)** A non-responsive equilibrium always exists. In any non-responsive equilibrium the manager accepts the project with probability one and the activist exits if and only if \( \theta \leq p_{NR} \), where \( p_{NR} \) is the share price conditional on “exit”, it is given by \( \min_p \{\varphi (\theta, p)\} \), and it decreases with \( \rho \).

Without voice, the option to exit does not change the manager’s incentives to invest in the project. This is because the activist does not observe the manager’s decision to take the project. Nevertheless, both the activist and the market maker correctly anticipate the approval of the project by the manager. Importantly, the market maker understands that for a given price \( p \), the activist exits if and only if \( v (\theta, a) = \theta < p \). Therefore, the market maker forms the “worst case” beliefs on the value of the firm conditional on the activist’s exit. This is why the price is set to be the \( \min_p \{\varphi (\theta, p)\} \). Finally, since in this benchmark the option of the activist to exit has no effect on the manager’s decision to approve the project, it is irrelevant from efficiency point of view. We conclude that \( \bar{V}_{NoVoice, Exit} = \bar{V}_{NoVoice, NoExit} = \mathbb{E} [\theta] \).

\(^{14}\) Most of the proofs also consider the case \( \beta < -\mathbb{E} [\theta] \). In this case the manager by default rejects the project and the activist’s challenge is to convince the manager to accept the project, despite the manager’s bias toward the project.
Voice Without Exit

Consider the benchmark case in which the activist cannot exit her investment in the firm. The activist incurs no cost by voicing her opinion and hence her utility is proportional to the firm’s value. Consistent with maximizing shareholders’ long term value, the activist would like the manager to accept the project if and only if 
\begin{equation}
0 \leq \theta,
\end{equation}
Let us define
\begin{equation}
z(x) \equiv \begin{cases} 
    z \colon \mathbb{E} [\theta | \theta < -z] = -x & \text{if } x > -\mathbb{E} [\theta] \\
    -\theta & \text{if } x \leq -\mathbb{E} [\theta] 
\end{cases}
\end{equation}
and note that $z(x) < x$, $z(x)$ strictly increases in $x$, and $\lim_{x \to -\theta} z(x) = -\theta$. The function $z$ will be useful in the characterization of equilibria.

**Lemma 4** Suppose the activist cannot exit. A responsive equilibrium exists if and only if $\beta \leq z^{-1}(0)$. In any responsive equilibrium the manager follows the first best decision rule.

Lemma 4 suggests that without the option to exit, voice is ineffective when the conflict of interests between the manager and shareholders is significant. In those cases, the manager will never find the activist’s advice credible, and therefore, will ignore it. However, when $\beta \leq z^{-1}(0) = -\mathbb{E} [\theta | \theta < 0]$ then despite the conflict of interests between shareholders and the manager, information is transmitted and the first best is obtained. Note that full revelation of information is not feasible in equilibrium, yet it is not necessary for efficiency to be realized. Quite the opposite, if information would be fully revealed, then the first best could not have been realized as the manager would approve the project when $\theta \in [-\beta, 0]$. Thus, in general, the value of the firm does not monotonically increase with the amount of information that is exchanged between the activist and the manager. Lemma 4 implies
\begin{equation}
\bar{V}_{Voice,\text{NoExit}} = \begin{cases} 
    \mathbb{E} [\theta] & \text{if } \beta \leq z^{-1}(0) \\
    \mathbb{E} [\theta] & \text{else} 
\end{cases}
\end{equation}

B Voice and Exit - Soft Shareholder Activism

In this subsection I analyze the strategic communication between the activist and the manager in light of the possibility of exit. The following lemma provides the first hint for the interaction between voice and exit in a responsive equilibrium.

**Lemma 5** In any responsive equilibrium $p^* > 0$ and the activist exits strategically if and only if $|\theta| \leq p^*$. 

14
The activist has incentives to exit and sell her holdings only if the share price that is set by the market maker over-values the firm. Lemma 5 suggests that in any responsive equilibrium the share price upon exit must be strictly positive. To see why, note that in any responsive equilibrium, by definition, the value of the firm depends on the activist’s voice. The activist can dictate the action that is taken by the manager by sending the appropriate message. If on the contrary the share price (upon exit) is negative, then the activist is strictly better off by keeping her holdings in the firm \((\Upsilon = \emptyset)\) and guiding the manager through the optimal decision rule \((\Theta = \Theta^{FB})\).\(^{15}\) In that case, the value of the firm is strictly positive with probability one. While the market maker does not observe the private message \(m\) or the managerial decision \(a\), in equilibrium, the market maker has rational expectations about the activist’s communication and exit strategies, as well as the manager’s approval strategy. The market maker understands that a negative price implies an efficient decision rule. Therefore, setting a negative price is inconsistent with the fair share value of the company, \(V(\Theta^{FB}, \emptyset) = \mathbb{E}[|\theta|] > 0\), yielding a contradiction. Since the share price upon exit is strictly positive, if \(|\theta| \leq p^*\) then the activist is strictly better off by selling her shares, a property which must be taken into account by the market maker when setting the price.

Since the stock price affects the incentives of the activist to exit, it may also feed back into the quality of communication between the activist and the manager. As I explain below, when the activist plans to exit she is less sensitive to the fundamentals of the firm and can communicate more freely with the manager. In equilibrium, all of these forces must be consistent with each other. Lemma 6 presents the set of communication strategies that the activist has incentives to follow, taking into account the market maker’s pricing decision.

**Lemma 6** Suppose the manager follows a threshold strategy \(\tau\). The implied share price conditional on exit is \(\pi(\tau)\), where \(\pi(\tau) \equiv \min_{p \geq -|\tau|} \{\varphi(v(\theta, 1_{\{\theta \geq \tau\}}), p)\}\). Moreover, let \(\bar{\tau} < 0\) be the unique negative solution of \(\pi(\tau) + \tau = 0\) and \(\tau > 0\) the unique positive solution of \(\pi(\tau) - \tau = 0\).\(^{16}\) Then:

(i) If \(\tau \in [\bar{\tau}, \tau]\) then \(\pi(\tau) > 0\).

(ii) If and only if \(\tau \in [\bar{\tau}, \bar{\tau}]\) then \(\tau \in [-\pi(\tau), \pi(\tau)]\).

To understand Lemma 6, suppose in equilibrium the manager approves the project if and only if \(\theta \geq \tau\) and the share price upon exit is give by \(p > 0\). As was mentioned before, the activist exits strategically if and only if \(\theta \in [-p, p]\). In equilibrium, the market maker’s expectations are consist with the manager and the activist’s behavior, and the share price is

\(^{15}\)If \(p^* = 0\) and \(\theta = 0\) the activist is indifferent and hence may exit. But since \(\theta = 0\) is a zero probability event, it will not change any of the results even if the activist chooses to exit in this case.

\(^{16}\)The Lemma is based on the assumption that
set equal to \( \pi (\tau) \). Indeed, in any equilibrium where \( \Theta = [\tau, \bar{\theta}] \) and \( \Upsilon = [-p, p] \) the price must be the solution of \( p = \varphi (v(\theta, 1_{\theta \geq \tau}), p) \). Following the same line of reasoning as in Lemma 3, the solution of this equation is the worst case scenario, \( \pi (\tau) \). Thus, if the manager follows a threshold strategy \( \tau \) then the short term price of the stock must be \( \pi (\tau) \). Part (i) of Lemma 6 proves that as long as \( \tau \in [\underline{\tau}, \bar{\tau}] \) the implied share price is strictly positive as required by Lemma 5. Part (ii) of Lemma 6 guarantees that if \( \tau \in [\underline{\tau}, \bar{\tau}] \) the activist indeed finds it optimal to follow a communication strategy that implements \( \Theta = [\tau, \bar{\theta}] \). To see why, note that when the activist strategically exits she becomes indifferent with respect to the long term value of the firm. Thus, the activist has weak incentives to follow any threshold \( \tau \in [-\pi (\tau), \pi (\tau)] \), even if \( \tau \neq 0 \). Figure 2 illustrates the determination of upper bound \( \bar{\tau} \) and lower bound \( \underline{\tau} \).

An immediate conclusion that follows from Lemma 6 is that there is no responsive equilibrium with threshold \( \tau \notin [\underline{\tau}, \bar{\tau}] \). The next proposition fully characterizes the equilibrium by imposing an additional condition that requires the manager has incentives to follow the proposed decision rule \( \tau \).

**Proposition 1** \( \text{If and only if } \tau \in [\underline{\tau}, \min \{ -z (\beta), \bar{\tau} \}] \text{ then a responsive equilibrium with threshold } \tau \text{ exists. A responsive equilibrium exists if and only if } \beta \leq z^{-1} (-\underline{\tau}). \)

The manager of the firm is biased toward the approval of the project. Therefore, the manager always follows the recommendation of the activist to approve the project. The binding incentive constraint is convincing the manager to reject the project. As long as the bias is not too high, there exists \( \tau \in [\underline{\tau}, \bar{\tau}] \) such that conditional on \( \theta < \tau \) the manager believes that rejection will be optimal, that is, \( \mathbb{E} [\theta | \theta < \tau] + \beta \leq 0 \). The higher is the bias, the lower must be threshold \( \tau \) in order to satisfy this constraint. Thus, in addition to requiring \( \tau \in [\underline{\tau}, \bar{\tau}] \), Proposition 1 reveals that a threshold strategy \( \tau \) can be sustained by a responsive equilibrium only if \( \tau \leq -z (\beta) \).
Given Lemma 2, it immediately follows from the first part of Proposition 1 that a responsive equilibrium exists if and only if \( \beta \leq z^{-1}(0) \). Since \( z^{-1}(\cdot) \) is an increasing function and \( \bar{\tau} < 0 \), the comparison between Proposition 1 and Lemma 4 reveals that voice is more effective with exit than without it. Counter to the intuition, the option of the activist to exit and sell her holdings enhances her ability to communicate with the manager and thereby affects his decision. The reason is that the activist becomes less sensitive to the performances of the firm. Indeed, if the activist believes the share is overpriced, she has the option to exit and therefore she becomes indifferent with respect to the decision of the manager to approve the project. This property facilitates the ability of the activist to credibly convey information and thereby overcome the disagreement with the manager.

Proposition 1 also implies that the most efficient threshold that can be supported by an equilibrium is \( \tau^* = \max \{-z(\beta), 0\} \). Thus, similar to Lemma 4, the first best \( FB_0 = 0 \) is obtained if and only if \( \beta \leq z^{-1}(0) \). However, when \( \beta \in (0, z^{-1}(\bar{\tau})) \) the second best threshold is \( SB_0 = -z(\beta) < 0 \). Thus, in equilibrium, the project is over-accepted by the manager. The next corollary provides welfare analysis.\(^{17}\)

**Corollary 1 (Welfare)** The least efficient equilibrium is non-responsive and the most efficient equilibrium is responsive. Moreover, \( V_{Voice, Exit} \) decreases in \( \beta \) and \( \rho \) and it satisfies,

\[
V_{Voice, Exit} = \begin{cases} 
E\left[ v(\theta, 1_{\{\tau \geq \max\{-z(\beta), 0\}\}}) \right] & \text{if } \beta \leq z^{-1}(\bar{\tau}) \\
E[\theta] & \text{else} 
\end{cases}
\]  

(11)

Note that \( E\left[ v(\theta, 1_{\{\tau \geq \max\{-z(\beta), 0\}\}}) \right] > E[\theta] \) if and only if \( E[\theta|\theta < \max\{-z(\beta), 0\}] < 0 \) which always holds. Thus, by comparing expression (11) with expression (10), we conclude that exit can facilitate communication and hence efficiency. Table 1 summarizes the ex-ante long run value of the firm that is obtained by the most efficient equilibrium under different regimes.

<table>
<thead>
<tr>
<th></th>
<th>No Voice</th>
<th>Voice Without Exit</th>
<th>Voice With Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta \in [0, z^{-1}(0)] )</td>
<td>( E[\theta] )</td>
<td>( E[\theta] )</td>
<td>( E[\theta] )</td>
</tr>
<tr>
<td>( \beta \in (z^{-1}(0), z^{-1}(\bar{\tau})) )</td>
<td>( E[\theta] )</td>
<td>( E[\theta] )</td>
<td>( E\left[ v(\theta, 1_{{\theta \geq -z(\beta)}}) \right] )</td>
</tr>
<tr>
<td>( \beta \in (z^{-1}(\bar{\tau}), \infty) )</td>
<td>( E[\theta] )</td>
<td>( E[\theta] )</td>
<td>( E[\theta] )</td>
</tr>
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</table>

\(^{17}\) The discussion that follows Corollary 1 ignores the welfare of the activist and the manager. The activist’s expected utility is given by \( E[v(\theta, 1_{\{\theta \geq \tau\}})] \) for any threshold \( \tau \). This is because the share is priced fairly in equilibrium and despite the activist’s opportunistic behavior when exiting. The manager’s expected utility is given by \( E\left[ v(\theta, 1_{\{\theta \geq \tau\}}) \right] + (Pr[\theta \geq \tau] - Pr[\theta < \tau]) \beta \). Thus, the manager would benefit by shifting the threshold \( SB_0 \) downward to \(-\beta\). Nevertheless, the most efficient equilibrium is Pareto efficient and hence can be sustained as a valid outcome of the game.
It immediately follows from Definition 2 and Table 1 that voice and exit exhibit complementarity.

More generally, Corollary 1 implies that the value of the firm increases with $\delta$ (decreases with $\rho$), where $\delta$ is the frequency of the activist’s liquidity shocks. Recall that because of the adverse selection between the activist and the market maker, exit generally reflects bad news on the company and its investment opportunities. Higher $\delta$ relaxes the adverse selection problem since the activist is more likely to exit for reasons which are unrelated to the value of the firm. All else equal, the negative price impact of exit diminishes. The option to strategically exit at better terms makes the activist less sensitive to the long run performances of the firm, and therefore, allows the activist to communicate more freely with the manager. The improved quality of communication enhances the efficiency of the manager’s decision making. In the Appendix I show that when $\delta \to 0$ then $\tau \uparrow 0$ and $\tilde{\tau} \downarrow 0$. Thus, the existence of a responsive equilibrium converges to the same conditions of the benchmark case of voice without exit when the frequency of liquidity shocks vanishes.

An interesting comparative static that is missing from Corollary 1 is whether shareholders are better off if the distribution of $\theta$ shifts upward in a first-order degree stochastic dominance manner. On the one hand, the manager is biased toward the approval of the project and he needs a stronger evidence that $\theta$ is low in order to reject the project. Thus, $z(\beta)$ is strictly higher for any $\beta$, and $|\tau^{SB}|$ increases. This implies a loss of efficiency. On the other hand, since $\theta$ has shifted upward, rejection is less likely to be the efficient decision in the first place. So the total effect on welfare is ambiguous. In the Appendix I provide examples where the direction of change in shareholders’ welfare can be either positive or negative as a response to such perturbation of the density function $f$.

Proposition 1 also has the following immediate yet, somewhat unexpected, implication.

**Corollary 2** If a fully revealing equilibrium exists, there is a threshold equilibrium which is strictly more efficient. A fully revealing equilibrium exists if and only if $-\beta \in [\tau, \tilde{\tau}]$.

Recall the discussion that follows Lemma 4: without exit a fully revealing equilibrium does not exist. By contrast, Corollary 2 suggests that with exit a fully revealing equilibrium may exist. Nevertheless, even if it exists, a fully revealing equilibrium is never the most efficient equilibrium. The activist can do strictly better by introducing noise into the communication with the manager by only revealing whether $\theta$ is greater or smaller than $\min \{-z(\beta), 0\}$, but without revealing the exact value of $\theta$. Recall that the binding incentive constraint is convincing the manager to reject the project. By pooling very low realizations of $\theta$ with intermediate realizations of $\theta$, the activist is able to persuade the manager to reject the project even when
$\theta \in [-\beta, \min \{-z(\beta), 0\}]$. Since $-z(\beta) > -\beta$, the implemented threshold under the latter strategy is strictly more efficient.

**Corollary 3 (The Likelihood of Exit)** At the most efficient equilibrium the ex-ante probability of exit decreases in $\beta$ and $\rho$ if $z(\beta) + \tau(\rho) \neq 0$. If $z(\beta) + \tau(\rho) = 0$ then this probability jumps upward if and only if in addition $\Pr[\theta \leq p_{NR}] > \Pr[|\theta| \leq -\tau]$.

Interestingly, almost everywhere a higher conflict of interests between shareholders and the manager reduces the likelihood the activist exits. Intuitively, with a higher bias the manager is less likely to follow the activist’s advice and less information is exchanged. Therefore, the long term value of the firm decreases and the market maker prices its shares at a lower level. All else equal, the incentives of the activist to exit are diminished and consequently her ability to influence the manager to make the efficient decision is harmed. The overall effect reduces the share price and the probability of exit.\(^{18}\) Note that without voice (or in any non-responsive equilibrium) the ex-ante probability of exit is invariant to $\beta$. Thus, it is through the channel of communication that the manager’s bias reduces the likelihood of exit. Nevertheless, there are special circumstances under which the probability of exit decreases with $\beta$. Recall that a responsive equilibrium exists if and only if $\beta \leq z^{-1}(-\tau)$. Thus, when $z(\beta) + \tau(\rho) = 0$ then an infinitesimal increase in $\beta$ changes the equilibrium from responsive to non-responsive. This change discretely shifts the ex-ante probability of exit by a magnitude that is proportional to $\Pr[\theta \leq p_{NR}] - \Pr[|\theta| \leq -\tau]$. If this term is positive then an increase in the manager’s bias makes voice completely ineffective and thereby increases the incentives of the activist to exit. In this case, the expected loss of the long term value of the firm is more significant then the depreciation of the short term share price, and hence the activist exits more often.

As one would expect, Corollary 3 shows that almost everywhere the probability of exit increases with the likelihood the activist experiences a liquidity shock. Indeed, with higher $\delta$ (lower $\rho$) the market maker is less sensitive to adverse selection and therefore, all else equal, sets the price at a higher level. With a higher stock price the activist has more opportunities to exit strategically. This dynamic enhances the effect of voice for a similar reason to what was mentioned above, and hence, increases the price even further. Overall, the likelihood of exit increases with $\delta$. Interestingly, when $\tau(\rho) + z(\beta) = 0$ the likelihood of exit can discretely fall at $\delta$ if in addition $\beta > \bar{\beta}$, where $\bar{\beta}$ is the unique solution of $\Pr[|\theta| \leq z(\beta)] = \Pr[\theta \leq p_{NR}]$.\(^{19}\) Recall that in a responsive equilibrium the decision making is more efficient than in a non-responsive

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\(^{18}\)The activist strategically exits if and only if the short term share price is higher than the long term value of the firm. The relative sensitivity of firm’s valuations to the manager’s bias determine the frequency of exit. As long as the activist can influence the manager by voicing her opinion, the short term price is relatively more sensitive and hence the activist will exit less often.

\(^{19}\)While $\Pr[\theta \leq p_{NR}]$ is independent of $\beta$, the term $\Pr[|\theta| \leq z(\beta)]$ strictly increases in $\beta$ from zero to one. Therefore, $\bar{\beta} \in (0, -\bar{\theta})$ and it is well defined.
equilibrium and hence not only the long term value of the firm is higher, but also the short term price. The former gives the activist more incentives to keep her holdings, while the latter gives the activist more incentives to exit. When $\beta$ is high the overall likelihood of exit is relatively small in a responsive equilibrium, which explains why the jump can be downward when $\beta > \beta^*$. We conclude that an increase in $\delta$ can reduce the ex-ante likelihood of exit.

III Shareholder Activism and Transparency

In the baseline model the actual decision made by the manager is unobservable to the activist and the market maker. Moreover, the message that is sent by the activist to the manager is private and hence unobservable to the market maker. When both of these assumptions hold, the framework is one of "No-Transparency". This section relaxes these assumptions and studies the effect of transparency on voice and exit.

Transparency of Actions

It is important to distinguish between two kinds of transparency: "Activism-Transparency" and "Market-Transparency". Under Activism-Transparency the activist directly observes the manager’s decision to approve the project before she decides whether to exit. The market maker, however, does not observe the manager’s decision before it sets the short term price of stock. Under Market-Transparency the manager’s decision to approve the project becomes public and is observable by the activist and the market maker before the former decides whether to exit. Under either of these regimes I maintain the assumption that the communication between the activist and the manager is private.

Definition 1 of a responsive equilibrium is trivially extended to the different kinds of transparency that are analyzed in this section. Regardless of the kind of action transparency, a non-responsive equilibrium always exists and its analysis coincides with the benchmark of exit without voice. Indeed, in a non-responsive equilibrium the activist’s message does not affect the manager’s decision and his decision is fully predicted by the market, even without observing it directly. In particular, the manager approves the project if and only if $\beta \geq -E[\theta]$ and regardless of the stock price. In anticipation of the manager’s behavior, the activist exits under exactly the same circumstances as described in Lemma 3. The next lemma states that under Activism-Transparency the nature of responsive equilibria does not change as well.

Lemma 7 The sets of equilibria under No-Transparency and Activism-Transparency are identical.
The reasoning behind Lemma 7 is the following. The manager does not have a direct utility from the short term share price and therefore his decision rule is not directly affected by the activist’s decision to exit. In equilibrium, the activist can perfectly predict the behavior of the manager. Since the market maker does not observe the manager’s decision, the circumstances under which the activist exits do not change. Therefore, the market maker has the same inference from the activist’s decision to exit, whether or not the activist directly observes the manager’s decision. This implies that set of equilibria is invariant to Activism-Transparency.

Unlike Activism-Transparency, Market-Transparency has a significant effect on soft shareholder activism. If the market maker observes the manager’s decision to approve the project, the share price conditional on exit will depend on $a$. Let $p_a$ be the share price conditional on exit and the manager’s actual action $a \in \{A, R\}$. In equilibrium, the activist takes $p_a$ as given. Thus, if the activist persuades the manager to accept the project, she will exit if and only if $p_A \geq \theta$. If the activist persuades the manager to reject the project, she will exit if and only if $p_R \geq -\theta$. The next proposition describes the set of equilibria under Market-Transparency.

**Proposition 2** There is $\beta^* < z^{-1}(-\theta)$ such that a responsive equilibrium under Market-Transparency exists if and only if $\beta \leq \beta^*$, where $\beta^* < z^{-1}(0)$ for certain density functions of $\theta$ and values of $\rho$. Moreover, any responsive equilibrium under Market-Transparency is also an equilibrium under No-Transparency, and it satisfies $p_A = p_R > 0$.

The immediate implication of Proposition 2 is that Market-Transparency limits the ability of the activist to communicate with the manager, when the manager is biased. Moreover, since the most efficient equilibrium is a responsive equilibrium, Market-Transparency also harms shareholders’ welfare. Even more striking is the result that with Market-Transparency, welfare can be lower than in markets where the activist cannot exit at all ($\beta^* < z^{-1}(0)$). That is, it may be better off to “lock in” the activist and eliminate his ability to trade, if the alternative is trade with Market-Transparency of managerial actions. In this sense, voice and exit could exhibit substitution under Market-Transparency.

Intuitively, when the market observes the manager’s action, the share price incorporates this

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20 With Activism-Transparency there are off-equilibrium events. Since the manager’s utility is invariant to the activist’s decision to exit $\sigma$ or the short term stock price $p$, the activist’s off-equilibrium beliefs or actions cannot change the set of equilibria.

21 One can interpret the decision of the activist to trade based on the observation of $a$ as her decision to time her trade before or after the manager is making his decision about $a$. For the reasons that are mentioned above, even with the option to time the trade, and to the extent that the market maker does not directly observe whether the activist trades before or after the manager has made his decision, the set of equilibria does not change. If the act of learning about $a$ is costly to the activist and is unobservable by the market maker, the activist will never choose to observe $a$ since she can fully predict it either way. One has to relax the assumption that the manager has no information that the activist does not have or that the market maker cannot directly observe the activist’s decision to learn about $a$ in order to get a different result.
information. In a responsive equilibrium, the activist is able to persuade the manager to take a particular action, and hence dictate the corporate decision. Therefore, in equilibrium, the share price conditional on exit must be invariant to the decision made by the manager. Otherwise, the activist will arbitrage the difference between these prices by sending the appropriate message. For this reason, under Market-Transparency the incentives of the activist to persuade the manager to approve the project are distorted, and consequently, the credibility of her advice and the effectiveness of voice are diminished. Overall, a responsive equilibrium is less likely to exist.

**Transparency of Voice**

The assumption that the message sent from the activist to the manager is private corresponds to behind the scenes communication that other market participants cannot observe but could be aware of. Is shareholder activism more effective if the communication is public rather than private? In this section I relax the assumption that the communication between the activist and the manager is private, and assume that the message $m$ is observed by the market maker as well. The assumption from Section I that the manager’s actions are not observable to the market or to the activist is maintained. The new framework is titled as “Voice-Transparency”.

As in Section II, a non-responsive equilibrium always exists. In this equilibrium, the manager ignores any message from the activist, and hence his decision is fully predictable. The activist may or may not, decide to exit based on her observation of $\theta$. However, she may still try to send a message that provides information to the market about $\theta$. Without the ability to influence the manager’s decision to approve the project, the activist is left with the objective of getting the highest price conditional on exit. The incentive to inflate the short term stock price prevents any revelation of information by the activist in such equilibrium. Therefore, a non-responsive equilibrium always exits when communication is public, and its characterization is given by Lemma 3. The next proposition proves that because a public message gives the activist incentives to manipulate prices directly, activism is severely impeded.

**Proposition 3** *The sets of equilibria under Voice-Transparency and Market-Transparency are identical.*

In any responsive equilibrium under Voice-Transparency the market infers from the activist’s message whether the manager will approve or reject the project. Therefore, given that the communication between the activist is public, whether or not the manager’s action is public does not change the set of responsive equilibria.\(^{22}\)

\(^{22}\)It would change, however, the set of non-responsive equilibrium.
If the activist can choose between sending a private message to the manager or communicating publicly, any equilibrium under Voice-Transparency can be implemented: the manager ignores any private messages from the activist, and the activist always randomizes when sending private messages. Thus, the option to communicate privately does not impede activism. Interestingly, the option to communicate privately actually extends the set of equilibria to the one that is obtained under No-Transparency. In light of Proposition 2, the option to communicate privately has social value. Basically, there is always an equilibrium in which the manager and the market maker ignore any public message sent by the activist, and the activist always randomizes when sending public messages.

IV Managerial Myopia

Managers are often sensitive to the short run performances of their company’s stock price. This could either be because of their compensation package which has stocks and options, or because of their career concerns and incentives to demonstrate executive talent. Either way, managerial myopia can play a significant role in the context of soft shareholder activism. To study the effect of myopia on the interaction of exit and voice, let us modify the preferences of the manager and assume the following,

\[ u_M = \omega p(\sigma) + v (\theta + \beta, a) \]  

(2’)

where \( \omega \geq 0 \) is the relative weight the manager puts on the short term stock price.

For what follows it is useful to note that exit always conveys bad news for the company and hence \( p(\sigma = 0) > p(\sigma = 1) \) for any \( \Theta \) and \( \Upsilon \). To see why, note that by definition, \( \theta \in \Upsilon \) implies \( v (\theta, 1_{\{\theta \in \Theta\}}) \leq p(1) \) or else the activist is strictly better off by not exiting. Therefore, \( E [v (\theta, 1_{\{\theta \in \Theta\}}) | \theta \in \Upsilon] \leq p(1) \) as well. From expression (4) it follows that \( E [v (\theta, 1_{\{\theta \in \Theta\}}) | \theta \in \Upsilon] \leq E [v (\theta, 1_{\{\theta \in \Theta\}}) | \theta \notin \Upsilon] \) and hence \( p(0) > p(1) \) as required. Thus, when \( \omega > 0 \) the option to exit opens up the possibility that the activist threatens to sell her holdings in the company if the manager does not defer to her view.

I start with the observation that under No-Transparency the set of equilibria is invariant to managerial myopia. Indeed, since both the activist and the market maker do not observe the manager’s action, the realized short term stock price is independent of the actual decision taken by the manager. Therefore, the manager’s incentives to approve the project are invariant to \( \omega \) and the analysis of Section II continues to hold. Hereafter, I assume Activism-Transparency.

Consider the benchmark cases with managerial myopia and under Activism-Transparency. In the first benchmark case, voice without exit, the set of equilibria is invariant to managerial myopia \( (V_{\text{Voice,NoExit}} (\omega) = V_{\text{Voice,NoExit}} (0) \) for any \( \omega \) and its characterization is given by
Lemma 4 in Section II. This is true for the same reason that was mentioned above. The next lemma considers the benchmark case of exit without voice. The analysis of this benchmark corresponds to the set of non-responsive equilibria with managerial myopia.

**Lemma 8 (Non-Responsive Equilibrium With Managerial Myopia)** A non-responsive equilibrium exists for any \( \omega \geq 0 \). There is \( \hat{\omega} \in (0, \infty) \) such that if and only if \( \Pr [-\theta < p_{NR}] < \Pr [\theta < p_{NR}] \) and \( \omega > \hat{\omega} \) then in any non-responsive equilibrium the manager rejects the project with a strictly positive probability.

As the proof of Lemma 8 demonstrates, with myopia, a non-responsive equilibrium does not have to be unique and sometimes the equilibrium must involve mixed strategies. Indeed, when comparing with Lemma 3, the set of non-responsive equilibria with myopia dramatically changes. The reason why myopia can change the set of non-responsive equilibrium is the following. When the manager considers his decision to approve the project, he realizes the activist’s exit strategy depends on this decision. Therefore, even though the market maker cannot condition the price on the manager’s decision, by changing the probability the activist exits, the manager also changes the price of the stock indirectly. In particular, if the manager’s myopia is significant (\( \omega > \hat{\omega} \)) and the activist is relatively more likely to exit when the project is approved than when it is rejected (\( \Pr [-\theta < p_{NR}] < \Pr [\theta < p_{NR}] \)), the attempt of the manager to minimize the likelihood the activist exits can crowd out his inherent bias toward the project’s approval. Indeed, under those circumstances, despite the manager’s bias for the project and the assumption that \( \mathbb{E}[\theta] + \beta \geq 0 \), an equilibrium in which the manager approves the project with probability one does not exist.

This result has welfare implications when the activist cannot voice herself. If \( \mathbb{E}[\theta] \in (-\beta, 0) \) then myopia may increase the likelihood the manager rejects the project and therefore has positive effect on shareholders’ welfare. However, if \( \mathbb{E}[\theta] > 0 \) then myopia actually harms shareholders. Thus, with myopia exit exerts power on the manager even when voice is idle. The next proposition shows that with exit and voice, the effect of myopia is always positive.

**Proposition 4** The set of all responsive equilibria strictly increases with \( \omega \) and decreases with \( \beta \). Moreover, a threshold equilibrium exists if and only if \( \beta \leq \bar{\beta} \equiv \max_{\tau \in [\xi, \eta]} \Psi (\tau; \omega, \rho) \), and the first best is obtained in equilibrium if and only if \( \beta \leq \hat{\beta} \equiv \Psi (0; \omega, \rho) \) where

\[
\Psi (\tau; \omega, \rho) = \frac{\omega}{2} \frac{\rho}{1 + \rho} \Pr [\theta < -\pi (\tau) | \theta < \tau] (\mathbb{E}[|\theta| | |\theta| > \pi (\tau)] - \pi (\tau)) - \mathbb{E}[\theta | \theta < \tau] \tag{12}
\]

Proposition 4 demonstrates that any responsive equilibrium without managerial myopia is also a responsive equilibrium with managerial myopia. The intuition for this result is that with myopia the manager has stronger incentives to follow the activist’s advice. If the manager
deviates and ignores the activist’s recommendation then the activist is strictly more likely to exit. As was explained above, exit always conveys bad news about the company. Thus, by ignoring the activist’s recommendation the manager depresses the short term price of the stock, to which he is sensitive. Different from the analysis of non-responsive equilibria in Lemma 8, the manager’s decision depends on the activist’s message and therefore the threat of exit exercises stronger discipline on the manager to follow the proposed action. For this reason, there are responsive equilibria with managerial myopia which do not exist without managerial myopia. This can be seen by noting that \( \beta > z^{-1}(0) \) and \( \tilde{\beta} > z^{-1}(\tilde{\tau}) \). Moreover, for any \( \beta \) there is \( \omega_\beta \in [0, \infty) \) such that the first best is obtained in equilibrium. In conclusion, managerial myopia increases the effectiveness of voice.\(^{23}\)

The next corollary shows that with managerial myopia there are special circumstances under which voice and exit exhibit substitution.

**Corollary 4** With managerial myopia, voice and exit exhibit substitution if and only if \( \beta \leq z^{-1}(0) \), \( \mathbb{E}[\theta] < 0 \), \( \omega > \tilde{\omega} \), and \( \Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}] \).

Intuitively, when \( \beta \leq z^{-1}(0) \) then the first best is obtained with voice and with or without exit. Recall that \( \nabla_{NoVoice, NoExit} = \mathbb{E}[\theta] \) and, per Definition 2, voice and exit exhibit complementarity if and only if \( \mathbb{E}[\theta] > \nabla_{NoVoice, Exit}(\omega) \). The conditions in Corollary 4 apply Lemma 8 for the case in which exit has strictly positive welfare implications by inducing the manager to reject the offer despite his bias to approve it. Thus, substitution stems from the effect of exit rather than from the effect of voice. Nevertheless, in all other cases, and in particular when the conflict of interests between shareholders and the manager is significant \( (\beta > z^{-1}(0)) \), voice and exit continue to exhibit complementarity.

## V Opportunistic Activism

In many cases activist investors are suspected of having a secret agenda that is conflicted with maximizing the value of the firm. Indeed, hedge funds are often blamed for being opportunistic and pursuing short term goals which are inconsistent with the long term value of the firm. To study this type of opportunism I keep the baseline model in Section II as is but modify the

\(^{23}\)Threshold \( \tilde{\beta} \) in Proposition 4 relates only to threshold equilibria. Thus, in principle, there could be non-threshold responsive equilibria even if \( \beta > \tilde{\beta} \). Recall that Lemma 2 argues that for any non-threshold managerial decision rule, there is a more efficient decision rule which can be characterized as a threshold. The reason why Lemma 2 does not apply in a setup with managerial myopia stems from the observation made before that in this setup the difference \( p(0) - p(1) \) plays a key role in the incentives of the manager’s to follow the activist’s advice. While \( p(0) \) and \( p(1) \) monotonically increase with the “efficiency” of the manager’s decision rule, depending on the properties of the density function \( f \), the difference \( p(0) - p(1) \) may not be monotonic. Therefore, it could be that with more efficient decision rule, the manager has fewer incentives to follow the activist’s advice.
activist’s preferences as follows,

\[ u_A = \sigma p(\sigma) + (1 - \sigma) v(\theta + \gamma, a) \] 

(3')

where \( \gamma \neq 0 \) is the activist’s bias. Note that if the activist exits and sells her holdings in the firm, she is completely indifferent with respect to the performances of the firm. Only when the activist keeps her holdings in the firm her bias matters. For example, if the hedge fund has cross-holdings in the target firm and in a potential set of acquirers, its deviation from maximizing target shareholders’ value persists only as long as the fund keeps her holdings in the target.

The analysis starts with the observation that without voice, the price upon exit is higher when the activist is biased than when it is unbiased. This is true regardless of the direction or magnitude of the bias. The reason for this seemingly counter-intuitive result comes from the observation that a biased activist will exit under different circumstances than an unbiased activist. That is, the activist’s private benefit relaxes the adverse selection problem that is embedded in her exit decision. To see why, recall that when the activist is unbiased, the market maker ascribes the “worst case beliefs” upon exit and hence the solution of the equation \( p = \varphi_\theta(p) \) is \( \min_p \{ \varphi_\theta(p) \} \). With the bias, however, the activist would like to exit if and only if \( |\theta + \gamma| \leq p \Leftrightarrow \theta \in [-p - \gamma, p - \gamma] \). Essentially, taking the price as given, the bias shifts the range of exit where the direction of the shift depends on the sign of the bias. This implies that the biased activist either exits too little or too much compared with the unbiased activist. For reasons that are unrelated to the firm’s fundamentals, the activist may strategically exit even if valuations are relatively high, or decide to keep her shares when valuations are relatively low. Either way, this dynamic pushes the price upon exit upward.

The decision making of the manager is not directly affected by the activist’s bias. Potentially, there is an indirect effect through the change of the activist’s ability to credibly convey information. Recall that for any \( \beta \) such that \( z(\beta) > 0 \) the second best decision rule is a threshold \(-z(\beta)\). According to Proposition 1, as long as \( \beta \leq z^{-1}(-\tau) \) this threshold can be implemented in equilibrium when the activist is unbiased. However, if \( \beta > z^{-1}(-\tau) \) then a responsive equilibrium does not exist and threshold \(-z(\beta)\) is not implementable. The next proposition shows that if \( \beta > z^{-1}(-\tau) \) the second best outcome can be obtained in equilibrium if and only if the activist is biased for the project’s approval.

**Proposition 5** Let \( \gamma^* \) be the lowest (in absolute terms) bias of the activist for which threshold \( \tau^* \equiv \max \{ -z(\beta), 0 \} \) is implementable in equilibrium. Then, \( \gamma^* \in (0, z(\beta)] \) if \( \beta > z^{-1}(-\tau) \) and \( \gamma^* = 0 \) otherwise. Moreover, \( \gamma^* \) increases in \( \beta \) and \( p \).

Proposition 5 implies that when the manager is sufficiently biased, voice can be effective
only if the activist is biased in the same direction as the manager. The smallest bias that is required from the activist, is never as large as the manager’s bias. Thus, the optimal bias is always in between the manager and shareholders, and it increases with the manager’s bias. Overall, a biased activist can improve welfare by communicating more information. One way to interpret this result is that ties between large shareholders and senior management are not necessarily harmful. For example, mutual funds that benefit from administrating the pension plan of their portfolio company might have an inherent bias toward the incumbent management. This result is in contrast to models in which voice is modeled as a costly action taken by the activist. In these models, in order to provide the activist with incentives to incur the personal cost of voice, the optimal bias would be the opposite to the bias of the manager.

As a final remark, the proof of Proposition 5 shows that when the bias of the manager is very large, there is an equilibrium with voice and exit in which the biased activist never exits for strategic reasons even though she has the option to exit.

Myopic Activism
Suppose the activist discounts the long term value of the firm in the following sense,

\[ u_A = \sigma p + (1 - \sigma) \xi v(\theta, a) \]  

(3’)

where \( \xi \in [0, 1] \). In that case, the activist would like to exit as long as \( \frac{p}{\xi} > v(\theta, a) \). Therefore, all else equal, the activist is more likely to exit. Similar to the analysis of Section II, this implies that prices will be higher and hence more information can be communicated. Similar to the reasoning that was presented above, the price is higher not only because more information is communicated, but also because exit by the myopic activist is not the worst case scenario.

VI Conclusion
This paper offers a new perspective on shareholder activism by focusing on what can be achieved when costly formal control cannot be obtained or exercised by shareholders. Two primary mechanisms are analyzed, voice and exit. Departing form the majority of the existing literature on shareholder activism, voice is modeled as a strategic transmission of information. This form of informal communication is a reflection of investors’ attempt to exercise activism by sending letters, calling senior executives, and meetings with board members, thereby providing their input and ultimately changing the strategic course of the company. The paper analyzes the conditions under which “soft” shareholder activism is an effective form of corporate governance. It highlights the synergetic nature of the interaction between voice and exit, the role of transparency in shareholder activism, and the effect of managerial myopia and opportunistic
activism on investors’ ability to voice themselves and incentives to exit.
References


VII Appendix

Proofs of Section II

Unless stated otherwise, all the results of Section I are proved for the general case where the activist’s utility function is given by $u_A(\theta, a, p, \sigma) = \sigma p + (1 - \sigma) v(\theta + \gamma, a)$ and $\gamma \in (-\bar{\theta}, -\underline{\theta})$.

Proof of Lemma 2. If a responsive equilibrium is non-threshold there are $\theta_1 < \theta_2$ such that $\theta_1 \in \Theta$ and $\theta_2 \not\in \Theta$. Since in a responsive equilibrium the activist can dictate $a$, according to (3), if $\theta \in [\theta_1, \theta_2]$ then she must be indifferent between approval and rejection and hence the activist exits with probability one in this interval. Let $p$ be the price upon exit in this equilibrium, then $[\theta_1, \theta_2] \subset \Upsilon$. Therefore, a non-threshold responsive equilibrium exists only if the activist can exit ($\rho < \infty$). Let $m_i$ be the message that is sent when the activist observes $\theta_i$ and $\Theta_i \equiv \{\theta : \mu(m_i|\theta) > 0\}$ the set of $\theta$ for which the activist sends message $m_i$. By definition, $\theta_i \in \Theta_i$. Note that $E[\theta|m_i] = E[\theta|\theta \in \Theta_i]$ and Lemma 1 implies

$$E[\theta|\theta \in \Theta_1] \geq -\beta \geq E[\theta|\theta \in \Theta_2] \quad (A1)$$

Consider an alternative equilibrium for which the communication strategy is identical to the original equilibrium, with the sole exception $\Theta'_i \equiv \Theta_i \cup \{\theta_j\} \setminus \{\theta_i\}$. Under the new strategy, the activist sends message $m_i$ when he observes $\theta_j$, $i \neq j$. Since $\theta_1 < \theta_2$ then $E[\theta|\theta \in \Theta'_1] > E[\theta|\theta \in \Theta_1]$ and $E[\theta|\theta \in \Theta'_2] > E[\theta|\theta \in \Theta_2]$. Thus, given Lemma 1 and (A1), the manager will have incentives to follow the activist’s recommendation under the new strategy. Note that if the activist would adopt this communication strategy, then the difference in expected value is $\Delta = (\theta_2 - \theta_1) - (\theta_1 - \theta_2) > 0$. Thus, the price under the new strategy is higher and the activist finds it weakly optimal to follow the new communication strategy, yielding an equilibrium which is strictly more efficient, as required. One can repeat this procedure as long as the equilibrium is non-threshold, eventually, converging to a threshold equilibrium. ▷

Lemma A.1 (Non-Responsive Equilibrium) Let $p_{NR}(\gamma)$ be the share price conditional on “exit” in equilibrium without voice, then:

(i) If $E[\theta] \geq -\beta$ then for any $\gamma$ and in any equilibrium the manager accepts the project with probability one and the activist exits if and only if $\theta + \gamma \leq p_{NR}(\gamma)$ where $p_{NR}(\gamma)$ is given by the solution of $p = \varphi(\theta, p - \gamma)$. If $\gamma = 0$ the equilibrium is unique and $p_{NR}(0) \equiv \min_p \{\varphi(\theta, p)\}$.

(ii) If $E[\theta] < -\beta$ then for any $\gamma$ and in any equilibrium the manager rejects the project with probability one and the activist exits if and only if $\theta + \gamma \geq -p_{NR}(\gamma)$ where $p_{NR}(\gamma)$ is given by the solution of $p = \varphi(-\theta, p + \gamma)$. If $\gamma = 0$ the equilibrium is unique and $p_{NR}(0) \equiv \min_p \{\varphi(-\theta, p)\}$.

(iii) $p_{NR}(\gamma) > p_{NR}(0)$ for any $\gamma \neq 0$.

(iv) $p_{NR}(\gamma, p)$ decreases in $p$. 

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Proof of Lemma A.1. As follows from the discussion that precedes Lemma 3, without voice the manager accepts the project in equilibrium if and only if $\mathbb{E}[\theta] \geq -\beta$. If the manager accepts (rejects) the project, then the value of the activist’s holdings is given by $\theta + \gamma (-\theta - \gamma)$ and hence she has strict incentives to exit if and only if $\theta + \gamma < p_{NV}(\gamma) (-\theta - \gamma < p_{NV}(\gamma))$. The market maker anticipates this behavior and hence conditional on exit it prices the share as $\varphi_\theta(p - \gamma) (\varphi_{-\theta}(p + \gamma))$ where $\varphi_\theta(p)$ is given by (8). In equilibrium, therefore, the price must satisfy $p = \varphi_\theta(p - \gamma) (p = \varphi_{-\theta}(p + \gamma))$. Note that this equation has a solution for any $\gamma$ as $\lim_{p \to \pm \infty} \varphi_\theta(p) = \mathbb{E}[\theta]$ (lim$_{p \to \pm \infty} \varphi_{-\theta}(p) = \mathbb{E}[-\theta]$). Finally, when $\gamma = 0$, the equation $p = \varphi_\theta(p) (p = \varphi_{-\theta}(p))$ has a unique fixed point given by $\min_p \{\varphi_\theta(p)\} (\min_p \{\varphi_{-\theta}(p)\})$. This follows from Proposition 1 in Archaya et al. (2010).

Consider the third part and suppose $\mathbb{E}[\theta] \geq -\beta$, a similar argument applies when $\mathbb{E}[\theta] < -\beta$. We argue that $\gamma \neq 0 \Rightarrow p_{NV}(\gamma) - \gamma \neq p_{NV}(0)$. Suppose on the contrary that there is $\gamma \neq 0$ such that $p_{NV}(\gamma) - \gamma = p_{NV}(0)$. If the manager accepts the project then $p_{NV}(\gamma) = \varphi_\theta(p_{NV}(\gamma) - \gamma) = \varphi_\theta(p_{NV}(0)) = p_{NV}(0)$. Therefore, $p_{NV}(0) = p_{NV}(\gamma)$ implying $p_{NV}(\gamma) - \gamma \neq p_{NV}(0)$, a contradiction. Therefore, $\varphi_\theta(p_{NV}(\gamma) - \gamma) \neq \varphi_\theta(p_{NV}(0))$. Since $\varphi_\theta(p_{NV}(0))$ is the unique minimum of $\varphi_\theta(p)$, for any $\gamma \neq 0$ we have $p_{NV}(\gamma) = \varphi_\theta(p_{NV}(\gamma) - \gamma) > \varphi_\theta(p_{NV}(0)) = p_{NV}(0)$ implying $p_{NV}(\gamma) > p_{NV}(0)$. Finally, the comparative static of $p_{NV}(\gamma)$ with respect to $\rho$ follows from the observation that $\varphi_\theta(p)$ and $\varphi_{-\theta}(p)$ decrease in $\rho$. 

Proof of Lemma 3. Lemma 3 is a special case of Lemma A.1 where $\gamma = 0$ and $\mathbb{E}[\theta] \geq -\beta$, and hence follows directly.

Lemma A.2 (Voice Without Exit) Suppose the activist cannot exit and let,

$$ k(x) \equiv \begin{cases} -\theta & \text{if } x \geq -\mathbb{E}[\theta] \\ k : \mathbb{E}[\theta|\theta > -k] = -x & \text{if } x < -\mathbb{E}[\theta] \end{cases} $$

(i) In any responsive equilibrium the project is accepted if and only if $\theta + \gamma \geq 0$.

(ii) A responsive equilibrium exists if and only if $\gamma \in [z(\beta), k(\beta)]$.

(iii) If exists, a non-responsive equilibrium is the most efficient equilibrium if and only if $\beta \leq -\mathbb{E}[\theta]$ and $\gamma \not\in [z(0), k(0)]$.

Proof of Lemma A.2. We start with several properties of the functions $k$ and $z$: $k(x)$ and $z(x)$ increase in $x$, $x \in [z(x), k(x)]$, if $x > -\mathbb{E}[\theta]$ then $[z(x), k(x)] = [z(x), \infty]$, if $x < -\mathbb{E}[\theta]$ then $[z(x), k(x)] = [-\infty, k(x)]$, and if $x = -\mathbb{E}[\theta]$ then $[z(x), k(x)] = [-\infty, \infty]$.

By Definition 1, if a responsive equilibrium exists then the activist can dictate the action taken by the manager. Since the activist cannot exit, she has strict incentives to persuade the manager to accept the project when $\theta + \gamma > 0$ and to reject it when $\theta + \gamma < 0$. This establishes the first part.

Note that for any $\gamma$ and $\beta > 0$ the condition $\gamma \in [z(\beta), k(\beta)]$ is equivalent to

$$ -\mathbb{E}[\theta|\theta < -\gamma] \geq \beta \geq -\mathbb{E}[\theta|\theta \geq -\gamma] \tag{A2} $$

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Suppose condition (A2) holds and consider an equilibrium in which the activist sends message $m_A$ if $\theta + \gamma \geq 0$ and a message $m_R \neq m_A$ otherwise. Conditional on $m = m_A$ the manager believes $\theta + \gamma \geq 0$. According to Lemma 1, the manager accepts the project if and only if $E [\theta | \theta + \gamma \geq 0] + \beta \geq 0$. Conditional on $m = m_R$ the manager believes that $\theta + \gamma < 0$. According to Lemma 1, the manager rejects the project if and only if $E [\theta | \theta + \gamma < 0] + \beta \leq 0$. Thus, if condition (A2) holds the manager will follow the activist’s recommendation. Given the manager’s expected behavior, it is in the best interest of the activist to follow the proposed communication strategy, so this is indeed a responsive equilibrium. To see the other direction, suppose a responsive equilibrium holds. Let $M_R$ be the set of all messages such that $a (m) < 1$ and $M_A$ be the set of all messages such that $a (m) = 1$. Since the equilibrium is responsive, neither set is empty. According to Lemma 1, if $a (m) = 0$ then $E [\theta | m] + \beta \leq 0$ and if $a (m) \in (0, 1)$ then $E [\theta | m] + \beta = 0$. Therefore, integrating over all $m \in M_R$ it follows that $E [\theta | M_R] + \beta \leq 0$. Similarly, integrating over all $m \in M_A$ implies $E [\theta | M_A] + \beta \geq 0$. Recall the activist has incentives to induce $a (m) < 1$ if and only if $\theta + \gamma \leq 0$. For this reason, $E [\theta | M_R] = E [\theta | \theta + \gamma \leq 0]$ and $E [\theta | M_A] = E [\theta | \theta + \gamma > 0]$. Overall, condition (A2) holds. This establishes the second part.

Last, in any responsive equilibrium shareholders’ welfare is given by $E \left[ (1_{\theta + \gamma \geq 0} - 1_{\theta + \gamma < 0}) \cdot \theta \right]$. In any non-responsive equilibrium the manager’s decision is independent of $\theta$. If the manager accepts the project then

$$E \left[ (1_{\theta + \gamma \geq 0} - 1_{\theta + \gamma < 0}) \cdot \theta \right] > E [\theta] \Leftrightarrow 0 > E [\theta | \theta < -\gamma] \Leftrightarrow \gamma > z (0)$$

If the manager rejects the project then

$$E \left[ (1_{\theta + \gamma \geq 0} - 1_{\theta + \gamma < 0}) \cdot \theta \right] = -E [\theta] \Leftrightarrow E [\theta | \theta > -\gamma] = 0 \Leftrightarrow k (0) > \gamma$$

Note that $\beta > -E [\theta] \Rightarrow k (\beta) = \infty$ and $z (0) < z (\beta)$. When $\beta > -E [\theta]$ the manager accepts the project in a non-responsive equilibrium. Hence, if a responsive equilibrium exists, $\gamma > z (\beta) > z (0)$ and it is always the most efficient equilibrium. If $\beta = -E [\theta]$ then any action can be taken by the manager in a non-responsive equilibrium. Thus, from the above calculations it is clear that any responsive equilibrium is more efficient than any non responsive equilibrium if and only if $\gamma \in [z (0) , k (0)]$. If $\beta < -E [\theta]$ then the manager rejects the project in a non-responsive equilibrium. Note that $\beta < -E [\theta] \Rightarrow z (\beta) = -\infty$ and $k (0) < k (\beta)$. Note that $E [\theta] < -\beta < 0$ and hence $z (0) = -\infty$ as well. Therefore, a responsive equilibrium is the most efficient one if and only if $\gamma \in [z (0) , k (0)] = (-\infty , k (0)]$. This concludes the claim. ■

**Proof of Lemma 4.** Lemma 4 is a special case of Lemma A.2 where $\gamma = 0$ and $E [\theta] \geq -\beta$, and hence follows directly. ■

**Proof of Lemma 5.** Suppose a responsive equilibrium exists. By definition, there are $m_A \neq m_R$ such that $a (m_A) = 1$ and $a (m_R) = 0$. Suppose by the way of contradiction $p^* \leq 0$. Since the action $a$ is not observable and the message $m$ is private, the market maker prices the stock conditional on exit, based on its expectation of the activist’s communication strategy in equilibrium, and the manager’s approval strategy in equilibrium. Therefore, in equilibrium,
from the activist’s point of view the price conditional on exit is fixed. Since \( p^* \leq 0 \) then the activist does not find it optimal to exit for any \( \theta \). Indeed, if \( \theta \geq 0 \) then the activist is better of by sending message \( m_A \) and keeping his holdings. If \( \theta < 0 \) the activist is better of by sending \( m_R \) and keeping his holdings. Either way, the activist never exits strategically (or only in cases where \( \theta = 0 \) which is a probability zero event). Thus, \( p = \mathbb{E} \left[ v(\theta, 1_{[\theta \geq 0]}) \right] \) is \( \mathbb{E} \left[ \left| \theta \right| \right] > 0 \), a contradiction. If \( p^* > 0 \) then whenever \( |\theta| < p^* \) the activist is strictly better off by exiting. Note that the argument does not hold for \( \gamma \neq 0 \). If \( \gamma \neq 0 \) then the price can be negative, in which case there is no strategic exit. Indeed, with a bias, the activist exits if and only if \( |\theta + \gamma| \leq p \). ■

**Proof of Lemma 6.** Extending Proposition 1 in Archaya et al. (2010) for a random variable \( v(\theta, 1_{[\theta \geq \tau]}), \) one can show that \( \pi(\tau) \) is the unique solution of the equation \( \varphi(v(\theta, 1_{[\theta \geq \tau]}), p) = p \). Therefore, \( \pi(\tau) \) is well defined. Consider several properties of \( \pi(\tau) \). First, note that \( \pi(0) \equiv \min_{\text{prob}} \{ \varphi(|\theta|, p) \} > 0, \pi(\theta) = \min_{\text{prob}} \{ \varphi(\theta, p) \} \in \mathbb{R}, \) and \( \pi(\theta) \equiv \min_{\text{prob}} \{ \varphi(-\theta, p) \} \in \mathbb{R}. \) Second, the random variable \( v(\theta, 1_{[\theta \geq \tau]}) \) FOSD \( v(\theta, 1_{[\theta \geq \tau']} ) \) if and only if \( |\tau| < |\tau'| \). It follows from (8) that for any given \( p, \varphi(v(\theta, 1_{[\theta \geq \tau]}), p) \) strictly decreases in \( |\tau| \). Therefore, \( \pi(\tau) \) strictly decreases in \( |\tau| \) as well, and \( \tau = 0 \) is the unique global maximizer of \( \pi(\tau) \). Third, note that

\[
\{ \theta : v(\theta, 1_{[\theta \geq \tau]}) \leq p \} = \{ \theta : \tau < \theta \wedge \theta \leq p \} \cup \{ \theta : \theta < \tau \wedge -\theta \leq p \} = \{ \theta : \tau < \theta \leq p \} \cup \{ \theta : -p < \theta < \tau \}
\]

and one can verify that for any \( p \geq -|\tau| \) we have \( \{ \theta : v(\theta, 1_{[\theta \geq \tau]}) \leq p \} = [\min \{ -p, \tau \}, \max \{ p, \tau \}] \).

Overall, (8) can be rewritten as

\[
\varphi(v(\theta, 1_{[\theta \geq \tau]}), p) = \frac{-\int_{\theta}^{\tau} \theta dF(\theta) + \int_{\tau}^{\max(\theta, p)} \theta dF(\theta) + \rho \left[ \int_{\min\{\theta, p\}}^{\max(\theta, p)} \theta dF(\theta) \right]}{1 + \rho \Pr[\theta \in [\min \{ -p, \tau \}, \max \{ p, \tau \}] outside of \{ \theta : \tau < \theta \leq p \} \cup \{ \theta : -p < \theta < \tau \}]
\]

and hence \( \varphi(v(\theta, 1_{[\theta \geq \tau]}), p) \) is continuous in \( \tau \in [\underline{\theta}, \bar{\theta}] \). Since \( \pi(\tau) \) is the unique minimum of \( \varphi(v(\theta, 1_{[\theta \geq \tau]}), p) \), it is continuous in \( \tau \) as well.

Note that \( \pi(0) > 0, \pi(\tau) + \tau \) increases in \( \tau < 0 \), and \( \pi(\tau) - \tau \) decreases in \( \tau > 0 \). Moreover, \( \lim_{\tau \downarrow 0} \pi(\tau) = \min_{\text{prob}} \varphi(\theta, p) < \mathbb{E}[\theta], \) and therefore, \( \lim_{\tau \downarrow 0} \pi(\tau + \tau) < \mathbb{E}[\theta + \theta]. \) Similarly, \( \lim_{\tau \uparrow 0} \pi(\tau) = \min_{\text{prob}} \varphi(-\theta, p) < -\mathbb{E}[\theta], \) and therefore, \( \lim_{\tau \uparrow 0} \pi(\tau - \tau) < -\mathbb{E}[\theta - \theta]. \) Under the assumptions \( \max \{ 0, -\mathbb{E}[\theta] \} < \bar{\theta} \) and \( \underline{\theta} < \min \{ 0, -\mathbb{E}[\theta] \} \) we get \( \lim_{\tau \downarrow 0} \pi(\tau + \tau) < 0 \) and \( \lim_{\tau \uparrow 0} \pi(\tau - \tau) < 0 \). By the intermediate value theorem, \( \tau \) and \( \bar{\tau} \) are well defined. Suppose \( \tau \in [\underline{\tau}, \bar{\tau}] \). Since \( \pi(\tau) + \tau \) is an increasing function over \( \mathbb{R}^- \) and \( \tau < 0 \) then it follows that \( \pi(\tau) + \tau > 0 \) implying \( \pi(\tau) > 0 \) and \( \tau \in [-\pi(\tau), 0] \subset [-\pi(\tau), \pi(\tau)]. \) Suppose \( \tau \in [0, \bar{\tau}] \). Since \( \pi(\tau) - \tau \) is a decreasing function over \( \mathbb{R}^+ \) then it follows that \( \pi(\tau) - \tau > 0 \) implying \( \pi(\tau) > 0 \) and \( \tau \in [0, \pi(\tau)] \subset [-\pi(\tau), \pi(\tau)] \) as required. Suppose \( \tau \in [-\pi(\tau), \pi(\tau)] \) then \( \pi(\tau) - \tau \geq 0 \) and \( \pi(\tau) + \tau \geq 0 \) implying \( \tau \in [\underline{\tau}, \bar{\tau}] \). ■

\(^{24}\)Note that if \( \max \{ 0, -\mathbb{E}[\theta] \} > \bar{\theta} > 0 \) then it is not guaranteed that \( \lim_{\tau \downarrow 0} \pi(\tau) - \tau < 0. \) If \( \lim_{\tau \downarrow 0} \pi(\tau) - \tau \geq 0 \) then \( \bar{\tau} = \bar{\theta}. \) Similarly, if \( \min \{ 0, -\mathbb{E}[\theta] \} < \underline{\theta} < 0 \) then it is not guaranteed that \( \lim_{\tau \uparrow 0} \pi(\tau) + \tau < 0. \) If \( \lim_{\tau \uparrow 0} \pi(\tau) + \tau > 0 \) then \( \underline{\tau} = \underline{\theta}. \) Overall, the same analysis follows.

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Proof of Proposition 1. Suppose \( \beta \leq z^{-1}(-\tau) \) and \( \tau^* \in [\tau, \min \{-z(\beta), \bar{\tau}\}] \), and note that since \( \beta \leq z^{-1}(-\tau) \) then the interval is not empty. Consider an equilibrium in which the activist sends message \( m_A \) if \( \theta \geq \tau^* \) and a message \( m_R \neq m_A \) otherwise. In this equilibrium the manager is advised to accept the project if and only if \( m = m_A \). If exists, the price must solves \( p = \varphi(v(\theta, 1_{\theta \geq \tau^*}), p) \), and following Proposition 1 in Archaya et al. (2010), the unique solution exists and is given by \( \pi(\tau^*) \). Since \( \beta \geq -\mathbb{E}[\theta] \) then \( \beta \geq -\mathbb{E}[\theta|\theta \geq \tau] \) for any \( \tau \) the manager follows the recommendation to accept the project. Conditional on \( m = m_R \) the manager believes that \( \theta < \tau^* \) and therefore follows the recommendation and rejects the project if and only if \( \mathbb{E}[\theta|\theta \leq \tau^*] \leq -\beta \). Note that \( \mathbb{E}[\theta|\theta \leq \tau^*] \leq \mathbb{E}[\theta|\theta \leq -z(\beta)] \) and by definition \( \mathbb{E}[\theta|\theta \leq -z(\beta)] = -\beta \). Thus, the manager has incentives to follow the recommendation and reject the project. Last, since \( \tau^* \in [\tau, \min \{-z(\beta), \bar{\tau}\}] \) then according to Lemma 6, \( \tau^* \in [\tau, \bar{\tau}] \) implies \( \pi(\tau^*) > 0 \) and \( \tau^* \in [-\pi(\tau^*), \pi(\tau^*)] \). Therefore, the activists finds it (weakly) optimal to follow this communication strategy. Overall, such a responsive equilibrium exists.

Given Lemma 2 it is sufficient to focus on threshold equilibria. Suppose by the way of contradiction that there exists a responsive equilibrium where \( \Theta = [\tau^*, \infty) \) and \( \tau^* \notin [\tau, \bar{\tau}] \). Recall the price must satisfy \( p^* = \pi(\tau^*) \). However, by Lemma 6, \( \tau^* \notin [-\pi(\tau^*), \pi(\tau^*)] \). If \( \tau^* < -\pi(\tau^*) \) then for any \( \theta \in [\tau^*, -\pi(\tau^*)] \) the activist is strictly better of by keeping his holdings in the firm and sending the manager a message that leads to rejection of the project. If \( \tau^* > \pi(\tau^*) \) then for any \( \theta \in [\pi(\tau^*), \tau^*] \) the activist is strictly better off by keeping his holdings in the firm and sending the manager a message that leads to approval of the project. Either way, \( \Theta \neq [\tau^*, \infty) \) leading to a contradiction.

Suppose by the way of contradiction that there exists a responsive equilibrium where \( \Theta = [\tau^*, \infty) \) and \( \tau^* \in (-z(\beta), \bar{\tau}] \). According to Lemma 1, \( \mathbb{E}[\theta|m] + \beta \leq 0 \) for any \( m \in M_R \). By integrating over all messages in \( M_R \) we get \( \mathbb{E}[\theta|M_R] + \beta \leq 0 \) as well, where \( \mathbb{E}[\theta|M_R] = \mathbb{E}[\theta|\theta \leq \tau^*] \). By definition, \( \mathbb{E}[\theta|\theta \leq -z(\beta)] = -\beta \). Therefore, \( \tau^* > -z(\beta) \) implies \( \mathbb{E}[\theta|\theta \leq \tau^*] > \mathbb{E}[\theta|\theta \leq -z(\beta)] \) yielding \( \mathbb{E}[\theta|\theta \leq \tau^*] + \beta > 0 \), a contradiction. \( \blacksquare \)

Proof of Corollary 1. We start by arguing that any responsive equilibrium is more efficient then a non-responsive equilibrium. Recall that in a non-responsive equilibrium the manager accepts the project with probability one and hence the ex-ante value of the firm is \( \mathbb{E}[\theta] \). A responsive equilibrium generates expected value higher than \( \mathbb{E}[\theta] \) if and only if \( \mathbb{E}[\theta|M_R] < 0 \). According to Lemma 1, \( m \in M_R \Leftrightarrow \mathbb{E}[\theta|m] < -\beta \). Therefore, it is necessary that \( \mathbb{E}[\theta|M_R] < -\beta < 0 \), as required.

According to Proposition 1, if \( \beta > z^{-1}(-\tau) \) then a responsive equilibrium does not exist. Therefore, \( \bar{V} = V = \mathbb{E}[\theta] \). Suppose \( \beta < z^{-1}(-\tau) \). Recall from the proof of Proposition 1 that \( v(\theta, 1_{\theta \geq \tau}) \) decreases in \( |\tau| \) and that the most efficient equilibrium has \( \tau^* = \max \{-z(\beta), 0\} \). Since \( z(\beta) \) is an increasing function and \( \tau^* \leq 0 \), then \( |\tau^*| \) increases with \( \beta \). Therefore, \( \bar{V} \) decreases in \( \beta \) as well. If \( \beta = z^{-1}(-\tau) \) then \( V \) drops from \( \mathbb{E}[v(\theta, 1_{\theta \geq \tau})] \) to \( \mathbb{E}[\theta] \) as \( \beta \) increases.

Consider the comparative static with respect to \( \rho \). It is immediate to see that if \( \beta < z^{-1}(-\tau) \) then \( \rho \) has no effect on \( \bar{V} \). Suppose \( \beta = z^{-1}(-\tau) \). In this case \( \bar{V} = \mathbb{E}[v(\theta, 1_{\theta \geq \tau})] > \mathbb{E}[\theta] \). Recall from the proof of Lemma 6 that \( \pi(\tau) \) is the unique solution of \( \varphi(v(\theta, 1_{\theta \geq \tau}), p) = p \).
for any \( \tau \). Since \( \varphi (v(\theta, 1_{\{\theta \geq \tau\}}), p) \) decreases in \( p \), so does \( \pi (\tau) \). Moreover, \( \varphi (v(\theta, 1_{\{\theta \geq \tau\}}), p) \) decreases in \( |\tau| \) and so does \( \pi (\tau) \). Recall from Lemma 6 that \( \pi (\tau) + \tau = 0 \) and \( \tau < 0 \). Therefore, at \( \tau = -\tau \), \( \pi (\tau) \) increases with \( \tau \). This implies that \( -\tau \) increases in \( p \) as well. Therefore, \( V \) drops to \( E [\theta] \).

Suppose \( \lim_{p \to -\infty} -\tau < 0 \). Since \( \pi (\tau) = -\tau \) and \( \pi (\tau) \) is the unique solution of \( \varphi (v(\theta, 1_{\{\theta \geq \tau\}}), p) = p \) for any \( \tau \), then \( \lim_{p \to -\infty} \pi (\tau) = E [\theta | \theta \in [-\tau, \pi (\tau)]] \), a contradiction. Therefore, it must be that \( \tau \to 0 \). A similar arguments proves that \( \lim_{p \to -\infty} -\tau = 0 \) as well. Therefore, \([\tau, -\tau] \) shrinks to zero as \( p \to -\infty \). \( \blacksquare \)

**Example.** Let \( \zeta (\Delta) \equiv \theta + \Delta \), \( \Delta > 0 \). Note that \( x \) such that \( E [\zeta | \zeta \leq -x] = -\beta \) satisfies \( x = z (\beta + \Delta) - \Delta \). Let \( x (\beta, \Delta) \equiv z (\beta + \Delta) - \Delta \) and Suppose \( \beta > \beta_0 \) where \( z (\beta_0) = 0 \). Note that \( \beta_0 > 0 \) and is unique. Then, for any \( \Delta \geq 0 \),

\[
W (\Delta) \equiv E [v (\zeta (\Delta), 1_{\zeta (\Delta) > -x (\beta, \Delta)})]
\]

\[
= - \Pr [\zeta (\Delta) < -x (\beta, \Delta)] E [\zeta (\Delta) | \zeta (\Delta) \leq -x (\beta, \Delta)]
+ \Pr [\zeta (\Delta) > -x (\beta, \Delta)] E [\zeta (\Delta) | \zeta (\Delta) > -x (\beta, \Delta)]
\]

\[
= E [\zeta (\Delta)] - 2 \Pr [\zeta (\Delta) < -x (\beta, \Delta)] E [\zeta (\Delta) | \zeta (\Delta) \leq -x (\beta, \Delta)]
= E [\zeta (\Delta)] + 2 \Pr [\zeta (\Delta) < -x (\beta, \Delta)] \beta
= E [\theta] + \Delta + 2 \Pr [\theta < -z (\beta + \Delta)] \beta
\]

Since \( \frac{\partial z (\beta + \Delta)}{\partial \Delta} = \frac{1}{f (-z (\beta + \Delta))} F (-z (\beta + \Delta)) > 0 \) then

\[
\frac{\partial W (\Delta)}{\partial \Delta} = 1 - 2 \beta f (-z (\beta + \Delta)) \frac{\partial z (\beta + \Delta)}{\partial \Delta}
= 1 - 2 F (-z (\beta + \Delta)) \frac{\beta}{\beta + \Delta - z (\beta + \Delta)}
\]

Thus,

\[
\frac{\partial W (\Delta)}{\partial \Delta} \big|_{\Delta = 0} = 1 - 2 F (-z (\beta)) \frac{\beta}{\beta - z (\beta)}
\]

and as \( \beta \downarrow \beta_0 \) then

\[
\frac{\partial W (\Delta)}{\partial \Delta} \big|_{\Delta = 0} \rightarrow 1 - 2 F (-z (\beta_0)) \frac{\beta_0}{\beta_0 - z (\beta_0)} = 1 - 2 F (0)
\]

Thus, depending on how zero is positioned relative to the median of \( \theta \), an increase in \( \Delta \) can decrease or increase the welfare of shareholders. \( \blacksquare \)

**Proof of Corollary 2.** A fully revealing equilibrium is a responsive equilibrium with threshold \( \beta \). According to Proposition 1 it exists if and only if \( \beta \in [\tau, \min \{-z (\beta), \tilde{\tau}\}] \). Recall \( \beta > z (\beta) \) and hence it is always the case \( \beta < -z (\beta) \). Therefore, A fully revealing equilibrium exists if and only if \( \beta \in [\tau, \tilde{\tau}] \). Note that \( \min \{-z (\beta), 0\} \in [-\beta, 0] \) and hence if \( -\beta \in [\tau, \tilde{\tau}] \) then \( \min \{-z (\beta), 0\} \in [\tau, \tilde{\tau}] \) as well. Therefore, according to Proposition 1, whenever a fully
revealing equilibrium exists, there also exists an equilibrium with threshold \( \min \{-z(\beta), 0\} \). Since \( 0 < \min \{-z(\beta), 0\} < -\beta \) then this equilibrium is strictly more efficient. ■

**Proof of Corollary 3.** If the equilibrium is responsive then at the most efficient equilibrium \( \eta_R = \delta + (1 - \delta) \Pr [v(\theta, 1_{\{\theta \geq \tau^{SB}\}}) \leq \pi(\tau^{SB})] \) and if the equilibrium is non-responsive then \( \eta_{NR} = \delta + (1 - \delta) \Pr [\theta \leq \rho_{NR}] \). Note that according to Lemma 6, \( \tau^{SB} \in [-\pi(\tau^{SB}), \pi(\tau^{SB})] \).

Therefore, \( \Pr [v(\theta, 1_{\{\theta \geq \tau^{SB}\}}) \leq \pi(\tau^{SB})] = \Pr [\theta \leq \pi(\tau^{SB})] \). We define \( \Delta \eta \equiv \frac{\eta_{R} - \eta_{NR}}{1 - \delta} = \Pr [\theta \leq -\tau] - \Pr [\theta \leq \rho_{NR}] \).

Consider the comparative static with respect to \( \beta \). If \( \beta \in [0, z^{-1}(0)] \) the equilibrium is responsive and \( \tau^{SB} = 0 \). Therefore, \( \frac{\partial \eta_{R}}{\partial \beta} = 0 \). If \( \beta \in [z^{-1}(0), z^{-1}(-\tau)] \) the equilibrium is responsive and \( \tau^{SB} = -z(\beta) \). Since \( \pi(\tau^{SB}) \) decreases with \( \beta \) then \( \frac{\partial \eta_{R}}{\partial \beta} < 0 \) as well. Finally, if \( \beta > z^{-1}(-\tau) \) the equilibrium is non-responsive. Since \( \rho_{NR} \) is independent of \( \beta \) then \( \frac{\partial \eta_{NR}}{\partial \beta} = 0 \).

Overall, if \( \beta \neq z^{-1}(-\tau) \) then \( \frac{\partial \eta_{R}}{\partial \beta} \leq 0 \). It is left to compare \( \eta_{R} > \eta_{NR} \) when \( \beta = z^{-1}(-\tau) \) (i.e. \( \tau^{SB} = \tau \)). By definition, \( \eta_{NR} - \eta_{R} = -(1 - \delta) \Delta \eta \). Thus, \( \eta_{NR} - \eta_{R} < 0 \) if and only if \( \Pr [\theta \leq -\tau] > \Pr [\theta \leq \rho_{NR}] \).

Consider the comparative static with respect to \( \delta \). Note that \( \frac{\partial \eta_{R}}{\partial \delta} = -\frac{\partial \eta_{R}(\delta)}{\partial \delta} > 0 \) and \( \frac{\partial \eta_{NR}}{\partial \delta} > 0 \).

If \( \beta > z^{-1}(-\tau(\delta)) \) then \( \tau(\delta) > -z(\beta) \) then the equilibrium is non-responsive. Thus \( \eta = \eta_{NR} \) for small values of \( \delta \) and \( \frac{\partial \eta_{NR}}{\partial \delta} = 1 - \Pr [\theta \leq \rho_{NR}] + (1 - \delta) \Pr [\theta = \rho_{NR}] \frac{\partial \rho_{NR}}{\partial \delta} > 0 \). Suppose \( \tau(\delta) < -z(\beta) \) then the equilibrium is responsive and \( \frac{\partial \eta_{R}}{\partial \delta} = 1 - \Pr [\theta \leq \pi(\tau^{SB})] + (1 - \delta) \Pr [\theta \in \{\pi(\tau^{SB}), -\pi(\tau^{SB})\}] \frac{\partial \pi(\tau^{SB})}{\partial \delta} \). Since \( \pi(\tau) \) is a global minimum, it follows from its definition that \( \frac{\partial \pi(\tau)}{\partial \delta} > 0 \) for any \( \tau \in [\tau, \overline{\tau}] \). Therefore, \( \frac{\partial \eta_{R}}{\partial \delta} > 0 \) as well. Overall, if \( \tau(\delta) \neq -z(\beta) \) then \( \frac{\partial \eta_{R}}{\partial \delta} > 0 \). It is left to compare \( \eta_{R} > \eta_{NR} \) when \( \tau(\delta) = -z(\beta) \) (i.e. \( \tau^{SB} = \tau \)). By definition, \( \eta_{NR} - \eta_{R} = -(1 - \delta) \Delta \eta \). Thus, \( \eta_{NR} - \eta_{R} < 0 \) if and only if \( \Pr [\theta \leq -\tau] > \Pr [\theta \leq \rho_{NR}] \). ■

**Proofs of Section III**

**Proof of Proposition 2.** We start by proving that \( p_A = p_R > 0 \) in any responsive equilibrium under Market-Transparency. The proof of this argument holds regardless of the manager’s short term compensation. First note that there is no responsive equilibrium in which \( \max \{p_A, p_R\} \leq 0 \). In that case, the activist would like the manager to accept the project if and only if \( \theta > 0 \), and would never exit unless she has a liquidity shock. This would imply that \( p_A = \mathbb{E} [\theta | \theta > 0] > 0 \) and \( p_R = \mathbb{E} [-\theta | -\theta > 0] > 0 \), yielding a contradiction. Suppose on the contrary a responsive equilibrium exists with \( p_A \neq p_R \). Neither \( M_A \) nor \( M_R \) are empty, and from the activist’s point of view, these prices are exogenous. If \( \theta > \max \{p_A, p_R\} \) then the activist strictly prefers the manager to accept the project. If \( \theta < -\max \{p_A, p_R\} \) the activist strictly prefers the manager to reject the project. Either way, the activist will not exit unless she has liquidity needs. If \( \theta \in [-\max \{p_A, p_R\}, \max \{p_A, p_R\}] \) the activist strictly prefers the manager to accept the project when \( p_A > p_R \) and strictly prefers the manager to reject the project if \( p_A < p_R \). In the former case the activist will exit for sure and get \( p_A \). In the latter case she will exit for sure and get \( p_R \). Either way, the activist will exit. Consider how the market maker would price the stock
conditional on the activist’s decision to exit and the manager’s decision \( a \). If \( p_A > p_R \) then the manager rejects the project if and only if \( \theta < -p_A \) and the activist never exits conditional on the manager’s rejection. Therefore, \( p_R = -E[\theta | \theta < -p_A] > p_A \) which contradicts the presumption that \( p_A > p_R \). If \( p_A < p_R \) then the manager accepts the project if and only if \( \theta > p_R \) and the activist never exit conditional on the manager’s approval. Therefore, \( p_A = E[\theta | \theta > p_R] > p_R \) which contradicts the presumption that \( p_A < p_R \). Either way, in any responsive equilibrium \( p_A = p_R > 0 \) as required.

Next, we prove that a responsive equilibrium under Market-Transparency exists if and only if there are \( p \) and \( \Theta \) that satisfy

\[
\Theta \supseteq [p, \overline{p}] \text{ and } \Theta \cap [\theta, -p] = \emptyset \tag{A3}
\]

\[
p = \varphi_A (p, \Theta) = \varphi_R (p, \Theta) > 0 \tag{A4}
\]

\[
E[\theta | \theta \notin \Theta] \leq -\beta \leq E[\theta | \theta \in \Theta] \tag{A5}
\]

where

\[
\varphi_a (p, \Theta) = \begin{cases} 
\frac{\mathbb{E}[\theta | \theta \in \Theta] + p \Pr[\theta < p | \theta \in \Theta] \mathbb{E}[\theta | \theta \in [-p, p] \cap \Theta]}{1 + p \Pr[\theta < p | \theta \in \Theta]} & \text{if } a = A \\
\frac{\mathbb{E}[-\theta | \theta \notin \Theta] + p \Pr[\theta > p | \theta \notin \Theta] \mathbb{E}[-\theta | \theta \in [-p, p] \cap \Theta]}{1 + p \Pr[\theta > p | \theta \notin \Theta]} & \text{if } a = R
\end{cases} \tag{A6}
\]

Suppose a responsive equilibrium exits. As was argued above, it is necessary that \( p_A = p_R = p^* > 0 \), \( \Theta^* \supseteq [p^*, \overline{p}] \), and \( \Theta^* \cap [\theta, -p^*] = \emptyset \). Therefore, condition (A3) must hold. Condition (A4) holds as well since in equilibrium it must be \( p^*_a = \varphi_a (p^*_a, \Theta^*) \). Last, since the equilibrium is responsive, \( -\beta \leq E[\theta | m] \) for any \( m \in M_A \) and \( E[\theta | m] \leq -\beta \) for any \( m \in M_R \). By integrating over all messages, it follows that \( E[\theta | M_A] = E[\theta | \theta \in \Theta^*] \) and \( E[\theta | M_R] = E[\theta | \theta \notin \Theta^*] \). Therefore, condition (A5) holds as required. Suppose conditions (A3), (A4), and (A5) hold for some \( p \) and \( \Theta \). Consider an equilibrium in which the manager sends \( m_A \) if \( \theta \in \Theta \) and \( m_R \neq m_A \) otherwise. As was explained above and since condition (A3) holds, the activist finds it weakly optimal to follow this communication strategy. At the same time, since condition (A5) holds the manager finds it optimal to follow the activist’s recommendation. Finally, (A4) ensures that under the proposed communication strategy and approval rule the implied exit strategy is consistent with \( p_A = p_R > 0 \). Thus, a responsive equilibrium indeed exists.

Next, according to the proofs of Proposition 1 and Lemma 6, a responsive equilibrium under No-Transparency exists if and only if there are \( p \) and \( \Theta \) such that conditions (A3) and (A5) hold, but instead of condition (A4), \( p \) and \( \Theta \) satisfy \( p = \varphi_{A|JR} (p, \Theta) \) where

\[
\varphi_{A|JR} (p, \Theta) = \frac{\mathbb{E}[v (\theta, 1_{\theta \in \Theta})] + \rho \Pr[\theta \in [-p, p]] \mathbb{E}[v (\theta, 1_{\theta \notin \Theta})] | \theta \in [-p, p]]}{1 + \rho \Pr[\theta \in [-p, p]]} \tag{A7}
\]

We argue that if \( p = \varphi_A (p, \Theta) = \varphi_R (p, \Theta) \) then \( p = \varphi_{A|JR} (p, \Theta) \) as well. Indeed, \( \varphi_a (p, \Theta) \) are
conditional expectations and for any $p$ and $\Theta$ that satisfy (A3), the following must hold

$$\varphi_{AJR}(p, \Theta) = \frac{\Pr[\theta \in \Theta] + \rho \Pr[\theta \in [-p, p] \cap \Theta]}{1 + \rho \Pr[\theta \in [-p, p]]} \varphi_A(p, \Theta)$$

(A8)

$$+ \frac{\Pr[\theta \notin \Theta] + \rho \Pr[\theta \in [-p, p] \setminus \Theta]}{1 + \rho \Pr[\theta \in [-p, p]]} \varphi_R(p, \Theta)$$

Thus, any responsive equilibrium under Market-Transparency is also an equilibrium under No-Transparency.

We show that if a responsive equilibrium under Market-Transparency exists then $\beta < z^{-1}(-\pi)$. Obviously, $\beta \leq z^{-1}(-\pi)$ since otherwise there is a responsive equilibrium under No-Transparency which contradicts Proposition 1. It is left to prove that $\beta \neq z^{-1}(-\pi)$. Suppose on the contrary a responsive equilibrium under Market-Transparency exists and $\beta = z^{-1}(-\pi)$. Conditions (A3) and (A5) hold and by definition of $z(\cdot)$ it must be $\Theta = [-\pi, \overline{\pi}]$. Recall $\pi = -\pi(\pi)$ where $\pi(\pi) = p_{AJR}$, and since $p_A = p_R = p_{AJR}$ then $\pi(\pi) = p_R$. Therefore, $\Theta = [-p_R, \overline{p_R}]$. If so, the project is rejected if and only if $\theta < -p_R \iff -\theta > p_R$ which implies that conditional on rejection the value of the firm is strictly greater than $p_R$. This yields a contradiction since $p_R$ must reflect the fair value of the firm conditional on the project being rejected. Thus, if a responsive equilibrium exists, $\beta < z^{-1}(-\pi)$.

Note that if a responsive equilibrium under Market-Transparency exists for some $\beta_0 < z^{-1}(-\pi)$, then it exists for any $\beta \in [0, \beta_0]$. Indeed, if we keep the same communication rule and exit rule it is only left to verify the manager has the same incentives to follow the activist’s recommendation. That is, that condition (A5) holds for any $\beta \in [0, \beta_0]$. Note that, $\beta \leq \beta_0$ implies $\mathbb{E}[\theta | \theta \notin \Theta] \leq -\beta$. Since $\beta \leq \beta_0 < z^{-1}(-\pi)$ then $-z(\beta) < \pi$ for any $\beta \in [0, \beta_0]$ and hence $-\beta = \mathbb{E}[\theta | \theta < \pi]$. Since condition (A3) must hold and the price must be greater than $\pi$ it follows that $\mathbb{E}[\theta | \theta < \pi] < \mathbb{E}[\theta | \theta \in \Theta]$. We conclude that condition (A5) holds for any $\beta \in [0, \beta_0]$ and this completes the argument. Therefore, there is $\beta^* < z^{-1}(-\pi)$ such that a responsive equilibrium under Market-Transparency exists if and only if $\beta \leq \beta^*$.

Finally, suppose $f$ is symmetric around zero and $\overline{\theta} = \infty = -\theta$. We prove that $\beta^* = z^{-1}(0)$. Conditions (A4) and (A6) imply

$$\int_{\pi}^\infty (\theta - p) dF(\theta) = \int_{-\infty}^{-p} (\theta + p) dF(\theta)$$

(A9)

Note that the symmetry of $f$ around zero implies $\int_{\pi}^\infty (\theta - p) dF(\theta) = -\int_{-\infty}^{-p} (\theta + p) dF(\theta)$ and hence (A.9) holds if and only if

$$\int_{\pi}^\infty (\theta - p) dF(\theta) + \int_{-\infty}^{-p} (\theta + p) dF(\theta) = 0$$

(A10)

Note that (A10) can be rewritten as $\int_{-p}^p \theta dF(\theta) = p \left[ \int_{\pi}^\infty dF(\theta) - \int_{-\pi}^{-p} dF(\theta) \right]$. The symmetry of $f$ around zero implies $\int_{-p}^p \theta dF(\theta) = 0$ and since $\int_{-p}^\infty dF(\theta) = \int_{-\infty}^{-p} dF(\theta)$
then it must be that $\int_{\Theta^c} dF(\theta) = \int_{\Theta^c} dF(\theta)$, which implies $\int_{\Theta^c} dF(\theta) = \frac{1}{2}$. Given condition (A5), if a responsive equilibrium exists then $\beta \leq -\mathbb{E}[\theta | \theta \notin \Theta]$. Suppose on the contrary that a responsive equilibrium exists and $\beta > z^{-1}(0)$. This implies $-\mathbb{E}[\theta | \theta < 0] < \beta$ and hence $\mathbb{E}[\theta | \theta < 0] > \mathbb{E}[\theta | \theta \notin \Theta]$. Since $\int_{-\infty}^{0} dF(\theta) = \frac{1}{2} = \int_{\Theta^c} dF(\theta)$ then $\int_{-\infty}^{0} \theta dF(\theta) > \int_{\Theta^c} \theta dF(\theta)$. At the same time, since $\int_{0}^{\infty} dF(\theta) = \frac{1}{2} = \int_{\Theta^c} dF(\theta)$ then if $\Theta^c \cap [0, \infty) = \emptyset$ then it must be $\int_{\Theta^c} \theta dF(\theta) > \int_{-\infty}^{0} \theta dF(\theta)$, a contradiction. Similarly, if $\Theta^c \cap [0, \infty) = \emptyset$ then it must be that $\Theta^c = [0, \infty)$ which implies $\int_{\Theta^c} \theta dF(\theta) = \int_{-\infty}^{0} \theta dF(\theta)$, a contradiction. Thus, $\beta^* \leq z^{-1}(0)$. Next, we will show that a responsive equilibrium exists when $\beta = z^{-1}(0)$. Consider an equilibrium where $\Theta = [0, \infty)$. The manager has incentives to follow this strategy since $\beta^* = z^{-1}(0)$ and the activist trivially will find it optimal. It is therefore left to verify that condition (A4) is satisfied. Note that according to (A6) and under the assumption that $f$ is symmetric around zero, $\varphi_A(p, \Theta) = \varphi_R(p, \Theta)$ for all $p$. Therefore, this constraint is not binding. The price will satisfy $\varphi_{A\cup R}(p, \Theta) = p$ which is the min$_p \{ \varphi((\theta|, p) \}$. Overall, a responsive equilibrium exists for all $\beta \leq z^{-1}(0)$ and hence $\beta^* = z^{-1}(0)$.

We argue that if $\mathbb{E}[\theta | \theta > 0] + \mathbb{E}[\theta | \theta < 0] < 0$ and $\bar{\theta} = \infty = -\theta$ then there exist $\delta' \in (0, 1)$ and $\beta' < z^{-1}(0)$ such that under Market-Transparency a responsive equilibrium does not exist. Suppose on the contrary that for any $\delta \in (0, 1)$ and $\beta \in (0, z^{-1}(0))$ a responsive equilibrium under Market-Transparency exists, and in this equilibrium, the manager accepts the project if and only if $\varphi_A(p, \Theta) = \varphi_R(p, \Theta)$ for all $p$. Let $\mu$ be the unique solution of $\mathbb{E}[\theta | \theta > x] + \mathbb{E}[\theta | \theta < x] = 0$ and note that $\mu > 0$ and $\mathbb{E}[\theta | \theta \leq \mu] < 0$. Let $\beta' = z^{-1}(-\mu) > 0$ and therefore $\beta' \in (0, z^{-1}(0))$. For any $\beta \in (z^{-1}(-\mu), z^{-1}(0))$ and $\delta \in (0, 1)$ the set $\Theta^{\delta, \beta}$ satisfies condition (A3). Condition (A5) implies $\mathbb{E}[\theta | \theta \notin \Theta^{\delta, \beta}] \leq -\beta = \mathbb{E}[\theta | \theta < -z(\beta)]$ for any $\delta$. Since $z^{-1}(-\mu) < \beta$ then $-z(\beta) < \mu$. Therefore,

$$\mathbb{E}[\theta | \theta \notin \Theta^{\delta, \beta}] < \mathbb{E}[\theta | \theta \leq \mu] < 0 \quad \text{(A11)}$$

Note that as $\delta \to 1$ conditions (A4) and (A6) imply

$$\mathbb{E}[\theta | \theta \notin \Theta^{\delta\to 1, \beta}] + \mathbb{E}[\theta | \theta \in \Theta^{\delta\to 1, \beta}] = 0 \quad \text{(A12)}$$

in conjunction with condition (A5) and $\beta > 0$ we get

$$\mathbb{E}[\theta | \theta \notin \Theta^{\delta\to 1, \beta}] < 0 < \mathbb{E}[\theta | \theta \in \Theta^{\delta\to 1, \beta}] \quad \text{(A13)}$$

Since $\mathbb{E}[\theta | \theta > \mu] + \mathbb{E}[\theta | \theta < \mu] = 0$ and given (A12) and (A13) then

$$0 < \mathbb{E}[\theta | \theta > \mu] < \mathbb{E}[\theta | \theta \in \Theta^{\delta\to 1, \beta}] \quad \text{(A14)}$$

Note that (A14) implies that $[\mu, \infty) \setminus \Theta^{\delta\to 1, \beta}$ is not empty (as otherwise $[\mu, \infty) \subset \Theta^{\delta=1, \beta}$ which contradicts (A14)). Note that $\sup \{ [\mu, \infty) \setminus \Theta^{\delta=1, \beta} \} > \mu > \sup \{ \Theta^{\delta=1, \beta} \setminus [\mu, \infty) \}$. Thus, relative to the pool $[\mu, \infty)$, the set $\Theta^{\delta=1, \beta}$ removes $\theta > \mu$ and adds $\theta < \mu$. This implies that $\mathbb{E}[\theta | \theta > \mu] > \mathbb{E}[\theta | \theta \in \Theta^{\delta=1, \beta}]$, a contradiction. We conclude that for $\delta$ sufficient close to one, a responsive equilibrium cannot exists, proving that $\beta^* < z^{-1}(0)$ is feasible. 

**Proof of Proposition 3.** Let $M$ be set of all (public) messages that are sent with a
Consider a non-responsive equilibrium under Market-Transparency. This equilibrium also exists under Voice-Transparency when the market and the manager completely ignore the activist’s message, and the activist’s message is indeed non-informative. Consider a non-responsive equilibrium under Voice-Transparency. Let $m_A$ be the activist’s message and hence conditional on $m = m_A$ it infers that the manager will accept (reject) the project. Therefore, $p(m) = p$ for any $m \in M$. This implies that the price does not respond to any information that is revealed by the activist, if any. Therefore, this non-responsive equilibrium is identical to equilibrium in which no information is revealed by the activist. Overall, the set of non-responsive equilibria under either form of transparency is identical.

Consider a responsive equilibrium under Market-Transparency and let $\Theta$ be the manager’s decision rule. Following Proposition 2, $p_A = p_R$ where $p_A$ is the solution of $p = \varphi_a(p, \Theta)$. Consider an equilibrium under Voice-Transparency in which the message sends message $m_A$ if $\theta \in \Theta$ and message $m_R \neq m_A$ otherwise. Integrating over condition (A5), the manager’s has incentives to follow the recommendation and approves the project if and only if $m = m_A$. The market maker observes $m$ and hence conditional on $m = m_A$ ($m = m_R$) it infers that $\theta \in \Theta$ ($\theta \notin \Theta$) and the manager will accept (reject) the project. Therefore, $p(m)$ must be the solution of $p = \varphi_a(p, \Theta)$. Effectively, the market maker has exactly the same information set as it would have under Market-Transparency. It is left to verify the activist has incentives to follow the proposed communication strategy. We suppose that off-equilibrium any message $m \notin \{m_A, m_R\}$ is interpreted as if $\theta \in \Theta$ with probability $\frac{1}{2}$ and $\theta \notin \Theta$ otherwise. Thus, since $p(m_A) = p(m_R)$ any other message yield exactly the same price conditional on exit. Thus, the incentives of the activist to send message $m_A$ or $m_R$ are solely determined by his incentives to change the manager’s decision and not to change the price conditional on exit. Since this strategy is incentives compatible under Market-Transparency, it has to be incentives compatible in this equilibrium as well.

Consider a responsive equilibrium under Voice-Transparency. Let $a(m) \in \{A, R\}$ be the manager’s decision given message $m \in M$. Let $M_A$ and $M_R$ be the set of messages that lead to approval and rejection in equilibrium, respectively. By definition, neither set is empty. We start by arguing that $m \in \arg \max_{m \in M_R} p(m) \cup \arg \max_{m \in M_A} p(m)$ is never sent with a strictly positive probability. Since $\rho < \infty$ there is a strictly positive probability that the activist exits, and hence it is a dominating strategy to send a message in arg $\max_{m \in M_R} p(m) \cup \arg \max_{m \in M_A} p(m)$. This implies that there are exactly two different prices conditional on exit, one for $m \in M_R$ and one for $m \in M_A$. We let these prices be $p_R$ and $p_A$, respectively. Essentially, there are only to effective messages: one that leads to approval and price $p_A$ and one that leads to rejection and prices $p_R$. For the same argument that is given in the beginning of the proof of Proposition 2, it must be that $p_A = p_R > 0$. Overall, it is immediate to see that any set $\Theta$ and $\Upsilon$ that emerge as equilibrium under Voice-Transparency, can also emerge as equilibrium under Market-Transparency.
Proofs of Section IV

Proof of Lemma 8. Consider a non-responsive equilibrium in which the manager chooses \( a^* \in \{ A, R \} \). The activist observes the actual decision \( a \) and strategically exits if and only if \( v(\theta, a) < p^*(1) \). Since the market maker does not observe \( a \), in equilibrium \( p^*(0) = \mathbb{E} [v(\theta, a^*) | v(\theta, a^*) > p^*(1)] \). If the manager chooses action \( a \) his expected utility is

\[
\omega [ (\delta + (1 - \delta) \Pr [v(\theta, a) < p^*(1))] p^*(1) + (1 - \delta) \Pr [v(\theta, a) > p^*(1)] p^*(0) + \mathbb{E} [v(\theta + \beta, a)]
\]

and the manager chooses \( a = A \) if and only if

\[
\frac{\rho}{1 + \rho} \omega \left( \Pr [-\theta < p^*(1)] - \Pr [\theta < p^*(1)] \right) (p^*(0) - p^*(1)) + \mathbb{E} [\theta] + \beta \geq 0 \quad (A15)
\]

and recall that \( p^*(0) - p^*(1) > 0 \) and by assumption \( \beta \geq -\mathbb{E} [\theta] \). Note that if \( a^* = A \) then \( p^*(0) = \mathbb{E} [\theta | \theta > p^*(1)] \) where \( p^*(1) = p_{NR} \) as given by Lemma 3. If \( a^* = R \) then \( p^*(0) = \mathbb{E} [\theta | \theta > p^*(1)] \) where \( p^*(1) = \pi(\infty) \equiv \lim_{\tau \to \infty} \pi(\tau) \). There are several cases to consider. First, if \( \Pr [-\theta < p_{NR}] \geq \Pr [\theta < p_{NR}] \) then a non-responsive equilibrium with \( a^* = A \) always exists. Second, suppose \( \Pr [-\theta < p_{NR}] < \Pr [\theta < p_{NR}] \). A non-responsive equilibrium in which the manager accepts the project exists if and only if

\[
\omega \leq \hat{\omega} \equiv \frac{\mathbb{E} [\theta] + \beta}{\frac{\rho}{1 + \rho} \left( \Pr [\theta < p_{NR}] - \Pr [-\theta < p_{NR}] \right) \left( \mathbb{E} [\theta | \theta > p_{NR}] - p_{NR} \right)}
\]

and note that \( \hat{\omega} > 0 \). Third, suppose \( \Pr [-\theta < p_{NR}] < \Pr [\theta < p_{NR}] \) and \( \omega > \hat{\omega} \). There is no equilibrium in which the manager accepts the project with probability one. Consider an equilibrium in which \( a^* = R \). According to (A15), such equilibrium exists if and only if

\[
\omega \geq \omega_0 \equiv \frac{\mathbb{E} [\theta] + \beta}{\frac{\rho}{1 + \rho} \left( \Pr [\theta < \pi(\infty)] - \Pr [-\theta < \pi(\infty)] \right) \left( \mathbb{E} [-\theta - \theta > \pi(\infty)] - \pi(\infty) \right)}
\]

It is left to show that if \( \omega > \hat{\omega} \) and either \( \omega \leq \omega_0 \) or \( \Pr [-\theta < \pi(\infty)] - \Pr [\theta < \pi(\infty)] \geq 0 \) then there is a mixed strategy equilibrium. Under a mixed strategy equilibrium, the manager accepts the project with probability \( x \in (0, 1) \) and let the price be \( p_x^*(\sigma) \). Since the market maker does not observe \( a \), it follows that \( p_x^*(1) = xp_{NR} + (1 - x) \pi(\infty) \) and \( p_x^*(0) = x\mathbb{E} [\theta | \theta > p_x^*(1)] + (1 - x) \mathbb{E} [-\theta - \theta > p_x^*(1)] \). The manager must be indifferent and hence (A15) must holds with equality for the assumed \( x \). Since with \( x = 1 \) the l.h.s of (A15) is negative (there is no equilibrium in which \( a = A \) with probability one) and with \( x = 0 \) it is positive (there is no equilibrium in which \( a = R \) with probability one), there is \( x \in (0, 1) \) that solves this condition.

\[ \square \]

Proof of Proposition 4. Consider a responsive equilibrium and let \( M_A \) and \( M_R \) be the set of messages that lead to approval and rejection of the project, respectively. Without the loss of generality, suppose \( M_A \cup M_R = [\underline{\theta}, \bar{\theta}] \). In a responsive equilibrium neither set is empty.

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Regardless of the message, the activist observes the actual decision \( a \) and strategically exits if and only if \( v(\theta, a) < p^*(1) \). Since the market maker does not observe \( a \) or \( m \), in equilibrium \( p^*(0) = \mathbb{E} [v(\theta, 1_{\theta \in \Theta}) | v(\theta, 1_{\theta \in \Theta}) > p^*(1)] \). If the manager chooses action \( a \) his expected utility conditional on message \( m \) is

\[
\omega [(\delta + (1 - \delta) \Pr [v(\theta, a) < p^*(1) | m]) p^*(1) + (1 - \delta) \Pr [v(\theta, a) > p^*(1) | m] p^*(0)] + \mathbb{E} [v(\theta + \beta, a) | m]
\]

Therefore, the manager chooses \( a = A \) if and only if

\[
\frac{\omega}{2} \frac{\rho}{1 + \rho} (\Pr [-\theta < p^*(1) | m] - \Pr [\theta < p^*(1) | m]) [p^*(0) - p^*(1)] + \mathbb{E} [\theta + \beta | m] \geq 0 \quad \text{(A16)}
\]

Note that if \( m \in \mathcal{M}_A \) then \( \theta \geq -p^*(1) \) with probability one. Otherwise, the activist is strictly better off sending message \( m \in \mathcal{M}_R \) and keeping her holdings in the firm. Therefore, for \( m \in \mathcal{M}_A \) the manager follows the recommendation and approves the project if and only if

\[
\frac{\omega}{2} \frac{\rho}{1 + \rho} \Pr [\theta > p^*(1) | m] [p^*(0) - p^*(1)] + \mathbb{E} [\theta + \beta | m] \geq 0 \quad \text{(A16a)}
\]

Similarly, if \( m \in \mathcal{M}_R \) then \( \theta \leq p^*(1) \) with probability one. Therefore, for \( m \in \mathcal{M}_R \) the manager follows the recommendation and rejects the project if and only if

\[
\frac{\omega}{2} \frac{\rho}{1 + \rho} \Pr [-\theta > p^*(1) | m] [p^*(0) - p^*(1)] - \mathbb{E} [\theta + \beta | m] \geq 0 \quad \text{(A16b)}
\]

Note that if both conditions (A16a) and (A16b) hold for some \( \omega_0 \geq 0 \) then since \( p^*(0) > p^*(1) \) they hold for any \( \omega > \omega_0 \). Therefore, a responsive equilibrium under \( \omega_0 \) (and not necessarily a threshold equilibrium) is also a responsive equilibrium under any \( \omega > \omega_0 \). By the same reasoning as above, the set of responsive equilibria decreases with \( \beta \).

Suppose \( \beta \leq \hat{\beta} \) and consider \( \tau \in [\bar{\tau}, \bar{\tau}] \) where the activist randomly chooses message in \( \mathcal{M}_A (\mathcal{M}_R) \) and sends it to the manager if \( \theta \in [\tau, \bar{\theta}] \) \( (\theta \notin [\tau, \bar{\theta}] \). Based on Lemma 6, \( \tau \in [-\pi(\tau), \pi(\tau)] \), \( p^*(1) = \pi(\tau) > 0 \), and \( p^*(0) = \mathbb{E} \|	heta\| | \theta > \pi(\tau) \). Therefore, the activist has (weak) incentives to follow the proposed communication strategy. Consider the manager’s incentives to follow threshold \( \tau \). The manager rejects the project upon \( m \in \mathcal{M}_R \) if and only if (A16b) holds, where the conditioning event is \( \theta < \tau \). Condition (A16b) holds if and only if \( \beta \leq \Psi(\tau; \omega, \rho) \).

Since \( \beta \leq \hat{\beta} \) condition (A16b) holds for any \( \tau \in [\bar{\tau}, \bar{\tau}] \). The manager approves the project upon \( m \in \mathcal{M}_A \) if and only if (A16a) holds, where the conditioning event is \( \theta \geq \tau \). If there is \( \tau_0 \in [\bar{\tau}, \bar{\tau}] \) such that \( \Psi(\tau_0; \omega, \rho) = \beta \) then for \( \tau = \tau_0 \) condition (A16a) holds as well. If there is no such \( \tau_0 \) then from the continuity of \( \Psi(\tau; \omega, \rho) \) in \( \tau \) and since \( \max_{\tau \in [\bar{\tau}, \bar{\tau}]} \{\Psi(\tau; \omega, \rho)\} \geq \beta \) then \( \min_{\tau \in [\bar{\tau}, \bar{\tau}]} \Psi(\tau; \omega, \rho) > \beta \) as well. It immediately follows that (A16a) holds for \( \tau = 0 \). Overall, if \( \beta \leq \hat{\beta} \) then a threshold equilibrium exists. This argument also proves that if \( \beta \leq \hat{\beta} \) then there is a responsive equilibrium with \( \tau = 0 \), the first best. Suppose \( \beta > \hat{\beta} \) and that on the contrary a threshold equilibrium exists. Per Lemma 6, it is necessary that \( \tau \in [\bar{\tau}, \bar{\tau}] \). Since the manager follows the recommendation to reject the project, \( \Psi(\tau; \omega, \rho) \geq \beta \). However, by assumption \( \beta > \max_{\tau \in [\bar{\tau}, \bar{\tau}]} \{\Psi(s; \omega, \rho)\} \) and hence a contradiction. A similar argument proves that if \( \beta > \hat{\beta} \) then the first best cannot be obtained.
Proof of Corollary 4. Note that if $\beta > z^{-1}(0)$ then without exit the only equilibrium is non-responsive and $\nabla_{Voice, NoExit} = \mathbb{E} [\theta] = \nabla_{NoVoice, NoExit}$. Since a non-responsive equilibrium always exists when voice is allowed, $\nabla_{Voice, Exit} \geq \nabla_{NoVoice, Exit}$. Since $\Psi(\tau; \omega, \rho)$ strictly increases with $\omega$, there is $\omega^* \in [0, \infty)$ such that the first best is obtained in equilibrium with voice and exit if and only if $\omega \geq \omega^*$. This implies that when $\beta > z^{-1}(0)$ then $\nabla_{Voice, Exit} > \nabla_{NoVoice, Exit}$ and voice and exit exhibit complementarity. When $\beta \leq z^{-1}(0)$ the first best is obtained with and without voice, that is, $\nabla_{Voice, NoExit} = \mathbb{E}[\theta] = \nabla_{Voice, Exit}$. Since $\nabla_{NoVoice, NoExit} = \mathbb{E}[\theta]$ then voice and exit exhibit complementarity if and only if $\nabla_{NoVoice, Exit}(\omega) \leq \mathbb{E}[\theta]$. It follows from Lemma 8, that $\nabla_{NoVoice, Exit}(\omega) > \mathbb{E}[\theta]$ if and only if $\mathbb{E}[\theta|\theta < 0] < -\beta < \mathbb{E}[\theta] < 0$, $\omega > \hat{\omega}$, and $\Pr[-\theta < p_{NR}] < \Pr[\theta < p_{NR}]$. Under those circumstances voice and exit exhibit substitution.

Proofs of Section V

Proof of Proposition 5. Following Proposition 1, regardless of $\gamma$ the most efficient threshold that can be obtained in any equilibrium is $\tau^* = \min \{0, -z(\beta)\}$. If $\beta \leq z^{-1}(-\tau)$ then $\tau^*$ is implementable in equilibrium when the activist is unbiased. Therefore, $\gamma^* = 0$ if and only if $\beta \leq z^{-1}(-\tau)$. Hereafter we assume $\beta > z^{-1}(-\tau)$ and focus on the smallest bias $\gamma$ that implements $\tau^* = -z(\beta)$ in equilibrium.

We start with the following observation. Let $\beta_0$ be a solution of $\mathbb{E}[v(\theta, 1_{\{\theta < -z(\beta)\}})] = 0$ if exists, and else define $\beta_0 = \infty$. Since $\mathbb{E}[v(\theta, 1_{\{\theta < -z(\beta)\}})]$ strictly decreases in $\beta$ and $\mathbb{E}[v(\theta, 1_{\{\theta < -z(\beta)\}})] \geq \mathbb{E}[\theta]$ for any $\beta > z^{-1}(0)$, it follows that $\beta_0 = \infty$ if $\mathbb{E}[\theta] \geq 0$ and else $\beta_0$ is unique by the intermediate value theorem. Note that $\beta_0 > z^{-1}(-\tau)$. According to Proposition 1 and Lemma 6, if $\beta = z^{-1}(-\tau)$ then the stock price conditional on exit in a responsive equilibrium is $\pi(\tau) = -\tau > 0$. Since $\pi(\tau) > 0$ implies $\mathbb{E}[v(\theta, 1_{\{\theta > \tau\}})] > 0$ for any $\tau$, then $\mathbb{E}[v(\theta, 1_{\{\theta < \gamma\}})] > 0$ which proves $\beta_0 > z^{-1}(-\tau)$.

Let $p_{\beta, \gamma}$ be the price conditional on exit in this equilibrium with threshold $\tau = -z(\beta)$. Note that $\gamma = \{\gamma : v(\theta + \gamma, 1_{\{\theta < -z(\beta)\}}) \leq p_{\beta, \gamma}\}$. We argue that $\gamma = \emptyset$ if and only if $p_{\beta, \gamma} \leq 0$. If $p_{\beta, \gamma} > 0$ then $\Pr[\theta \in \gamma] > 0$. If $p_{\beta, \gamma} \leq 0$ the activist has strict incentives to send message $m \in M_R$ if $\theta < -\gamma$ and $m \in M_A$ otherwise. Since the proposed equilibrium is responsive, neither set is empty, and the manager will follow this recommendation. This strategy will guarantee the activist a long term value that satisfies $\Pr[v(\theta + \gamma, 1_{\{\theta < -z(\beta)\}}) > 0] = 1$. Therefore, $\gamma = \emptyset$.

We argue that a responsive equilibrium with $\gamma = \emptyset$ and $\tau = -z(\beta)$ exists if and only if $\gamma = z(\beta)$ and $\beta \geq \beta_0$. Suppose such equilibrium exists. The market maker’s pricing strategy must be consistent with $\gamma = \emptyset$. Therefore, the fair price of the stock is $p_{\beta, \gamma} = \mathbb{E}[v(\theta, 1_{\{\theta < -z(\beta)\}})]$. Since $\gamma = \emptyset$ implies $p_{\beta, \gamma} \leq 0$ it is required that $\mathbb{E}[v(\theta, 1_{\{\theta < -z(\beta)\}})] \leq 0$ and hence $\beta \geq \beta_0$. Moreover, note that in any responsive equilibrium without exit, the activist has strictly incentives to implement threshold $-\gamma$. Therefore, it must be $\gamma = z(\beta)$. By construction, if $\gamma = z(\beta)$ and $\beta \geq \beta_0$ then there is an equilibrium with $\gamma = \emptyset$ and $\tau = -z(\beta)$. We conclude that if $\gamma = \emptyset$ and $\tau = -z(\beta)$ then $\gamma = z(\beta) \geq z(\beta_0)$.

We argue that if $\tau = -z(\beta)$ and $\gamma \neq \emptyset$ in a responsive equilibrium then $\gamma = [-p_{\beta, \gamma} - \gamma, p_{\beta, \gamma} - \gamma]$. Note that $\gamma \neq \emptyset$ implies $p_{\beta, \gamma} > 0$. Moreover, the biased activist has incentives to follow a
communication strategy that implements threshold \(-z(\beta)\) if and only if
\[
-z(\beta) \in [-p_{\beta,\gamma} - \gamma, p_{\beta,\gamma} - \gamma]
\] (A17)

Noting that \(v(\theta + \gamma, 1_{\theta>z(\beta)}) \leq p\) if and only if \(\theta \in [-p - \gamma, p - \gamma]\) completes the argument. Consider a responsive equilibrium where \(\tau = -z(\beta)\) and \(\Upsilon \neq \emptyset\). Given (A17) the price \(p_{\beta,\gamma}\) must be strictly positive and the solution of
\[
p = \frac{\mathbb{E}[v(\theta, 1_{\theta>z(\beta)})] + \rho \Pr[\theta \in [-p - \gamma, p - \gamma]] \mathbb{E}[v(\theta, 1_{\theta>z(\beta)}) | \theta \in [-p - \gamma, p - \gamma]]}{1 + \rho \Pr[\theta \in [-p - \gamma, p - \gamma]]}
\] (A18)

In what follows we prove that there is \(p_0 > 0\) and \(\gamma_0\) that satisfy \(p_0 + \gamma_0 = z(\beta)\) and (A18). Let \(\gamma = z(\beta) - p\) then for any \(p > 0\) the r.h.s of (A18) becomes
\[
\zeta(p, \beta) = \frac{\mathbb{E}[v(\theta, 1_{\theta>z(\beta)})] + \rho \Pr[\theta \in [-z(\beta), 2p - z(\beta)] \mathbb{E}[\theta | \theta \in [-z(\beta), 2p - z(\beta)]]}{1 + \rho \Pr[\theta \in [-z(\beta), 2p - z(\beta)]]}
\] (A19)

and (A18) implies \(p = \zeta(p, \beta)\). We note several properties of \(\zeta(p, \beta)\) that follows from its definition. First, \(\lim_{p \to 0} \zeta(p, \beta) = \mathbb{E}[v(\theta, 1_{\theta>z(\beta)})]\). Second, \(\lim_{p \to \infty} \zeta(p, \beta) = \mathbb{E}[v(\theta, 1_{\theta>z(\beta)})] + \rho \Pr[\theta > z(\beta)] \mathbb{E}[\theta | \theta > z(\beta)]\). Third, \(\zeta(p, \beta)\) is continuous in \(p\) and \(\beta\). Forth, \(\zeta(p, \beta)\) decreases in \(p\) if and only if \(2p - z(\beta) < \mathbb{E}[v(\theta, 1_{\theta>z(\beta)})]\) if and only if \(p < p_{\min}(\beta) \equiv z(\beta) + \frac{\mathbb{E}[v(\theta, 1_{\theta>z(\beta)})]}{2}\). This means that \(\zeta(p, \beta)\) has a unique minimum obtained at \(p_{\min}(\beta)\). Since \(v(\theta, 1_{\theta>z(\beta)}) > -z(\beta)\) then \(p_{\min}(\beta) > 0\). We conclude that if \(\beta < \beta_0\) then \(\mathbb{E}[v(\theta, 1_{\theta>z(\beta)})] > 0\) and hence there exists a strictly positive solution to \(p = \zeta(p, \beta)\). If \(\beta \geq \beta_0\) then \(\mathbb{E}[\theta] < 0\). Since \(\mathbb{E}[\theta | \theta < -z(\beta)] < \mathbb{E}[\theta | \theta > -z(\beta)]\), this implies \(\mathbb{E}[\theta | \theta < -z(\beta)] < 0\). Note that \(\beta \geq \beta_0\) also implies \(\mathbb{E}[v(\theta, 1_{\theta>z(\beta)})] < 0\). Therefore \(\mathbb{E}[\theta | \theta > -z(\beta)] < \frac{\mathbb{E}[\theta | \theta < -z(\beta)]}{\mathbb{E}[\theta | \theta > -z(\beta)]}\). Overall, since \(\mathbb{E}[\theta | \theta < -z(\beta)] < 0\) and \(\frac{\mathbb{E}[\theta | \theta < -z(\beta)]}{\mathbb{E}[\theta | \theta > -z(\beta)]} > 0\) then \(\mathbb{E}[\theta | \theta > -z(\beta)] < 0\) as well. Last, \(\lim_{p \to \infty} \zeta(p, \beta) < \mathbb{E}[v(\theta, 1_{\theta>z(\beta)})]\) if and only if \(\mathbb{E}[\theta | \theta > -z(\beta)] < \mathbb{E}[v(\theta, 1_{\theta>z(\beta)})]\), which holds if and only if \(0 > \mathbb{E}[\theta | \theta > -z(\beta)] + \mathbb{E}[\theta | \theta < -z(\beta)]\). Since both components are negative, and since \(\zeta(p, \beta)\) has a unique minimum, that \(\zeta(p, \beta) = p\) has no positive solution if \(\beta \geq \beta_0\). This immediately implies that if \(\beta \geq \beta_0\) then \(\Upsilon = \emptyset\) and hence \(\gamma^* = z(\beta)\). We conclude that if \(\beta < \beta_0\) then the equation \(p = \zeta(p, \beta)\) has a strictly positive solution. We let \(\gamma_0 = \min_{p > 0, p = \zeta(p, \beta)} |z(\beta) - p|\). That is, if there are multiple solutions we choose the one that minimizes \(|z(\beta) - p|\).

We argue that if \(\beta < \beta_0\) then \(\gamma^* = \gamma_0\). We let \(\xi(\gamma, p, \beta)\) be the r.h.s of (A.18). Note that \(\xi(z(\beta) - p, p, \beta) = \zeta(p, \beta)\) for any \(p > 0\) and \(\beta\) in this range. Suppose on the contrary there is \(\gamma_1\) such that \(\gamma_1 + p_1 > z(\beta), |\gamma_1| < |\gamma_0|\), and \(p_1 = \xi(\gamma_1, p_1, \beta)\). We note several properties of \(\xi(\gamma, p, \beta)\) that follows directly from its definition. First, \(\lim_{\gamma \to 0} \xi(\gamma, p, \beta) = \mathbb{E}[v(\theta, 1_{\theta>z(\beta)})]\). Second, \(\xi(\gamma, p, \beta)\) is continuous in \(p \geq 0, \gamma\), and \(\beta \geq 0\) (one can see that by a direct calculation of \(\mathbb{E}[v(\theta, 1_{\theta>z(\beta)}) | \theta \in [-p - \gamma, p - \gamma]]\)). Thus, from the intermediate value theorem, if \(\mathbb{E}[v(\theta, 1_{\theta>z(\beta)}) | \theta \in [-p - \gamma, p - \gamma]] \geq 0\) then for any \(\beta\) and \(\gamma\) there is a positive solution to \(p = \xi(\gamma, p, \beta)\). Let us define \(\kappa(\gamma) \equiv \gamma + p(\gamma)\) where \(p(\gamma)\) is the highest solution of \(p = \xi(\gamma, p, \beta)\).
Note that $\kappa(\gamma)$ is continuous in $\gamma$ and $\kappa(\gamma_1) = \gamma_1 + p_1 > z(\beta)$. From Proposition 1 we know that $\kappa(0) < z(\beta)$ for $\beta > z^{-1}(-\tau)$. Thus, from the intermediate value theorem there is $\gamma_2$ such that $\kappa(\gamma_2) = z(\beta)$, $|\gamma_2| < |\gamma_1| < |\gamma_0|$, and $p_2 = \xi(\gamma_2, p_2, \beta)$. This contradicts the fact that $\gamma_0$ is the lowest absolute value that satisfies $p + \gamma = z(\beta)$ and $p = \xi(\gamma, p, \beta)$.

Let $p_\beta^* = \arg\min_{p > 0, p = \zeta(p, \beta)} |z(\beta) - p|$. We argue if $\beta \geq z^{-1}(-\tau)$ then $\frac{\partial p_\beta^*}{\partial \beta} < 0$. Since $p_\beta^*$ satisfies $p = \zeta(p, \beta)$, the implicit function theorem implies $\frac{\partial p_\beta^*}{\partial \beta} = -\frac{\partial \zeta(p, \beta)}{\partial p} |_{p = p_\beta^*}$ and (A19) implies that $\frac{\partial \zeta(p, \beta)}{\partial p} |_{p = p_\beta^*} < 0$ for any $p$. Thus, $\frac{\partial p_\beta^*}{\partial \beta} < 0$ if and only if $\frac{\partial \zeta(p, \beta)}{\partial p} |_{p = p_\beta^*} < 1$. Note that

$$\frac{\partial \zeta(p, \beta)}{\partial p} |_{p = p_\beta^*} = \frac{2\rho f(2p_\beta^* - z(\beta))}{1 + f(2p_\beta^* - z(\beta))} (p_\beta^* - z(\beta)).$$

This implies that if there is $\beta'$ for which $p_{\beta'}^* \leq z(\beta')$ then since $z(\beta)$ increases in $\beta$, for any $\beta > \beta'$ we have $p_{\beta}^* \leq z(\beta)$. For $\beta = z^{-1}(-\tau)$ we know from Proposition 1 that $p_{\beta}^*_{z^{-1}(-\tau)} = z(z^{-1}(-\tau)) = -\tau$ and hence for any $\beta > z^{-1}(-\tau)$ we have $p_{\beta}^* \leq z(\beta)$ and $\frac{\partial p_{\beta}^*}{\partial \beta} < 0$. This implies that $\gamma^*$ increases with $\beta$.

Last, we show that when $\beta > z^{-1}(-\tau)$ then $\gamma^*$ strictly increases in $\rho$. If $\beta \geq \beta_0$ then $\gamma^* = z(\beta)$ and is independent of $\rho$. Suppose $\beta \in (z^{-1}(-\tau), \beta_0)$. By the implicit function theorem,

$$\frac{\partial p^*(\rho)}{\partial \rho} = -\frac{\partial \zeta(p, \beta)}{\partial p} |_{p = p_\beta^*} < 0$$

and $\frac{\partial \zeta(p, \beta)}{\partial \rho} < 0$. Therefore, $\frac{\partial p^*(\rho)}{\partial \rho} < 0 \iff \frac{\partial \zeta(p, \beta)}{\partial \rho} |_{p = p_\beta^*} < 1$. But we have shown above that $\frac{\partial \zeta(p, \beta)}{\partial \rho} |_{p = p_\beta^*} < 0$ for $\beta > z^{-1}(-\tau)$ which completes the proof.