DOES A RISE IN MAXIMAL FINES INCREASE OR DECREASE THE OPTIMAL LEVEL OF DETERRENCE?

Avraham D. Tabbach  
Tel-Aviv University

The economic literature on crime and law enforcement shows that the optimal level of deterrence always increases when maximal fines rise. This paper shows that this view may be incorrect. In particular, if the gains from crime can be disgorged, as is usually the case in reality, then increasing the maximal fine may reduce the optimal level of deterrence. This may happen if offenders’ wealth is less than the monetary value of the harm that offenders cause.

1. INTRODUCTION

Many private and business activities generate external harm. For example, oil refineries generate air pollution, overloaded trucks damage highways, messenger services obstruct traffic as they double park, and individuals sometimes litter the streets or speed while driving. To control these harms, the government usually expands resources to apprehend offenders and impose monetary sanctions on them. Since the pioneering work of Becker (1968), there has been an extensive literature exploring the features of optimal law enforcement, in particular of the optimal probability and magnitude of fines and the optimal level of deterrence.1 This literature gives rise to several basic results. It shows that, under certain conditions, fines should be set to their maximum level, traditionally interpreted as offenders’ wealth, and that the expected fine should be less than the external harm resulting from the offense, so that some degree of under-deterrence prevails. The explanations of these results are now well-known. If fines, which are assumed to be a socially costless transfer, were not maximal, any level of deterrence,

---

1 See, for example, Garoupa (1997), Polinsky and Shavell (1999, 2007).
including the optimal level, could be achieved at lower costs by increasing the fine to its maximum level and reducing the probability of punishment proportionally. This is Becker's argument. Similarly, if the expected fine were equal to the harm, it would be possible to lower the probability of punishment and save enforcement costs without affecting much deterrence, because the marginal offenders would derive gains that were approximately equal to the harm and therefore impose zero net social harm.2 This is Polinsky and Shavell's point. These results have been qualified in the literature on various grounds, including marginal deterrence, risk aversion, avoidance efforts, and more (see, for example, Garoupa, 1997, Polinsky and Shavell, 1999, 2007).

The literature on crime and law enforcement also explores the consequences of relaxing the fine constraint, for example, imagining that the wealth of potential offenders has been increased. This literature demonstrates that as the maximal fine rises, the optimal fine should rise as well, but the optimal probability of punishment may either fall or rise depending on the degree of under-deterrence (Garoupa, 2001). This latter, less familiar result can be explained as follows. If the level of under-deterrence is substantial, greater deterrence can and should be achieved not only by increasing the optimal fine, but also by increasing the optimal probability of punishment. The additional gains from increasing the probability of punishment can now be cost-justified, because as the optimal fine increases, the deterrent value of enforcement expenditures increases as well. To illustrate, when the optimal fine is equal to 400, any 1% increase in the probability of punishment increases the level of deterrence by 4 (1% times 400), while if the optimal fine is equal to

---

2 Clearly, if the expected fine were greater than the harm, reducing the probability of punishment would increase social welfare because it would also reduce over-deterrence.
500, any 1% increase in the probability of punishment has an impact on deterrence of 5
(1% times 500).³

Finally, the literature on crime and law enforcement shows that as the maximal fine
increases, the optimal level of deterrence always increases and approaches first-best
deterrence, that is, a deterrence level which induces offenders to commit harmful acts if
and only if their gains exceed the harm (Garoupa 2001). The explanation is this: as the
maximal fine rises, the pre-fine-increase optimal level of deterrence can be achieved by
increasing the fine and reducing the probability of punishment proportionally. However,
since the deterrent value of enforcement expenditures is greater, it is never socially
desirable to reduce the probability of punishment all the way down so as to maintain the
pre-fine-increase optimal level of deterrence. Instead, since some degree of under-
deterrence prevails, it is socially desirable to reduce the degree of under-deterrence and
achieve greater deterrence. This result is so intuitive that it is sometimes regarded as a
"folk theorem" (Garoupa 1997).

This paper shows that this "folk theorem" is not generally correct. In particular, if the
gains from crime can be disgorged, as is the case in many real situations, then increasing
the maximal fine may paradoxically decrease the optimal level of deterrence and
consequently increase the optimal level of crime. This may happen if the maximal fine,
which for clarity will be understood not to include the disgorgement of gains, is less than
the monetary value of the harm resulting from the offense.

The explanation for this surprising result, which is discussed in greater detail in Section
3, lies with the impact of an increase in the maximal fine on the deterrent value of
³ If the level of under-deterrence is not substantial, then the optimal probability of punishment should
decrease as maximal fines rise. The reason is that much deterrence is gained by the mere fact that the
optimal fines are increased. See further discussion in Section 3.2.
enforcement expenditures after the fine is increased and the probability of punishment reduced to maintain the pre-fine-increase optimal level of deterrence. As noted, in the standard law enforcement model, in which offenders' gains are not disgorged, the deterrent value of any 1% increase in the probability of punishment is simply 1% times the level of the optimal fine. In contrast, if offenders' gains from harmful acts can be disgorged, the deterrent value of enforcement expenditures is markedly different. Not only is it greater than 1% times the optimal fine (for the obvious reason that the probability of punishment applies not only to fines but also to the disgorgement of gains) but it is actually an increasing convex function of the probability of punishment itself. So, for example, if the optimal fine is 400 and the probability of punishment is 1/3, the deterrent value of enforcement expenditures is 9; if the probability of punishment is 1/2, the deterrent value is 16; and if the probability of punishment is 2/3, then the deterrent value is 36. Compare these values with the deterrent value in the standard model which is 4 regardless of the probability of punishment.

Now, as the maximal fine increases, the pre-fine-increase optimal level of deterrence can be achieved by increasing the optimal fine to the new maximum level and reducing the probability of punishment appropriately. The increase in the optimal fine increases the deterrent value of enforcement expenditures. In contrast, the decrease in the probability of punishment reduces the deterrent value of enforcement expenditures. If the latter effect is greater than the former effect, then the deterrent value of enforcement expenditures at the pre-fine-increase optimal level of deterrence will be reduced. This implies that it would be socially desirable to lower the enforcement expenditures, so as to achieve a lower level of deterrence.
It should be observed that an increase in the maximal fine unequivocally increases social welfare or, equivalently, reduces the social costs of crime and law enforcement. The reason is that as the maximal fine rises, any level of deterrence can be obtained at lower enforcement costs. However, as this paper argues, it might nevertheless be socially desirable to achieve less deterrence, because the possible savings in enforcement costs may outweigh the additional social harm associated with less deterrence and more crime.

This paper does not argue that it is common that the optimal level of deterrence falls as maximal fines rise. However, the possible positive relationship between maximal fines and optimal crime levels should not be dismissed for several reasons. First, in many situations, offenders' gains from harmful acts take a monetary or monetary-like form and therefore can be confiscated or disgorged. For example, the gains from all business activities generating external harm are monetary in nature and therefore can be subject to disgorgement. This is in contrast to the standard law enforcement model which assumes that the benefits from harmful acts are non-monetary in nature and therefore cannot be disgorged. Second, as discussed extensively by Bowles et al. (2000, 2005), legislation enabling courts to confiscate or remove illegal gains has grown rapidly across a wide range of countries within both civil and common law systems; thus, the possibility to disgorge illegal gains is now a standard feature of the law. Third, the possibility that a rise in maximal fines would lead to lower levels of optimal deterrence might seem unrealistic because, as pointed out, it requires that maximal fines (offenders' wealth) would be less than the harm caused by the offense. In reality however, this might be a very common situation. Usually, if offenders' wealth is substantially less than the value of the harm, a substantial degree of under-deterrence will result, unless other forms of
punishment such as imprisonment are employed. Indeed, under circumstances of low wealth levels in comparison to harm, the use of imprisonment is generally justified (see Shavell, 1985, Posner, 1985). However, as will be explained in Section 2, if the gains from harmful acts can be disgorged, first-best or approximately first-best behavior can be achieved even if offenders' wealth is substantially less than the harm.

This paper analyzes a simple model of crime and law enforcement which assumes that the gains from harmful acts are monetary or monetary-like in nature and therefore can be disgorged. The model follows a model proposed by Bowles et al. (2000). However, Bowles et al. (2000) focus on deriving the optimal fine and the optimal level of disgorgement of gains in their analysis. They neither explore the optimal probability of punishment or level of deterrence, nor analyze how these variables change when the maximal fine rises. This paper provides such an analysis and shows that, aside from the ambiguous, counter-intuitive effect a rise in maximal fines has on the optimal level of deterrence, the other canonical results of the standard law enforcement model qualitatively carry over to the present model. For example, even if the gains from crime can be disgorged, optimal law enforcement is characterized by some degree of under-deterrence, that is, by a deterrence level which induces certain offenders to commit harmful acts even though the gains they derive are less than the harm done.4

Finally, this paper also compares the optimal law enforcement schemes with and without disgorgement of gains, a task which is not fully conducted by Bowles et al. (2000). It shows, for example, that the possibility of disgorging offenders' gains not only increases social welfare, but also unambiguously increases the optimal level of

---

4 In the standard model, under-deterrence is usually equivalent to the statement that the expected fine (for all individuals) is less than the harm. In the present model, as will be explained in Section 2, the expected punishment is different for different individuals. Therefore, such a statement is meaningless or unimportant.
deterrence, bringing it closer to first-best deterrence. Interestingly then, fines and
disgorgement of gains affect optimal deterrence differently: When the level of the
maximal fine increases, the optimal level of deterrence may either increase or decrease,
while if a greater fraction of offenders' illegal gains can be disgorged, the optimal level of
deterrence is always higher.

The paper proceeds as follows. Section 2 develops a simple model of crime and law
enforcement incorporating the option to disgorge the gains that offenders derive from
crime, and characterizes the optimal law enforcement scheme. Section 3 then analyzes
how optimal law enforcement should change as the maximal fine rises and derives the
main results of this paper. Section 4 compares the optimal law enforcement schemes with
and without the possibility to disgorge offenders' gains. This section shows that as the
fraction of disgorging illegal gains increases, the level of optimal deterrence always
increases. Section 5 summarizes the analysis and the Appendix provides formal proofs
for the results.

2. THE MODEL

Risk-neutral individuals (or firms) contemplate whether or not to commit a harmful act,
causing harm of \( h \). Each individual obtains monetary or monetary-like gains \( g \), which
are assumed to be randomly chosen from a continuous uniform distribution function with
a density function \( b(g) = 1 \) and a cumulative distribution function \( B(g) = g \) on the
support \([0,1]\). Assume that \( h < 1 \), so that some harmful acts are socially desirable in the
sense that the gains for certain individuals exceed the harm. These assumptions however are not crucial for our qualitative results.5

If an individual does commit the harmful act, he will face some probability of being fined and will also risk losing his illicit gains. For clarity, let us define fines as not including the disgorgement of gains. The maximum feasible fine $f$, so defined, is constrained to $f_{\text{max}}$. Traditionally, $f_{\text{max}}$ is interpreted as the level of wealth of individuals above a subsistence level. Alternatively, it can be interpreted as the maximal fine which is allowed by law, for example, for constitutional considerations. Under this interpretation, $f_{\text{max}}$ serves as an exogenous legal or constitutional constraint on the level of fines.

In addition to being fined, offenders risk losing the gains they derive from the harmful act. For clarity, let us use the term "punishment" to mean both the fine and the disgorgement of the illicit gains. Assume then that "punishment", so defined, must be uniform for all potential offenders in the sense that it takes the form $f + \eta g$, where $f \leq f_{\text{max}}$ and $\eta [0,1]$ is the fraction of the illicit gains that can be confiscated. This will allow the existence of a unique solution to the social problem. Since $g$ is a random variable taking values in the range 0 to 1, punishment will differ among individuals, unless $\eta = 0$; and the maximum feasible punishment for each individual must be less than $f_{\text{max}} + g$. As usual, fines are assumed to be a socially costless transfer: they entail no administration costs, and the costs imposed on offenders are completely offset by the

---

5 The working-paper version of this paper uses a generalized form of the distribution of benefits and derives qualitatively similar results to those derived here (see Tabbach, 2008).
revenues obtained by the government. The same, it is assumed, applies to the
disgorgement of the illicit gains.\(^6\)

The costs of apprehending offenders with probability \(p\) are assumed to be given by the
function \(c(p) = cp\), so that total costs are proportional to the probability of punishment,
and marginal costs are constant and equal to \(c\).\(^7\)

Social welfare or, equivalently, the social cost of crime and law enforcement, is the sum
of gains obtained by those individuals who commit the harmful act, less the harm done,
less enforcement costs. To determine social welfare, observe that individuals will commit
the harmful act if and only if the gains they derive from doing so exceed the expected
punishment they face, that is, iff

\[
1. g \geq p(f + \eta g),
\]

which means that the level of deterrence is determined by:

\[
2. g = \frac{p}{1 - np} f.
\]

Hence, social welfare can be formulated as:

\[
3. SW = \int_{\frac{1}{g}}^{1} (g - h)dg - cp.
\]

---

\(^6\) Observe that these two assumptions need not be strictly so with respect to the disgorgement of gains. First, while fines are socially costless to administer, disgorgement of gains may nevertheless be socially costly (see the treatment in Bowles et al. (2000). Second, sometimes the loss to offenders is not offset by the gains to the government from disgorgement. For example, if the illicit gains are monetary-like in nature and take the form of illegal goods such as drugs, confiscation has no value in the hands of the government (the police will take steps to destroy the illegal goods). However, in many cases, for example, with respect to stolen goods, the assumptions in the text will be valid.

\(^7\) The consequences of assuming that \(c(p)\) is a strictly convex function are discussed briefly in footnotes 13, 14, and 18.
The social problem is to choose the probability of punishment \( p \), the magnitude of fines \( f \), and the fraction of disgorgement of illegal gains \( \eta \) that maximize (3). The solution to this problem is characterized in the following proposition.

**Proposition 1:** (1) The optimal fine and level of disgorgement are maximal, \( f^* = f_{\text{max}} \) and \( \eta^* = 1 \), so the optimal punishment for each individual is maximal and equal to \( f_{\text{max}} + g \). (2) The optimal probability of punishment, assuming it is positive, satisfies the first order condition \( c p f g h = -2 \max f^* \), where \( g^* \), is the optimal level of deterrence, and it is such that there is some degree of under-deterrence, that is, \( g^* < h \). This also implies that \( p^* < \frac{h}{f_{\text{max}} + h} \).

**Proof:** See Appendix A.

The explanation of Proposition 1 is as follows. The fine should be set at its maximum level, because otherwise it would be possible to increase the fine and reduce the probability of punishment such that the same level of deterrence, including the optimal level of deterrence, would be obtained at lower enforcement costs. The same argument applies, as Bowles et al. (2000) show, with respect to the level of disgorgement, which should be maximal as well.8

---

8 A sketch of the proof that \( \eta^* = 1 \) is as follows. Suppose to the contrary that \( \eta^* < 1 \), \( f^* = f_{\text{max}} \), and \( p^* > 0 \). The optimal level of deterrence is thus given by \( g^* = \frac{p^*}{1 - \eta^* p^* f_{\text{max}}} \) (equation 2). Increase \( \eta \) to \( \eta_l \) and change \( p \) to \( p_l = \frac{p^*}{1 + p^* (\eta_l - \eta^*)} \), so that deterrence remains the same, \( \frac{p_l}{1 - \eta_l p_l} f_{\text{max}} = g^* \). Since \( \eta_l > \eta^* \), then \( p_l < p^* \), implying that enforcement costs are saved. Therefore, social welfare increases, and \( \eta^* < 1 \) could not be optimal.
The optimality of some degree of under-deterrence (Proposition 1(2)), which is not discussed or proven by Bowles et al. (2000), stems from the equality between the marginal benefits and costs of increasing the probability of punishment at the optimum. The marginal costs are of course positive for any $p > 0$, so the marginal benefits must be positive as well. But the marginal benefits of increasing the probability of punishment are the benefits resulting from greater deterrence, that is, from the reduced number of harmful acts committed. If these marginal benefits are positive, then it must be that at the optimum some socially undesirable harmful acts are committed. Put differently, at the optimum, some offenders must commit the harmful act even though the gains they derive are less than the harm. This precisely means that the optimal solution is characterized by some degree of under-deterrence. Formally, $p^*$ (throughout asterisks denote optimal choices), assuming it is positive, should satisfy the first order condition

\[(4) \quad (h - g^*) \frac{f_{\max}}{(1 - p^*)} = c, \]

where

\[(5) \quad g^* = \frac{p^*}{1 - p^*} f_{\max} \]

is the gain-threshold which determines the optimal level of deterrence, obtained after substituting $f^* = f_{\max}$ and $\eta^* = 1$ in equation 2. Since the RHS of (4), which reflects the marginal costs of $p$, is positive, $c > 0$, it follows that the LHS of (4), which reflects the marginal benefits of $p$, should be positive as well, so that $h - g^* > 0$. This implies that:

\[(6) \quad g^* < h. \]

Precisely the condition that means that under-deterrence is optimal. Moreover, substituting (5) into (6), one obtains that:
Proposition 1 (2) can be viewed as a generalization of the standard result in the economic literature regarding the optimality of under-deterrence.\footnote{As is well known, standard optimal law enforcement models which disregard the possibility of confiscating the offenders’ gains (since they are non-monetary in nature) show that the optimal expected fine is less than harm, \( p^* f^* < h \), so that under-deterrence prevails, and also that the optimal probability of punishment is less than the harm divided by offenders’ wealth, \( p^* < \frac{h}{f_{\max}} \). See, for example, Garoupa (1997), Polinsky and Shavell (1999, 2007).}

Before proceeding, let us show how first-best behavior (in the sense that offenders will commit the harmful act if and only if the gains from doing so exceed the harm) can be achieved in the present model at the lowest costs. This will help us understand better the role that the disgorgement of gains plays in law enforcement and the differences between the present and the standard law enforcement models. In addition, it will facilitate understanding of the conditions under which a rise in the maximal fine leads to less deterrence. Punishment should be maximal, that is, \( f_{\max} + g \), for exactly the same reason discussed above. The probability of punishment, in contrast, should be set at \( \frac{h}{f_{\max} + h} \), so that expected punishment is given by:

\[
\frac{f_{\max} + g}{f_{\max} + h} h.
\]

This expected punishment would guarantee, at the lowest costs, that individuals for whom \( g \leq h \) are deterred, while individuals for whom \( g > h \) are not deterred. In contrast to the standard law enforcement model, in which the expected punishment is invariant to offenders' gains, in the present model the expected punishment increases with the gains
that offenders derive from the harmful act. It is less than the harm for individuals for whom $g < h$, and more than the harm if the reverse is true. Nevertheless, the expected punishment increases with the gains from the harmful act, but at a lower rate (i.e., less steeply) than the increase of the gains themselves, as illustrated in Figure 1.\textsuperscript{10} Therefore, first-best behavior can be obtained if the expected punishment is equal to the harm for individuals for whom $g = h$, which is achieved by setting the expected punishment according to (8). This too is illustrated in Figure 1.

\textbf{Figure 1: How First-Best Deterrence Can Be Achieved}

Observe that in contrast to the standard law enforcement model in which first-best or approximately first-best behavior cannot be achieved if $f_{\text{max}}$ is substantially less than the

\textsuperscript{10} The gains from the harmful act are increased at a rate of 1 with the gains, while the expected punishment is increased at a rate $\frac{h}{f_{\text{max}} + h}$ with the gains, which is less than 1.
harm, in the present model first-best or approximately first-best behavior can be obtained in similar circumstances. For example, if the maximal fine is 400 and the harm is 800, first-best behavior can be induced by setting the probability of punishment at $\frac{2}{3}$ ($\frac{2}{3} = 800/400$, equation 2); if the maximal fine is 400 and the harm is 1600, the probability of punishment which will induce first-best behavior is $\frac{4}{5}$ ($\frac{4}{5} = 1600/400$). The possibility to achieve first-best or approximately first-best behavior even if the maximal fine is very low in comparison to the harm is due precisely to the fact that offenders' gains from the harmful act can be disgorged.

Observe finally that if offenders' wealth is greater than the harm, then first-best behavior can be achieved with a probability of punishment which is less than 1/2. Moreover, the optimal probability of punishment that characterizes optimal law enforcement in this case will be necessarily less than 1/2 (see equation 7). On the other hand, if offenders' wealth is less than the harm, then first-best behavior is obtained with a probability of punishment which must be greater than 1/2. However, the optimal probability of punishment that characterizes optimal law enforcement may be either higher or lower than 1/2 (see again equation 7), depending, among other things, on the costs of enforcement. These observations will play a major role in the following analysis.

---

11 Appendix A contains the second-order condition which imposes some restrictions on the optimal values of $p^*$.
3. MAXIMAL FINES RISE

Suppose that the maximal fine rises. This can be interpreted as an increase in the wealth of potential offenders, or, perhaps more realistically, as relaxing a legal or constitutional constraint on the level of the maximal fine. How will optimal law enforcement change?

The optimal fine level should be clearly increased and set equal to the new maximal level, since maximal fines are optimal in the present model. It is also quite obvious that social welfare increases as the maximal fine rises, because any level of deterrence can be achieved by increasing the punishment and reducing the probability of punishment appropriately so as to save enforcement costs. The interesting questions are therefore how an increase in the maximal fine affects the optimal level of deterrence and how it alters the optimal probability of punishment. Let us analyze these questions in turn.

3.1. The Optimal Level of Deterrence

**Proposition 2:** As the maximal fine increases, the optimal level of deterrence increases (decreases) if the optimal probability of punishment is less (greater) than 1/2.

*Proof:* See Appendix B

Proposition 2 states that, contrary to commonly held views, the optimal level of deterrence may decrease as the maximal fine rises. To understand this surprising result, consider the consequences of increasing the maximal and, accordingly, optimal fine to $f_1$ ($> f_{\text{max}}$) and reducing the probability of punishment to $p_1 (< p^*)$, so as to maintain the
pre-fine-increase optimal level of deterrence $g^*$. Recall that prior to the increase in the maximal and optimal fines, the optimal level of deterrence was determined by the equality between the marginal costs and benefits of increasing the probability of punishment:

$$ (h - g^*) \frac{\max f}{(1 - p^*)^2} = c. \quad (4) $$

At the optimal level of deterrence, the costs of increasing $p$ slightly, say, by 1%, were equal to the benefits from so doing in terms of greater deterrence and, accordingly, reduced net harm. The critical question is how these marginal costs and benefits change after increasing the optimal fine and reducing the probability of punishment appropriately. More precisely, the critical question is whether

$$ (h - g^*) \frac{f_i}{(1 - p_i)^2} \geq c. \quad (9) $$

If the marginal benefits of increasing the probability of punishment (the LHS of 9) are now greater than the marginal costs (the RHS of 9), then greater deterrence would be socially desirable. If, however, the reverse is true, then less deterrence would be socially optimal.

Observe that the marginal costs of increasing the probability of punishment are unaffected (the RHS of 9 is equal to the RHS of 4). This is due to our simplified assumption that the costs of enforcement are proportional to the probability of punishment. In addition, the gains from greater deterrence, for example, from a 1% increase in the probability of punishment, would be

---

12 This means that $p_i$ should satisfy $\frac{p_if_i}{1 - p_i} = \frac{p^*f^*}{1 - p^*} = g^*$.

13 If marginal costs of enforcement expenditures are increasing, then the reduction in the probability of punishment implies that the marginal costs are now lower.
increase in the level of deterrence, in terms of reduced net social harm, are also the same \((h-g^*)\). This is due to the fact that by construction the level of deterrence is held at its pre-fine-increase level. The critical question is therefore whether

\[
(10) \quad \frac{f_1}{(1-p_i)^2} \gg \frac{f_{\text{max}}}{(1-p^*)^2}.
\]

The terms \(\frac{f_1}{(1-p_i)^2}\) and \(\frac{f_{\text{max}}}{(1-p^*)^2}\) reflect the impact of a slight increase in the probability of punishment on the level of deterrence or, in Garoupa's (2001) words, the *deterrent value of enforcement expenditures*. This deterrent value (for any combination of \(p\) and \(f\)) is given by:

\[
(11) \quad \frac{\partial g}{\partial p} = \frac{f}{(1-p)^2}.
\]

As is evident in equation (11), an increase in the level of the maximal and, consequently, optimal fines increases the deterrent value of enforcement expenditures for any given probability of punishment. This should be very intuitive. However, as equation (11) also reveals, the deterrent value of enforcement expenditures also depends on the level of \(p\) itself. Indeed, the deterrent value of enforcement expenditures increases as the probability of punishment increases, and does so in increasing rates. To illustrate, if \(f = 400\), then the deterrent value of a 1% increase in the probability of punishment is only 4.04 for \(p = 0.01\); it becomes 9 for \(p = 1/3\); it rises to 16 for \(p = 1/2\); and it reaches 36 for \(p = 2/3\). The reason for this lies with the fact that the probability of punishment affects not only the fine, but also the gains offenders derive from the harmful act (see equation 1). As \(p\) increases, a greater fraction of gains are eventually disgorged, and this has a greater impact on the level of deterrence. Graphically, this greater impact
can be seen by observing that an increase in the probability of punishment increases the slope of the expected punishment curve and also shifts it up. Therefore, the new level of deterrence (point $A_1$ at which the expected punishment and the gains curves cross) increases in greater proportions than the increase in $p$ (see Figure 1).

Since the probability of punishment that is required to maintain the level of deterrence decreases as the maximal fine rises, that is, since $p_1 < p^*$, the deterrent value of enforcement expenditures, on this account, is decreased. Therefore, we have identified two opposing effects: On the one hand, the deterrent value of enforcement expenditures is increased because the optimal fine is higher (the fine effect); on the other hand, the deterrent value of enforcement expenditures tends to be lower because the probability of punishment which is necessary to achieve the pre-fine-increase optimal level of deterrence is lower (the probability-of-punishment effect). The total effect depends on the relative magnitudes of these opposing effects.

Proposition 2 implies that these opposing effects cancel out exactly if $p^* = 1/2$, while the fine effect dominates the probability-of-punishment effect if $p^* < 1/2$, and the reverse is true if $p^* > 1/2$.

To illustrate this numerically, suppose that the optimal fine is 400 and the optimal probability of punishment is 1/2. The optimal level of deterrence is therefore $400 \left(\frac{1/2}{1-1/2}\right)$, equation 5), and the deterrent value of a 1% increase in the probability of punishment is $16 \left(\frac{400}{(1-1/2)^2}\right)$ X1%, equation 11). Suppose now that the maximal and optimal fines increase from 400 to 500, that is, by 25%. In such a case, to achieve the same level of deterrence, the probability of punishment should be decreased to 0.44
It can be easily verified that the deterrent value of a 1% increase in the probability of punishment remains 16 \( \left( \frac{500}{(1-0.44)^2} \right) \times 1\% \). Therefore, the marginal benefits of increasing the probability of punishment and, consequently, the level of deterrence will not change. Suppose however that the optimal probability of punishment were only \( 1/3 < 1/2 \). The optimal level of deterrence therefore would be \( 200 \left( \frac{1/3}{1-1/3} \right) \) and the deterrent value of a 1% increase in the probability of punishment would be 9 \( \left( \frac{400}{(1-1/3)^2} \right) \times 1\% \). To maintain the same level of deterrence, after the optimal fine increases from 400 to 500, the probability of punishment should be reduced to 0.286 \( \left( \frac{0.286}{1-0.286} \right) \) and the deterrent value of a 1% increase in \( p \) would increase from 9 to 9.8 \( \left( \frac{500}{(1-0.286)^2} \right) \times 1\% \). Consequently, the marginal benefits of increasing the probability of punishment and the optimal level of deterrence would increase as well. Finally, suppose that the optimal probability of punishment were \( 2/3 > 1/2 \). The optimal level of deterrence and the deterrent value of enforcement expenditures would be \( 800 \left( \frac{2/3}{1-2/3} \right) 400 \) and 36 \( \left( \frac{400}{(1-2/3)^2} \right) \times 1\% \), respectively. An increase in the optimal fine from 400 to 500 accompanied by a decrease in the probability of punishment to 0.615 \( \left( \frac{0.615}{1-0.615} \right) 500 = 800 \) to maintain the same level of deterrence, would reduce the deterrent value of a 1% increase in the probability of punishment from
36 to 33.7 \left( \frac{500}{(1-0.615)^2} \right) \times 1\% \). In such a case, the marginal benefits of raising the probability of punishment and, as a result, the optimal level of deterrence would go down.

Proposition 2 states that the optimal level of deterrence will increase or decrease as the maximal fine rises, if the optimal probability of punishment is less than or greater than 1/2.\[^{14}\] As pointed out in Section 2, if the level of the maximal fine is greater than the harm, the optimal probability of punishment is definitely less than 1/2, implying that in those cases the optimal level of deterrence will increase with maximal fines, as is commonly expected. Therefore, a necessary but not sufficient condition for the optimal level of deterrence to decrease with maximal fines is that \( f_{\text{max}} < h \). This condition, which can be interpreted, for example, as a situation in which offenders' wealth is less than the harm, is not necessarily uncommon in reality. Indeed, in many situations, potential offenders cause great harm and possess in comparison very little wealth, so they are effectively judgment-proof. In standard law enforcement models which disregard the possibility of disgorging offenders' gains, it is observed that if offenders' wealth is substantially less than the harm, substantial under-deterrence will result, unless other forms of punishment such as imprisonment are utilized. Indeed, a severe problem of under-deterrence justifies the use of expensive forms of punishment such as imprisonment (see Shavell, 1985, Posner, 1985). However, if offenders' gains can be disgorged, a substantial level of under-deterrence does not necessarily result even if offenders' wealth is substantially less than the harm (as demonstrated in Section 2). Indeed, if offenders' wealth is substantially less than the harm, then in order to achieve a

\[^{14}\] If the costs of enforcement are convex with the probability of punishment, then there is an additional force to increase the optimal level of deterrence as maximal fines increase. The reason is that at the pre-fine-increase level of deterrence, the marginal costs of increasing the probability of punishment are lower.
significant level of deterrence, the probability of punishment should be relatively high. For example, if offenders' wealth is 400 and the harm is 800, then first-best deterrence requires that the probability of punishment will be \( \frac{2}{3} \) (\( \frac{2/3}{1-2/3} = \frac{400}{400} = \frac{800}{800} \)), while if offenders' wealth is 400 but the harm is 1600, the probability of punishment should be \( \frac{4}{5} \) (\( \frac{4/5}{1-4/5} = \frac{1600}{1600} \)).

As noted, \( f_{\text{max}} < h \) is a necessary but not a sufficient condition for the optimal level of deterrence to decrease as maximal fines rise. In addition, it should be the case that the optimal level of deterrence is rather high, which requires that the costs of enforcement are relatively low. Otherwise, if the costs of enforcement are relatively high, the optimal probability of punishment would be relatively low (Indeed it may even drop to zero).

### 3.2. Optimal Probability of Punishment

**Proposition 3:** As the maximal fine increases, the optimal probability of punishment increases (decreases) if \( g^* < \frac{h}{2} \) (\( g^* > \frac{h}{2} \)).

**Proof:** See Appendix C

Proposition 3 parallel Garoupa's (2001) result in a model without disgorgement of gains, and therefore can be viewed as a generalization of his result.\(^{15}\) The explanation is also similar. To understand Proposition 3, consider the impact of increasing the maximal

\(^{15}\) More precisely Proposition 3 parallels Garoupa's (2001) Proposition 2, assuming in Garoupa's (2001) that the benefits from the harmful acts are distributed uniformly in the range 0 to 1. More generally, Proposition 3 in the working-paper version of this paper parallels Garoupa's (2001) proposition precisely.
fine and, accordingly, the optimal fine to $f_i$ ($> f_{\text{max}}$) on the marginal costs and benefits from enforcement expenditures evaluated at $p^*$. More precisely, consider whether

$$\left(h - \frac{p^* f_i}{1 - p^*}\right) \frac{f_i}{(1 - p^*)^2} \geq c.$$

If the marginal benefits of increasing the probability of punishment (the LHS of 12) are now less than the marginal costs (the RHS of 12), then the optimal probability of punishment should be decreased. If, however, the reverse is true, then the optimal probability of punishment should be increased.

Observe first that the marginal costs are, of course, unaffected (the RHS of 12 is equal to the RHS of 4). Therefore, the critical question is whether the marginal benefits of enforcement expenditures increase or decrease, that is, whether

$$\left(h - \frac{p^* f_i}{1 - p^*}\right) \frac{f_i}{(1 - p^*)^2} \geq \left(h - \frac{p^* f_{\text{max}}}{1 - p^*}\right) \frac{f_{\text{max}}}{(1 - p^*)^2}.$$

As the optimal fine rises, the deterrent value of enforcement expenditures is increased, that is, $\frac{f_i}{(1 - p^*)^2} \geq \frac{f_{\text{max}}}{(1 - p^*)^2}$. This is the fine effect discussed above. On the other hand, the increase in the optimal fine increases the level of deterrence, $\frac{p^* f_i}{1 - p^*} > \frac{p^* f_{\text{max}}}{1 - p^*} = g^*$, which means that the gains from further deterrence are reduced,

$$\left(h - \frac{p^* f_i}{1 - p^*}\right) < \left(h - \frac{p^* f_{\text{max}}}{1 - p^*}\right).$$

The reason for this is that, as the level of deterrence increases, the marginal offenders impose less net harm, because the gains they derive from committing the harmful act are greater. To illustrate, if the gains threshold that determines the level of deterrence is 100 and the harm is 200, then the marginal offender imposes net harm of 100 $(200 - 100)$. As the level of deterrence is increased, say, to 150,
the marginal offender imposes net harm of only 50 (200 – 150). Therefore, whether the optimal probability of punishment should increase or decrease as the maximal fine rises depends on the relative magnitude of these two opposing effects. If the level of under-deterrence is low, then the reduction in the gains from more deterrence dominates the increased deterrent effect of enforcement expenditures, and calls for reducing the optimal probability of punishment. This possibility is most vivid if the level of under-deterrence is very low, so that an increase in the level of fines will lead to over-deterrence. In such a case, there will be actually losses from further deterrence, so the optimal probability of punishment should definitely decrease. On the other hand, if the level of under-deterrence is substantial, then the increase in the deterrent value of enforcement expenditures outweighs the reduction in the gains from more deterrence. Therefore, the social planner should achieve a higher level of deterrence not merely by increasing the optimal fines, but also by increasing the optimal probability of punishment.

Proposition 3 implies that the above opposing effects cancel out exactly if $g^* = \frac{h}{2}$, while the reduction in the level of deterrence dominates the increase in the deterrent value of enforcement expenditures if $g^* > \frac{h}{2}$, and the reverse is true if $g^* < \frac{h}{2}$.

To illustrate this numerically, suppose that the maximal fine is 400 and the harm is 200. If the probability of punishment is 0.2, the optimal level of deterrence will be 100 ($\frac{0.2}{1-0.2} \times 200$ equation 5), which is exactly half the level of harm. The deterrent value of enforcement expenditures is 6.25 ($\frac{400}{(1-0.2)^2}$ X1% equation 11), and the marginal benefits from greater deterrence are 100 (200 – 100). Suppose now that the maximal fine
increases from 400 to 420, that is, by 5%. The deterrent value of enforcement expenditures will rise from 6.25 to 6.56 that is, by 5%. The marginal benefits from greater deterrence will go down from 100 to 95 (200 – 105), that is, by 5% as well. Therefore, the marginal benefits of enforcement expenditures will not change and, consequently, the optimal probability of punishment will be unaffected. Suppose, however, that the probability of punishment were 0.25. Then, the optimal level of deterrence would be 133.33 \left( \frac{0.25}{1-0.25} \right) 400, which is more than half the level of harm (200). The deterrent value of enforcement expenditures would be 7.11 \left( \frac{400}{(1-0.25)^2} \right) X1\%, and the marginal benefits from deterrence would be only 66.67 (200 – 133.33). Suppose that maximal fines increased from 400 to 420, that is, by 5%. The deterrent value of enforcement expenditures would increase also by 5% (from 7.11 to 7.47). In contrast, the marginal benefits from greater deterrence would decrease from 66.7 to 60 (200 – 140), which is by 10%. Therefore, the marginal benefits of enforcement expenditures and, consequently, the optimal probability of punishment would go down. Suppose, finally, that the probability of punishment were 0.1, such that the optimal level of deterrence was 44.4 \left( \frac{0.1}{1-0.1} \right) 400, which is substantially less than half the level of harm (200). The deterrent value of enforcement expenditures would be 4.94 \left( \frac{400}{(1-0.1)^2} \right) X1\%, and the marginal benefits from deterrence 155.6 (200 – 44.4). If maximal fines increased from 400 to 420, that is, by 5%, the deterrent value of enforcement expenditures would rise by 5% as well (from 4.94 to 5.19). However, the marginal benefits from deterrence would be reduced from 155.6 to 153.33 (200 – 46.66), which is by less than 2%. In this case, the
marginal benefits of enforcement expenditures would rise, and, as a result, the optimal probability of punishment would increase as well.

It is interesting to note that the complementarity between maximal fines and the optimal probability of punishment is less likely to arise in the present model than in the standard model in which offenders' gains cannot be disgorged, even though the effect itself takes the same form in the two models. The reason for this, as the next section shows, is that the optimal level of deterrence is always higher with disgorgement than without it.

4. DISGORGEMENT VERSUS NO-DISGORGEMENT

The previous section analyzes how an increase in the maximal fine affects optimal law enforcement if offenders' gains from the harmful act can be disgorged. A natural, analogous question is how the possibility to disgorge offenders' gains affects optimal law enforcement. This question can be interpreted in two ways: one, as a question concerning the comparison between optimal law enforcement with and without disgorgement of offenders' gains, and second, as an inquiry of how increasing the fraction of gains that can be physically (or legally) disgorged affects optimal law enforcement.16

Bowles et al. (2000) provide an important analysis of the removal of illegal gains. However, as pointed out in the Introduction, they focus in their analysis on deriving the optimal fine and the optimal level of disgorgement of gains, assuming alternatively that disgorgement of gains is socially costless or socially costly. Bowles et al. (2000) also suggest that disgorgement of gains is socially desirable and that it allows for the

---

16 The latter view assumes that there is some physical or legal constraint on the fraction of illegal gains that can be disgorged and examines the consequences of relaxing such a constraint; for example, imagine that law enforcement gets better at recovering a larger fraction of offenders' gains. Nevertheless, this paper does not examine the costs of disgorging offenders' gains or possible measures of offenders to dispose of their wealth or of their gains.
achievement of greater deterrence, but they do not provide a formal proof or a satisfactory explanation with respect to the latter claim.

It should be clear that the possibility of disgorging offenders' gains increases social welfare, because this option allows the social planner to achieve any level of deterrence at lower enforcement costs (see also Bowles et al., 2000). It is also clear that offenders' gains should be disgorged to the maximum extent possible, assuming that disgorgement, like levying fines, is a socially costless transfer (see Bowles et al., 2000, and Proposition 1(1)). Again, the interesting questions which are not formally discussed by Bowles et al. (2000), concern how the possibility of disgorging offenders' gains or a larger fraction of these gains affects the optimal level of deterrence and the probability of punishment.

To facilitate the analysis, observe that if the fraction of offenders' gains that can be disgorged is constrained to $\eta_{\text{max}} < 1$, the optimal law enforcement scheme will be characterized by an analogous proposition to Proposition 1: The optimal punishment should be maximal, that is, $f^* = f_{\text{max}}$ and $\eta^* = \eta_{\text{max}}$; The optimal probability of punishment should satisfy $(h - g^*) \frac{f_{\text{max}}}{(1 - \eta^* p^*)^2} = c$, where $g^* = \frac{p^*}{1 - \eta^* p^*} f_{\text{max}}$. Some degree of under-deterrence prevails $g^* < h$, implying also that $p^* < \frac{h}{f_{\text{max}} + \eta^* h}$.

4.1 Optimal Level of Deterrence

**Proposition 4:** As the fraction of offenders' gains that can be disgorged increases, the optimal level of deterrence increases and gets closer to first-best deterrence. The optimal level of deterrence is therefore higher with disgorgement than without it.

**Proof:** See Appendix D.
At a first blush, Proposition 4 is not surprising at all. It seems quite natural that the possibility of disgorging offenders' gains or a larger fraction of such gains should increase the optimal level of deterrence. However, in light of proposition 2, this result deserves explanation.

When the fraction of offenders' gains that can be disgorged is increased, the pre-disgorgement-increase optimal level of deterrence can be achieved at lower enforcement costs by increasing the level of disgorgement and reducing the probability of punishment appropriately. At that point, the marginal costs of increasing the probability of punishment are unchanged, since enforcement costs are assumed to be proportional to the probability of punishment. In addition, by construction, the marginal benefits from increasing the level of deterrence, in terms of reduced social harm, are also unaffected. Again, the question of whether the optimal level of deterrence should increase or decrease boils down to the change in the deterrent value of enforcement expenditures after disgorgement is increased and the probability of punishment appropriately reduced.

When only a fraction of offenders' gains can be disgorged, the deterrent value of enforcement expenditures (for any given combination of $p$ and $f$) is given by:

\[
\frac{\partial g}{\partial p} = \frac{f}{(1-\eta p)}.
\]

As equation (14) reveals, the deterrent value of enforcement expenditures depends on the product of $p$ and $\eta$, that is, on $\eta p$. Therefore, an increase in $\eta$ "compensated" by a proportional decrease in $p$ will leave the product $\eta p$ unchanged, and consequently, will not affect the deterrent value of enforcement expenditures. For example, an increase of 25% in the fraction of gains that are disgorged, accompanied by a 20% decrease of the
probability of punishment, will keep the product $\eta p$ and therefore the deterrent value of enforcement expenditures unchanged.

However, as can be demonstrated, as $\eta$ increases, the reduction in the probability of punishment which is required to maintain the pre-disgorgement-increase optimal level of deterrence is less than proportional. The reason is that the probability of punishment affects not only the gains offenders derive from the harmful act, but also the fine imposed on them. To illustrate this numerically, suppose that $p^* = 0.2$, $\eta^* = 0.4$, and $f_{\text{max}} = 400$; then, the level of deterrence is given by approximately $87 \left( \frac{0.2 	imes 400}{1 - 0.2 	imes 0.4} \right)$. Now suppose that $\eta$ is increased by 25% from 0.4 to 0.5. To maintain the level of deterrence, the probability of punishment should be reduced from 0.2 to 0.196 ($\frac{0.196 	imes 400}{1 - 0.196 	imes 0.5} = 87$), that is by only 2%, which is clearly less than 20%. Therefore, as $\eta$ increases and $p$ is reduced to maintain the pre-disgorgement-increase optimal level of deterrence, the product $\eta p$ is actually increased, which means that the deterrent value of enforcement expenditures is unequivocally increased. Put differently, the increase in the deterrent value of enforcement expenditures as a result of an increase in the fraction of gains that are disgorged (the disgorgement effect) always dominates the reduction in the deterrent value of enforcement expenditures on account of the lower probability of punishment (the probability of punishment effect) which is necessary to maintain the

---

17 To see this, consider an increase of $100r\%$ in the fraction of the disgorgement of gains. A proportional reduction in the probability of punishment means that $p$ is reduced by $\frac{100r}{1 + r}\%$. However, the necessary reduction in the probability of punishment that will keep the level of deterrence unchanged is $\frac{100p \eta r}{1 + p \eta r}\%$, which is clearly less than proportional, since $p \eta < 1$. 

28
deterrence level. Since the deterrent value of enforcement expenditures is higher at the pre-disgorgement-increase optimal level of deterrence, the marginal benefits of increasing the probability of punishment are higher and the optimal level of deterrence should definitely increase.\textsuperscript{18}

4.2. Optimal Probability of Punishment

**Proposition 5:** As the fraction of offenders' gains that can be disgorged increases, the optimal probability of punishment increases (decreases) if \( g^* < \frac{2h_3}{h} \) (\( g^* > \frac{2h_3}{h} \)).

*Proof:* See Appendix E.

The explanation of Proposition 5 is completely analogous to that of Proposition 3, and therefore, is omitted. It should be noted that the complementarity between the disgorgement of offenders' gains and the optimal probability of punishment may seem more or less likely than the complementarity between maximal fines and the optimal probability of punishment, because it requires that \( g^* < \frac{2h_3}{h} \) rather than \( g^* < \frac{h}{2} \). However, it should be remembered that the level of deterrence is generally higher as the fraction of the gains from the harmful acts which is disgorged is higher (Proposition 4).

4. CONCLUSION AND SUMMARY

This paper analyzes the optimal law enforcement scheme if the gains from harmful acts are monetary or monetary-like in nature and therefore can be subject to disgorgement. It

\textsuperscript{18} In fact, there is another reason why the optimal level of deterrence would increase. If enforcement costs are convex with the probability of punishment, then the costs of enforcement will be lower at the pre-disgorgement-increase optimal level of deterrent, which would also justify attaining a higher optimal level of deterrence.
also compares this optimal law enforcement scheme to the optimal law enforcement scheme that prevails if offenders' gains cannot be disgorged (for example, because the gains are non-monetary in nature). An important issue which is analyzed concerns the effects of increasing the maximal fine. This issue is significant because it can describe situations in which legal constraints to the maximum level of fines are relaxed, or, alternatively, situations in which offenders' resources are increased. The main results of the analysis are summarized in Tables 1 and Table 2.

**Table 1: The Characteristics and Comparison of Optimal Law Enforcement with and without Disgorgement of Illegal Gains**

<table>
<thead>
<tr>
<th></th>
<th>Fines</th>
<th>Enforcement</th>
<th>Deterrence</th>
<th>Social Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Disgorgement</td>
<td>$f_{\text{max}}$</td>
<td>$p^* &lt; h/ f_{\text{max}}$,</td>
<td>Under-deterrence</td>
<td>Lower</td>
</tr>
<tr>
<td>Disgorgement</td>
<td>$f_{\text{max}}$</td>
<td>$p^* &lt; h(f_{\text{max}} + h)$,</td>
<td>Under-deterrence</td>
<td>Higher</td>
</tr>
<tr>
<td></td>
<td>same</td>
<td>higher/lower</td>
<td>Lower</td>
<td></td>
</tr>
<tr>
<td></td>
<td>same</td>
<td>lower/higher</td>
<td>Higher</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2: The Effects on Optimal Law Enforcement of Increasing the Maximal Fine**

<table>
<thead>
<tr>
<th></th>
<th>Fines</th>
<th>Enforcement</th>
<th>Deterrence</th>
<th>Social Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Disgorgement</td>
<td>Higher</td>
<td>Higher/Lower</td>
<td>Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>Disgorgement</td>
<td>Higher</td>
<td>Higher/lower</td>
<td>Higher/Lower</td>
<td>Higher</td>
</tr>
</tbody>
</table>

As is evident in Table 1, the possibility of disgorging offenders' gains increases social welfare and the optimal level of deterrence. In addition, it does not affect the main qualitative results associated with the standard model in which disgorgement of gains is not an option. However, as evident from Table 2, an increase in the maximal fine may
paradoxically *decrease* the optimal level of deterrence and consequently *increase* the optimal level of crime if offenders' gains can be disgorged.

**Appendix A - Proof of Proposition 1.**

Define the Lagrangian function as $L = SW + \lambda(f_{\text{max}} - f) + \mu(1 - \eta)$, where $\lambda$ and $\mu$ are the Lagrangian multipliers. The optimal $f^*$, $\eta^*$, and $p^*$ should satisfy the *Kuhn-Tucker* first-order-conditions:

(A1) \[ L_p = (h - g^*) - \frac{f^*}{(1 - \eta^* p^*)^2} - c = 0 \]

(A2) \[ L_f = (h - g^*) - \frac{p^*}{1 - \eta^* p^*} - \lambda = 0 \]

(A3) \[ L_\eta = (h - g^*) - \frac{p^*^2 f^*}{(1 - \eta^* p^*)^2} - \mu = 0 \]

(A4) \[ L_\lambda = f_{\text{max}} - f \geq 0 \quad \lambda \geq 0 \quad \text{and} \quad \lambda(f_{\text{max}} - f) = 0 \]

(A5) \[ L_\mu = 1 - \eta \geq 1 \quad \mu \geq 0 \quad \text{and} \quad \lambda(1 - \eta) = 0 \]

Where \( g^* = \frac{p^* f^*}{1 - \eta^* p^*} \)

Suppose that $\lambda = 0$. From (A3) it follows that $h = g^*$. However, this implies that $L_p < 0$, which contradicts (A1). Therefore, the optimal solution must be $f^* = f_{\text{max}}$ and $\lambda^* > 0$. Suppose that $\delta = 0$. From (A3) it follows that $h = g^*$. However, this implies again that $L_p < 0$, which contradicts (A1). Therefore, the optimal solution must be $\eta^* = 1$ and $\mu > 0$. Assuming an interior solution, it follows from (A3) that $p^*$ should satisfy:
(A6) \((h - g^*) \frac{f_{\text{max}}}{(1 - p^*)^2} = c\),

which implies that:

(A7) \(g^* = \frac{p^*}{1 - p^*} f_{\text{max}} < h\) and \(p^* < \frac{h}{f_{\text{max}} + h}\).

The second-order condition requires that:

(A9) \(L_{pp} = SW_{pp} = \frac{f_{\text{max}}}{(1 - p^*)^2}[2h - f_{\text{max}} - 2p^*(h + f_{\text{max}})] < 0\), or

(A9) \(p^* < \frac{2h - f_{\text{max}}}{2(h + f_{\text{max}})}\).

Observe that the second-order condition is satisfied for any \(p^*\) if \(f_{\text{max}} \geq 2h\). Otherwise, the second-order condition restricts the possible \(p^*\). For example, if \(f_{\text{max}} = h\), then \(p^*\) must be greater than 1/4. If \(f_{\text{max}} = 2h/3\), then \(p^*\) must be greater than 2/5. And if \(f_{\text{max}} = h/2\), then \(p^*\) should be greater than 1/2.

Appendix B - Proof of Proposition 2:

To examine how the level of deterrence changes with the maximal fine, we inquire:

(B1) \(\frac{dg^*}{df_{\text{max}}} = \frac{\partial g^*}{\partial f_{\text{max}}} + \frac{\partial g^*}{\partial p^*} \frac{dp^*}{df_{\text{max}}}\)

Substituting \(\frac{\partial g^*}{\partial f_{\text{max}}} = \frac{p^*}{1 - p^*} \frac{\partial g^*}{\partial p^*} = \frac{f_{\text{max}}}{(1 - p^*)^2}\), and \(\frac{dp^*}{df_{\text{max}}} = \frac{SW_{pp}}{SW_{pp_{\text{max}}}}\), and rearranging, we have that:

(B2) \(\text{sign}\left[\frac{dg^*}{df_{\text{max}}}\right] = \text{sign}[f_{\text{max}} SW_{pp_{\text{max}}} - p^*(1 - p^*)SW_{pp}]\)

Substituting
(A9) $SW_{pp} = \frac{f_{\max}}{(1 - p^*)^2} [2h - f_{\max} - 2p^*(h + f_{\max})]$ and

(B3) $SW_{p^*} = \frac{1}{(1 - p^*)^2} [h - 2g^*],$

and rearranging, we obtain that:

(B4) $\text{sign}[\frac{dg^*}{df_{\max}}] = \text{sign}[(h - g^*)(1 - 2p^*)].$

Proposition 2 follows immediately.

**Appendix C - Proof of Proposition 3:**

To examine how the optimal probability of punishment $p^*$ changes with the maximal fine, we inquire the sign of $\frac{dp^*}{df_{\max}}$. By the Implicit Function Theorem:

(C1) $\frac{dp^*}{df_{\max}} = -\frac{SW_{p^*}}{SW_{pp}}.$

Noting that, by the second-order condition (A9), $SW_{pp} > 0$, and substituting (B3), it follows that:

(C2) $\text{sign}[\frac{dp^*}{df_{\max}}] = \text{sign}[h - 2g^*].$

Proposition 3 immediately follows.

**Appendix D - Proof of Proposition 4:**

To examine how the optimal level of deterrence changes with changes in the fraction of the gains that can be disgorged, let us assume that $f^* = f_{\max}$, $\eta^* = \eta$, and $p^*$ satisfies the first order condition:
(D1) \[ SW_p = (h - g^*) \frac{f_{\max}}{(1 - \eta^* p^*)^2} - c = 0, \]

Where \[ g^* = \frac{p^* f_{\max}}{1 - \eta^* p^*}. \]

The effect is then given by:

(D2) \[ \frac{dg^*}{d\eta^*} = \frac{\partial g^*}{\partial \eta^*} + \frac{\partial g^*}{\partial p^*} \frac{dp^*}{d\eta^*} \]

Substituting \[ \frac{\partial g^*}{\partial \eta^*} = \frac{p^* f_{\max}}{(1 - \eta^* p^*)^2}, \quad \frac{\partial g^*}{\partial p^*} = \frac{f_{\max}}{(1 - \eta^* p^*)^2}, \quad \text{and} \quad \frac{dp^*}{d\eta^*} = \frac{SW_{pq}}{SW_{pp}} \]

rearranging, we have that:

(D3) \[ \text{sign} \left[ \frac{dg^*}{d\eta^*} \right] = \text{sign}[SW_{pq} - p^* SW_{pp}] \]

Substituting

(D4) \[ SW_{pq} = \frac{g^*}{(1 - \eta^* p^*)^2} [2h - 3g^*] \]

and

(D5) \[ SW_{pp} = \frac{g^*}{p^*} \frac{1}{(1 - \eta^* p^*)^2} [2h \eta^* - f - 3g^* \eta] \]

into (D3) and rearranging, we have that:

(D6) \[ \text{sign} \left[ \frac{dg^*}{d\eta^*} \right] = \text{sign} \left[ \frac{g^* (h - g^*)}{(1 - \eta^* p^*)} \right] \]

Since \( \eta^* p^* < 1 \), Proposition 4 immediately follows.

Appendix E – Proof of Proposition 5:

By the Implicit Function Theorem:

(E1) \[ \frac{dp^*}{d\eta^*} = -\frac{SW_{pq}}{SW_{pp}}. \]
Noting that $-SW_{pp} > 0$ (SOCs) and substituting (D4), we have that:

\[(E2) \quad \text{sign}\left[\frac{dp^*}{d\eta^*}\right] = \text{sign}[2\eta - 3g^*].\]

Proposition 5 follows.

REFERENCES


